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Volatility under Bounded Rationality

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The ARCH model shares with the related literature on risk and return one common thing: *rational- expectation paradigm*. In particular, market prices should reflect investors' rational forecasts based on the best available information. When new information arrives, the market's expectations change. Therefore, prices fluctuate. Thus, price volatility is due to information arrivals and hence, volatility can be forecast, based on the up-to-date information. However, when the available information is too complex, rational expectation may no longer hold. *Bounded rationality* should be added into our frame work to study risk and return, so that, we can gain a better understanding of market volatility.

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1. Introduction

Market volatility is a time-varying phenomenon. And the challenge for econometricians is how to forecast the mean and variance of asset returns in this context. The ARCH model was introduced by Engle (1982) to meet such a challenge.

The key feature of the ARCH model is the *conditional variance*, which depends on past forecast errors. This concept implies a strong relationship between *volatility and return* - an idea that in fact, is not new. It can be dated back to the works by Markowitz (1952) and Tobin (1958), who associated risk with the variance in the value of a portfolio and the implications by Sharpe (1964) for the case, where investors are concerned with the risk associated with their portfolio as a whole. This theory is called Capital Asset Pricing Model (CAPM) that views the excess return of an asset as a kind of compensation for amplified variance of the returns to the portfolio, when an individual purchases this asset. Later, Black and Scholes (1972) and Merton (1973) developed a model to evaluate the pricing of options. This model is also consistent with CAPM. In particular, options can be viewed as insurance, whose price depends upon the risks, being measured by the variance of asset returns (Engle, 2004).

Thus, the conditional variance equation of the ARCH model has a very strong base, theoretically. Empirically however, there exist several findings that are inconsistent with some important implications drawn from the model.

First, the key feature of the ARCH model is conditional variance such that, high volatility should correspond to high expected returns. If such a relation were linear, *the* *mean of the return equation would depend upon the past squared returns*, exactly in the same way that the conditional variance depends on past squared returns (Engle, 2004)[2].

This conjecture has been undergone serious tests and empirical evidence on this measurement has been mixed. For instance, Engle (2004) wrote: "While Engle *et al.* (1987) find a positive and significant effect, Chou, Engle and Kane (1992), and Glosten, Jagannathan and Runkle (1993), find a relationship that varies over time and may be negative because of omitted variables". As Engle himself said, such an inconsistency is a challenge to better modeling of the risk return trade-off.

Second, another important implication of the ARCH model is found in determining the value at risk, often abbreviated as VaR. The ARCH/GARCH model can estimate one-day 99% VaR by using the 99% *asymptotic confidence interval* for the one-step-ahead forecast errors. Obviously, the accurate estimate of standard deviation for the following day is crucial. And this is precisely what is claimed as one of the advantages of the ARCH model over the simple least squares model.

An empirical test for this claim can be found in Bollerslev (1986), although he studied inflation rather than VaR. In the paper, plots of the actual inflation rate and asymptotic confidence intervals for the one-step-ahead forecast errors are given for the predictions of the model estimated by the least squares and the GARCH (1,1) model.

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[2] The GARCH conditional variance is: $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1} + ... + \alpha_q \varepsilon_{t-q} + \beta_1 \sigma_{t-1}^2 + ... + \beta_q \sigma_{t-q}^2$. The linear relation means: $\sigma_t^2 = \varepsilon_t^2 + u_t, \forall t,$ where $u_t \sim N(0, \sigma_u^2)$. Insert the latter into the former, take the expectation, and collect terms, we have: $E(\varepsilon_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2$. Here, $\varepsilon_t = p_t - p_{t-1}$, is forecast error and p_t is asset price. For the return $r_t = \ln(p_t / p_{t-1})$, we just replace ε_t and p_t by r_t and $\ln(p_t)$, respectively.

According to Greene (1993, Ch. 19), “In periods of very volatile inflation (the late 1940s and early to mid-1950s), the simple least squares regression is a visibly better predictor. The effect is reversed in the more stable period of the late 1950s to early 1970s.” Greene’s comment raises the question whether the least squares could outperform the ARCH model, at least in highly volatile periods.

Third, the ARCH model might over-estimate the unconditional variance of returns which could lead to poor volatility forecast. How such a possibility could happen?

The ARCH model shares with the CAPM and all related literature reviewed above one common thing: *rational-expectation paradigm*. Given the relevant information is fragmentally distributed among investors; the distribution of forecast errors should have fat tails in periods of price discovery. However, these periods should be time limited, *if* investors are rational, so that the forecast is certain to eventually revert to less extreme volatilities (Engle (2004). This is precisely where the volatility clustering phenomenon has found its base in rational-expectation paradigm, so the ARCH model goes.

