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Abstract

We develop a model of optimal pattern of economic development that is first rooted in physical capital accumulation and then in technical progress. We study an economy where capital accumulation and innovative activity take place within a two sector model. The first sector produces a consumption good using physical capital and non skilled labor. Technological progress in the consumption sector is driven by the research activity that takes place in the second sector. Research activity which produces new technologies requires technological capital and skilled labor. New technologies induce an endogenous increase of the Total Factor Productivity of the consumption sector. Physical and technological capital are not substitutable while skilled and non skilled labor may be substitutable.

We show that under conditions of the adoption process of new technologies, the optimal strategy for a developing country consists in accumulating physical capital first; postponing the importation of technological capital to the second stage of development. This result is due to a threshold effect from which new technologies begin to have an impact on the productivity of the consumption sector. However, we show that once a certain level of wealth is reached, it becomes optimal for the economy to import technological capital to produce new technologies.

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1 Introduction

The growth performance of the East Asian newly industrialized economies (NIEs) gave rise to a broad and diversified literature aiming at explaining the reasons for such a long lasting period of expansion. On one hand, the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. According to these authors, Asian countries have adopted technologies previously developed by more advanced economies (assimilation view) and "the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy" (Pack [1992]). Implicitly, they admit that the TFP is one of the main factors of growth in accordance with the thesis developed by Solow [1957]. [1] On the other hand, supporters of the accumulation view stress the importance of physical and human capital accumulation in the Asian growth process. According to this standard growth view, poorer countries should grow faster than wealthier ones during their first stage of development. This result is rooted in the assumption of decreasing returns to scale on capital accumulation that induce a catching-up process compatible with conditional convergence (Cass [1965]). As pointed out by Krugman [1994] "the newly industrializing countries of the Pacific Rim have received a reward for their extraordinary mobilization of resources that is no more than what the most boringly conventional economic theory would lead us to expect. If there is a secret to Asian growth, it is simply deferred gratification, the willingness to sacrifice current satisfaction for future gain".

Besides this theoretical debate, on empirical grounds the continuous development of growth accounting analysis gives us an insight into the respective role of assimilation and accumulation on Asian growth process. In a first wave of empirical studies, Young [1994, 1995], Kim and Lau [1994, 1996] found that the postwar economic growth of the NIEs was mostly due to growth in input factors (physical capital and labor) with no increase in the total factor productivity. [2] Moreover, the hypothesis of no technical progress cannot be rejected for the East Asian NIEs (Kim and Lau [1994]). Consequently, accumulation of physical and human capital seems to explain the

[1] Solow, in this paper, used US data from 1909 to 1949 and showed that the capital intensity contributed for one eight to the US economic growth. The remainder was due to increased productivity

[2] Krugman [1997] wrote that Larry Lau and Alwyn Young works suggested that Asian growth could mostly be explained by high saving rates, good education and the movement of underemployed peasants into the modern sector.

major part of the NIEs' growth process. Krugman's [1994] interpretation of these results is very pessimistic since, according to him, the lack of technical progress will inevitably bound the growth engine of East Asian NIEs as a result of the diminishing returns affecting capital accumulation.

However, this pessimistic view is challenged by a second series of works (Collins and Bosworth [1