More specifically, provided that most investors are sophisticated, i.e. they are able to learn and process the relevant information and act optimally upon it, forecast errors can never be *too far* out of line for a *too long* period. In other words, forecast errors should be clustered in time. Furthermore, although future returns are extremely unpredictable, investors' expectations are assumed not to be *systematically biased* and they use all relevant information in forming their forecasts. Therefore, forecast errors should be symmetrically distributed around the mean with fat tails. These features of forecast errors - volatility clustering and fat tails - are precisely the characteristics for which an ARCH model is designed (Engle, 2004). Thus, the ARCH conditional variance equation is consistent with rational expectation. So long as rational expectation assumption holds, the ARCH model will give an accurate measurement of volatility.

Suppose, however, the information set is too complex, beyond the limits of rational investors in formulating and solving problems and in processing information. Under such a *bounded rationality*, investors may employ the use of heuristics to make decisions rather than rational forecasts (Simon, 1957, Williamson, 1981). They do this because of the complexity of the situation, and their inability to process and compute the expected return of every alternative option (Simon, 1991).

Put differently, the more complex the situation, the smaller the fraction of sophisticated, well-informed investors, who are able to form rational forecasts; and the greater the fraction of uninformed investors, who tend to act irrationally. Therefore, the market's expectations, which are a kind of weighted average of the expectations of all investors, should be inclined toward heuristic beliefs of uninformed ones. Thus, the weighted average may not reflect the best available information and more volatile than what is justified by changes in fundamentals (Milgrom and Robert, 1992, p. 469).

More importantly, volatility now can *asymmetrically* respond to past forecast errors. Large forecast errors seem to feed market emotions, so the forecast errors that move in *the same* direction tend to appear at a much higher frequency than the ones that move in the reverse direction (see Figure 1).

Volatility asymmetry phenomenon may be the essential that causes the ARCH model to overestimate volatility in periods of economic turmoil. If the problem of volatility asymmetry can last, the distribution of forecast errors will have too fat tails, while forecast errors tend to fall in only *one side* of the confidence interval at a high frequency. By using the same chain of reasoning, we can explain why the simple least squares model can outperform the ARCH model in highly volatile periods. The bottom line is that, in these periods, rational expectation hypothesis no longer hold.

The above suggests that when adding the bounded rationality hypothesis into our framework to study risk and return, we can gain a better understanding about market volatility[3]. This is the paper's main purpose.

In what remains, the paper is organized as follows: Sections 2 introduces our approach to understand volatility asymmetry under bounded rationality. Section 3 presents the forces under which, volatility asymmetry is reduced, and the market's expectations are adjusted to reflect available information, if time is left to unfold. Sections 4 and 5 present a price adjustment model. Section 6 concludes the paper.

2. Volatility Asymmetry

As already said, in highly volatile periods, the least squares can outperform the ARCH model, according to Greene. This could only be plausible if rational expectation no longer holds. Let us now investigate this matter in detail.

To begin with, let us rethink the conditional variance and its implications for the distribution of forecast errors.

Without loss, let us consider the simplest form of ARCH (1):

$$y_t = \varepsilon_t, \;\; \varepsilon_t \mid \varepsilon_{t-1} \sim N(0, \sigma_t^2) \qquad (1)$$

[3] The ARCH family has become so large. For example, Engle and Rangel (2008) proposed the Spline-GARCH model to investigate macroeconomic causes of low frequency volatility; Chou (2005) developed the CARR (p, q) model to study the dynamic dependencies in time series of high-low asset prices. Nevertheless, the ARCH family's members still share the same underlying feature: rational expectation paradigm.

where, $y_t = p_t - p_{t-1}$, is the change in security price p_t; time series y_t is assumed to be stationary; furthermore, the conditional variance has the following specification:

$$\sigma_t^2 = Var(\varepsilon_t \mid \varepsilon_{t-1}) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \tag{2}$$

Therefore, the unconditional variance is:

$$Var(\varepsilon_t) = E(Var(\varepsilon_t \mid \varepsilon_{t-1})) = \alpha_0 + \alpha_1 Var(\varepsilon_{t-1})$$

Bollerslev presented the conditions needed to ensure the stability of higher moments of the normal distribution. Thus, the maximum-likelihood estimate of the conditional variance asymptotically converges to the unconditional one:

$$\hat{\sigma}_t^2 \overset{a}{\approx} Var(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1} = const \tag{3}$$

where, $\hat{\sigma}_t^2$ is the estimate of the conditional variance in the model (1) - (2).

The correspondence of an ARCH (1) model in the least-squares is an AR (1), such that, the error term is defined as follows: $\varepsilon_t = \rho\varepsilon_{t-1} + .u_t$, where $u_t \overset{iid}{\sim} N(0, \sigma_u^2)$. Thus, the unconditional variance is:

$$Var(\varepsilon_t) = \frac{\sigma_u^2}{1-\rho^2} = const \tag{3'}$$

Comparing the two unconditional variances in (3) and (3'), we can see a similarity in the structure, but there are some differences in estimate of parameters, which reveal some insight about how an ARCH (1) differs from an AR (1) in estimate of standard deviation.

For an ARCH (1), in the short run, the conditional variance equation is estimated, using the up-to-date information. In the long run, the estimate of conditional variance asymptotically converges to the unconditional one, and hence, it reflects more accurately changes in fundamentals, when temporal shocks taper off. Thus, an ARCH can provide an accurate forecast of standard deviation for everyday or every period, despite time-varying variance of returns. For least squares, the problem is that volatility is estimated by the sample standard deviation of forecast errors. So the trouble is what is the right period to use? If it is too long, then it will not be so relevant for today and if it is too short, it will be very noisy (Engle, 2004).

Now, let us turn to the case when the information set is too complex so that, most investors face their limits in processing information. In this context, price discovery period may be too long and too volatile.

More importantly, under bounded rationality, false beliefs can feed market emotions in a self-fulfilling manner, so volatility now can asymmetrically respond to past forecast errors, as already mentioned. Therefore, there exist too many successive moves in *the same direction,* observed by Cootner (1964) and Lo and MacKinlay (1999)[4].

Remind that, the random error ε_t is normally distributed. Many successive moves of forecast errors in the same direction can be possible, only if the self-fulfilling

[4] It is clear that our concept of asymmetrically volatility response to past forecast errors differs from the one implied by the Exponential GARCH or EGARCH model of Dan Nelson.

prophecy has caused successive shifts in securities' value far and further from their intrinsic worth.

The above paragraph suggests some idea about how to measure volatility under bounded rationality. To pave a way toward this end, let us compare again the asymptotic confidence intervals estimated by the least squares and the ARCH model in the context of volatility asymmetry.

By construction, the ARCH model (1)-(2) should provide the distribution of forecast errors, which is concentrated at zero with fat tails. This implies the asymptotic confidence interval can be very large, should the period under study be very volatile. Furthermore, suppose the economy experiences volatility asymmetry problem, as in a bubble period. Such a large confidence interval can even hardly capture tomorrow's forecast error ε_{t+1}, since it tends to move far and further to reach a right extreme value.

Instead, for the least squares, the best predictor for tomorrow price $\hat{p}_{t+1}$ is today price p_t, such that: $E(\hat{p}_{t+1} \mid p_t) = p_t$. Thus, the least squares forecast can overcome the volatility asymmetry problem by shifting the mean $E(\hat{p}_{t+1} \mid p_t)$ accordingly. Further, the estimate of the standard deviation is unbiased and consistent. Therefore, once shifting the mean in according to the market's expectations, the least squares' asymptotic confidence interval can still capture tomorrow's forecast error, especially when the direction of price moves is not reversed. Such a conjecture is consistent with Greene's comment. That is, the least squares can outperform the ARCH/GARCH model in highly volatile periods.

But does that make the least squares more useful in financial practice? Obviously, the answer is no! The model itself cannot predict a price reversal. Subsequently, the model can cause too many forecast errors, which are unacceptable in financial practice. Thus, a natural extension of the least squares model is to upgrade its capacity to predict price/trend reversal. To develop such a framework, we must first investigate how asset prices fluctuate under bounded rationality.

3. Market Adjustments

Investors often purchase or sell stocks because they constantly calculate the trade-off between risk and return from holding those assets. For a single stock, if they expect a rise in the rate of return higher than enough for compensating the risk, they tend to buy more shares of that stock. At new higher prices, the expected return is lower so it is consistent with the risk. In contrast, a rise in risk should lead owners to sell some shares. At new lower prices, the expected return is driven up to match with the risk increased (Merton, 1980, and Engle, Lilien and Robins, 1987).

As already mentioned, if most investors are sophisticated, they can form rational expectations, based on the best available information. Thus, prices can never be too far out of line for too long, so volatility is time varying, but can be forecast, using up-to-date information.

However, not all investors are sophisticated. If the information is too complex, the fraction of investors acting emotionally tends to grow in proportion with the magnitude of informational complexity. Under such bounded rationality, the relation between risk and expected return can no longer be stable.

To be specific, suppose an investor sees others repeatedly buy shares of the stock at successively higher prices than his estimate of the value. He may suspect that these people have better information or forecasts and consequently raise his valuation. But his action is noticed by others, which will further reinforce the belief that the stock may have a higher value. Thus, the perception spreads by contagion (Young, 1998). The point is that, if this perception is not based on rational forecasts, but on employing heuristics, it can be false. Even so, it still causes itself to become true, due to positive feedback between belief and behavior (Robert K. Merton, 1968). This tendency is often called the herd among uninformed investors.

Fortunately, not all investors are uninformed or unsophisticated to keep following the herd of others. At successively higher stock prices, informed investors can predict that, the expected rate of return has become too low, or the risk of holding shares of this stock has become too high. Their rational forecasts should lead them to sell some shares. Subsequently, the course now can be reversed, whereby people tend to lower their valuation. As before, this process spreads by contagion. Thus, prices should go down and approach the security' true worth. It follows that the risk is reduced and the expected return is driven up, just enough to re-establish the correspondence between risk and return.

However, under bounded rationality, this possibility may have only a slim chance. Given the information used for making decision is too complex, there may have too few knowledgeable, sophisticated investors. More likely, many people tend to act on their beliefs rather than on rational forecasts which they are unable to form or understand. Thus, it is harder for the course to be reversed toward rational forecasts.

Even when some very influential, well-informed investors sell off risky assets, the others would follow suit and revise their valuation downward. However, because this majority of investors are still facing bounded rationality, their downward valuation tends to turn into overreactions, causing prices to drop, more than what is justified by rational forecasts. At these so low prices, the expected return is driven up, more than enough to compensate for the risk. The uninformed investors then will view this as evidence confirming their self- fulfilling beliefs. The weighted average of expectations of all investors is hence quickly tipped back to the self-fulfilling beliefs by contagion or by the herd (David Scharfstein and Jeremy Stein, 1990). Security prices go up high again.

Thus, rises in prices now appear at a higher frequency than falls in prices. This is the volatility asymmetry we have mentioned, but now it is applied for hyperemotional periods. Bubbles and sudden crashes could be the result.

Fortunately, there are some adjustment mechanisms embodied in the market, which can often work out to correct false beliefs and cool down the situation.

In the context we are discussing, volatility asymmetry causes the overall level of risk in the stock market to keep rising. In response, interest rates, gold prices go up sharply, or hard currencies are appreciated. Such market responses can work as a filter that gradually wipes out wrong beliefs, reducing volatility. How does such a mechanism work?

It is clear that self-fulfilling beliefs tend to widen the gap between securities prices and their intrinsic value. The larger the gap, the riskier it is to hold these assets. Again sophisticated investors will be the first group, who sell off some of those securities and place their wealth in a safe haven. As gold prices increase more frequently, others tend to sell off their stocks too, as they fear for risk increased. This tendency causes overall securities prices decreased or bubbles burst. To survive, firms with weak fundamentals have to rely more on debt than on equity. These firms then suffer from volatile earning and large interest expenses. When such news reaches the public, beliefs of investors now are inconsistent with facts *if* their beliefs are wrong. Therefore, false beliefs are wiped out one by one, which reinforces the process of learning whereby uninformed investors become more informed. Subsequently, market overreactions become less frequent or volatilities are reduced to less extreme.

Thus, volatility is just a mirror of the complexity of the learning process, whereby volatility reduces when the number of informed investors increases. In other words, volatility clustering is simply cluster of the market' expectations, which remain a weighted average of the expectations of all investors, weighing the expectations of informed investors in proportion to the magnitude of their transactions.

In the short run, such a magnitude is insignificant. Securities prices are very volatile because self-fulfilling beliefs feeding market emotions and because of market overreactions. In the long run, when this magnitude becomes significant, securities prices approach their true worth and therefore, volatilities are reduced.

4. The Model

We have shown that, in the short run, there still have some forces that correct false perceptions, and in the long run, these forces will become strong enough to wipe them out. Thus, the weighted average will eventually converge to the state that reflects better rational expectations (that Graham Benjamin (1965) called "weighing mechanism").

This rule forms our concept of two-step forecast: First, we build a model to predict how the weighted average may evolve in a few steps ahead. Second, based on this short-term trend forecast, one can go back in time to infer how the mean of returns may shift accordingly in the next period. This two-step forecast can provide an accurate estimate of the asymptotic confidence interval, despite volatility clustering and volatility asymmetry.

To formalize this dynamic process, let us first remind that market price at time *t*, p_t follows a random walk process, say:

$$p_t = p_{t-1} + \varepsilon_t, \quad \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2) \tag{6}$$

where, ε_t is random shock at time *t*.

Furthermore, the first order difference series $y_t = p_t - p_{-1}$ is a stationary process:

$$y_t = \varepsilon_t; \quad \varepsilon_t \overset{iid}{\sim} N(0,\sigma^2) \tag{6'}$$

Hence, the best predictor for the today price is yesterday price: $E(\hat{p}_t \mid p_{t-1}) = p_{t-1}$.

We then immediately have the following result:

$$E(\hat{p}_t) = E(E(\hat{p}_t \mid p_{t-1})) = E(p_{t-1}) = \pi \tag{7}$$

where π is the company's intrinsic worth that no one knows exactly!

To understand how the weighted average may evolve in the short term, we need to add one more equation.

Let us denote $m_t = \frac{1}{m}\sum_{\tau=t-m+1}^{t} p_\tau$. The first order difference series $z_t = m_t - m_{t-1}$ is assumed to be stationary and it follows an *ARMA (p, q)* process[5]:

$$z_t = \beta' x_t + \rho_1 z_{t-1} + \rho_2 z_{t-2} + ... + \rho_p z_{t-p} + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + ... + \theta_q u_{t-q} ,$$

$$u_t \overset{iid}{\sim} N(0, \sigma_u^2) \tag{8}$$

Here, vector x_t represents some macroeconomic trends, such as technical progress or inflation, which are *publicly known* at time *t.* The parameters associated with the first *p* lagged terms of dependent variable reflect how slowly or quickly the market processes available information. Likewise, the parameters associated with the last *q* terms reflect the noise of new information arrived from the *q* most recent moments. Thus, equation (8) is a natural way to express the weighted average of expectations of all investors.

Without loss, let us consider an $AR(1)$ process: $z_t = \beta' x_t + \rho z_{t-1} + u_t$. Given that, the least squares predictor $\hat{m}_t$ satisfies the following condition:

$$E(\hat{m}_t \mid m_{t-1}, x_t) = m_{t-1} + \beta' x_t /(1-\rho) \tag{9}$$

We shall have: $E(\hat{m}_t) = E(E(\hat{m}_t \mid m_{t-1}, x_t)) = \pi + \beta' x_t /(1-\rho)$. Hence, one can denote:

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[5] Take a notice that, $z_t = \frac{1}{m}\sum_{\tau=t-m+1}^{t} y_\tau = \frac{1}{m}\sum_{\tau=t-m+1}^{t} \varepsilon_\tau \equiv \nu_t$. Therefore, $\mathrm{cov}(\nu_t, \nu_s) \neq 0, \forall t, s \ni: |t-s| < m$. That is another way to say, $z_t \sim ARMA(p, q)$.

$$\pi_t \equiv \pi + \beta' x_t \ /(1-\rho) \tag{10}$$

where π_t is the company's intrinsic worth calculated at time *t*.

The additional term $\beta' x_t$ reflects how macroeconomic trends may work in favor of the firm. However, most economists believe that in the long run, no firm can outperform its competitors, when all temporal shocks taper off. Accepting that, we assume the following asymptotic condition:

$$\beta' x_t \to 0 \Rightarrow \pi_t \to \pi, when \ \ t \to \infty \tag{11}$$

Equations (8) to (10) allow us to predict how the weighted average may evolve in the short term. Such a trend prediction should help to infer how price p_t will move in the next few days. Condition (11), on the other hand, means that in the long run, the dynamic path of the weighted average can help to predict which stock prices are underpriced or overpriced. These are our next subjects of study.

5. Adjustment Process

It is useful to distinguish between two types of price adjustments in the short term: *Price reversal* is the case, where price series p_t returns to and remains near mean m_t for some time *t;* and *trend reversal* is when trend m_t reverses to the opposite direction.

Now, let us state these concepts formally.

Definition 1: we shall say that the market has been locked into the stage of *price reversal*, if time series y_t successively returns to a confidence interval, for some *t*.

Recall that the time series p_t is a random walk process. Thus, time series of price change $y_t = p_t - p_{t-1}$ could reach up to five-sigma deviation in some circumstances. However, because the series y_t is stationary, it tends to return to a confidence interval, when temporal shocks taper off.

For definiteness, in what follows, we will consider one-sigma confidence interval estimated by regression (6').

Lemma 1: *When a market has been locked into the stage of price reversal, prices* p_t *tend to return to and remain near the mean* m_t.

Proof: When the market has been locked into the stage of price reversal, by definition, at least for some *t*, the variable y_t must actually stay inside one-sigma confidence interval. In other words, the absolute value of price change, $|y_t| = |p_t - p_{t-1}| < \sqrt{\hat{\sigma}^2_{LS}}$, where $\sqrt{\hat{\sigma}^2_{LS}}$ is the standard deviation estimated by (6').

Provided that the size of the moving average m_t (already denoted by m) is small enough, we shall have: $|m_t - p_t| < \sqrt{\hat{\sigma}^2_{LS}}$, at least for some time *t,* until a new shock arrives and tips p_t out of the one-sigma-deviation from m_t □

One can see that, *Lemma 1* provides nothing new from a practical point of view (see Figure 1). But it suggests some techniques to predict price reversal and trend reversal. In our framework, these techniques are similar. Thus, we focus only on the technique used to predict trend reversal. This trend forecast is important to estimate volatility under volatility asymmetry.

Definition 2: We shall say *trend-turning point* is the moment that thereafter, price series p_t reverses its tendency, such that, the short-term trend m_t also reverses its direction.

We now propose our main theme:

Proposition 1: *In a very general condition, a trend-turning point can accurately be forecast in a few steps ahead.*

Proof: Without loss, assume prices have been dropping for a while, (although price reversals may occur from time to time). As already noticed, after a trend-turning point has been reached, price must keep rising for some steps. That is the series y_t has to jump from left extreme values to cross the mean, and then it reaches the right side of the confidence interval (see Figure 1).

According to Lemma 1, the moments that condition, $|y_t| = |p_t - p_{t-1}| < \sqrt{\hat{\sigma}_{LS}^2}$, must be very brief, even *just one*. Otherwise, we should be locked in the state of price reversal. In other words, right after the trend-turning point, we shall have, $y_t = p_t - p_{t-1} > \sqrt{\hat{\sigma}_{LS}^2}$, at a high frequency. Remind that, before the turning point, we should also have, $y_t = p_t - p_{t-1} < -\sqrt{\hat{\sigma}_{LS}^2}$, at a high frequency (due to volatility asymmetry).

Thus, exactly at the turning point, there is a sudden change in the tendency of prices p_t , that *marks* on the evolving of trend series m_t, such that, the latter will also exhibit *trend reversal* , even with some lagged moments in time. It is clear that, the more volatile the period under study, the greater the magnitude of reversal and therefore, the sharper the mark.

To re-emphasize, the change in price tendency must be durable enough. Otherwise, we shall return to the state of price reversal, as already suggested.

The important message here is that, if we can predict a trend reversal of series m_t a few steps ahead, we can go back in time and infer that the turning point of price tendency is sure to eventually happen.

The next question is how can we accurately predict *trend reversal* of time series m_t? This question leads us to investigate the least squares estimate of asymptotic standard deviation once more.

From the regression (6'), we have: $Var(y_t) = \sigma^2$. We still assume that series (8) follows AR (1) process. Therefore, we have: $Var(z_t) = \sigma^2 / [m(1-\rho^2)]$.

Obviously, the smaller the term *m*, the easier it is to detect trend reversal of time series m_t. On the other hand, the greater the term *m* and the smaller the term ρ, the more accurate the prediction of the trend reversal, since the standard error of the regression (8) is smaller[6] . (See Figure 2, for illustration).

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[6] Forecast errors of regression (8) is: $e_{z.t} = \hat{z}_t - z_t$ and recall, $s_z^2 \approx \frac{1}{T}\sum_t e_{z,t}^2 \to \sigma^2 / [(1-\rho^2)m]$. The greater the term *m* and the smaller the term ρ is, the smaller the standard errors of regression (8), and hence, the more accurate it is the forecast of the trend -reversing point. The latter is due to the Law of Large Number, applied for the sample mean of $e_{z,t}^2 = |\hat{m}_t - m_t|^2$.

Now, suppose the *trend-reversal* has been predicted. By going back in time, one can infer that the *turning point* of price series p_t is certain to happen in the next few days □

This trend forecast allows to improve the accuracy of estimate of volatility, since we should know the direction to which the mean of returns will be shifted.

Next, recall that the smaller the term ρ, the faster the market's capability to process information. This implies that a majority of investors have become more informed through leaning. This process is reinforced by market filter which has wiped out false beliefs, pushing securities prices closer to their true worth.

Thus, the smaller is the term ρ, the more likely the series y_t will remain within a confidence interval for some time. We then return to the state of Efficient Market Hypothesis (EMH), such that price p_t changes very little and runs along a short-term trend m_t that may be up or down, depending on the market's expectations.

Proposition 2: *When false beliefs are wiped out one by one, market gradually converges to the state of EMH, where prices p_t slightly fluctuate around its intrinsic value π_t, which is approximately equal to m_t.*

In the state of EMH, by definition, time series price p_t tends to return and stays near mean m_t and hence, price reversals should appear at a high frequency. That is, the following condition tends to hold: $| p_t - m_t | < 2\sqrt{\hat{\sigma}^2_{LS}}$. Furthermore, due to the unbiased estimate of the least squares, we shall have $m_t = E(\hat{m}_t) = \pi_t$, according to (10). Thus, under the state of EMH, we have: $| p_t - \pi_t | < 2\sqrt{\hat{\sigma}^2_{LS}}$ □

The EMH state can last as long as no new shocks arrive to shake up the market. In other words, if time is left to unfold, securities prices should never be too far out of their intrinsic worth. We then state that:

Proposition 3: *In the long run, when all uncertainties resolve, securities prices tend to converge to companies' intrinsic value*

Proof: In the short run, price $p_t \to m_t$, whenever the weighted average is tipped toward rational expectations. In the long run, when all temporal shocks taper off, the weighted average: $m_t = E(\hat{m}_t) = \pi_t \to \pi$. Combining the two, we have: $p_t \to \pi$ □

Conclusions

The approach presented here is simple, yet powerful enough to predict price/trend reversal in the short term. Therefore, the model can provide an accurate estimate of volatility of asset returns under volatility clustering and volatility asymmetry. The model can also estimate which securities are underpriced or overpriced, given the life time of companies under consideration is long enough. Through experiments, one can form the rule to increase the accuracy of the estimate process. Thus, a venture is opened for an experimental study on volatility. It is simple, yet efficient in financial practice.

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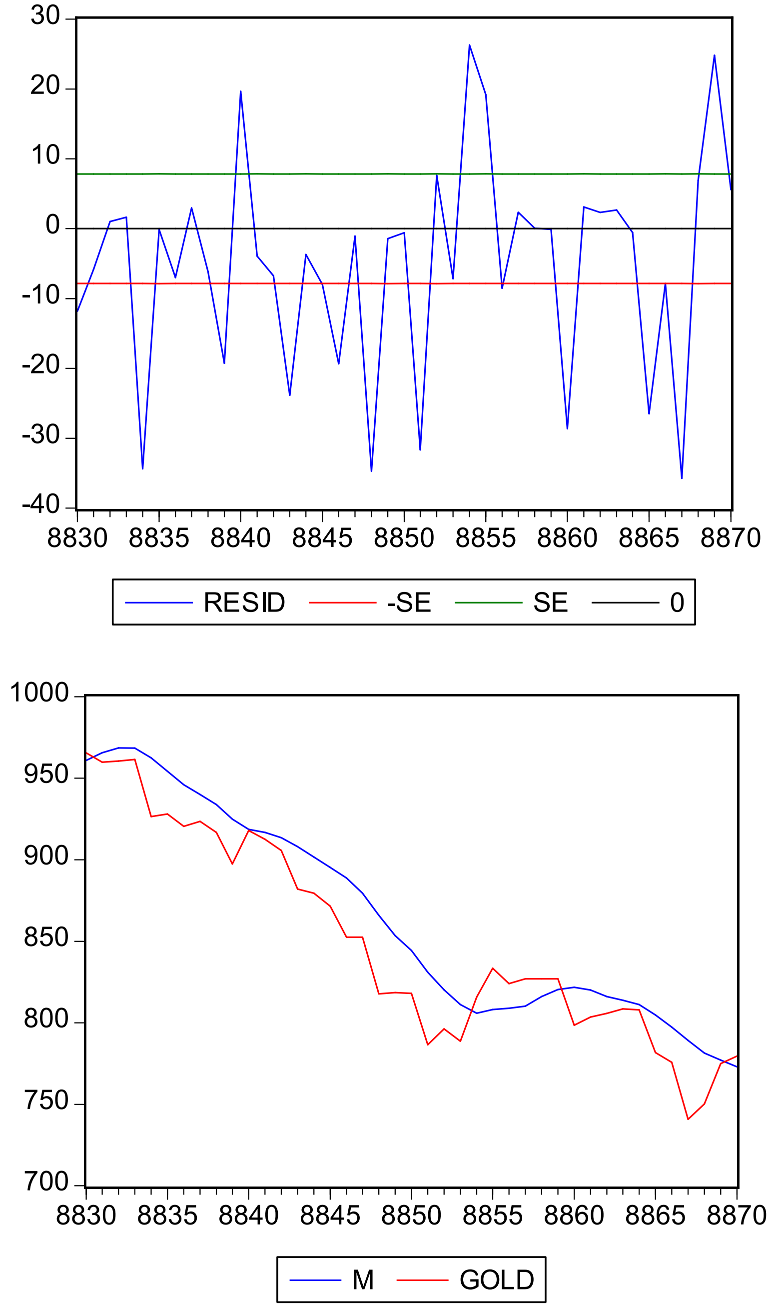
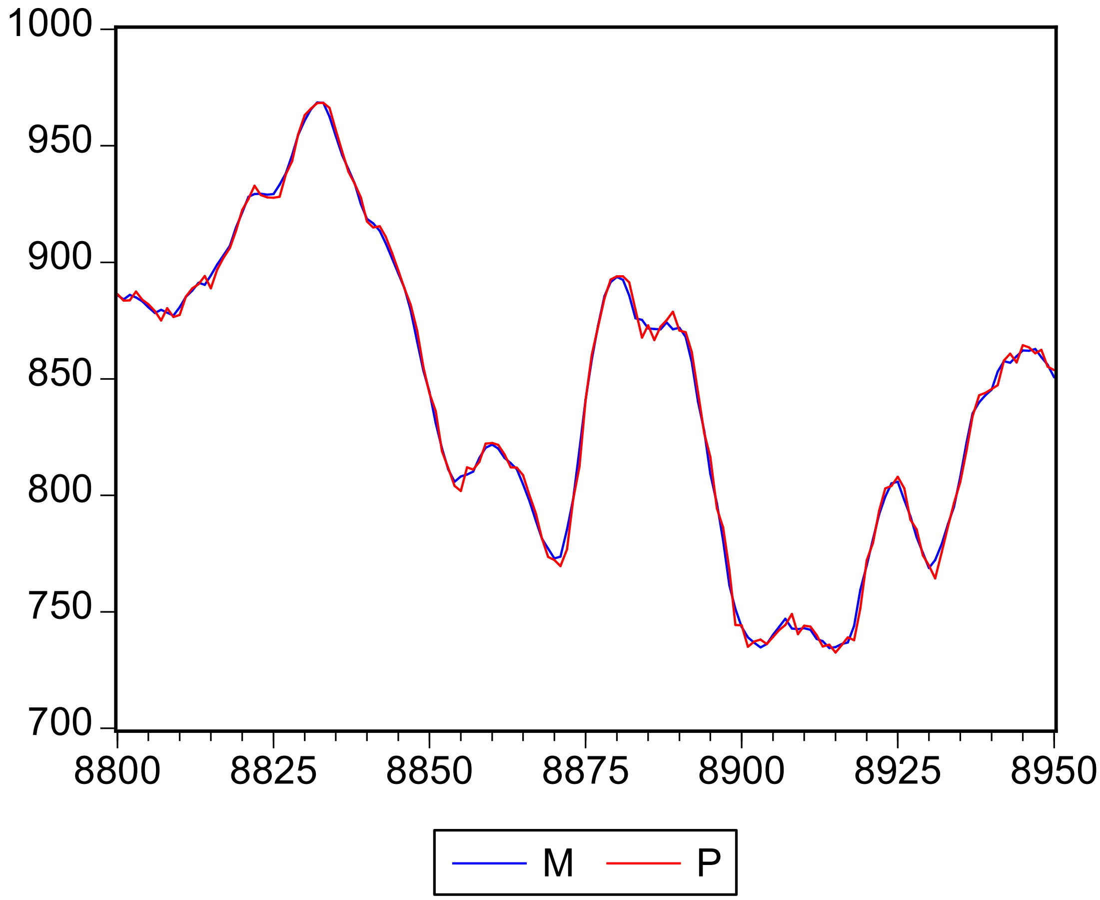
Figure 1: Volatility Asymmetry



Forecast errors tend to move in the same direction when price (GOLD) tumbles. In *price reversal* state, GOLD remains close to trend M (8856-64). During a short period of *trend reversal* (8851-55), forecast errors jump from one side to the opposite side of the asymptotic confidence interval. (The sample size N= 2500 obs.)

Figure 2: In-sample-trend forecast P for trend M of price series at London Gold Fix:



The five-step-ahead forecast captures the trend reversal that occurs at the point 8949.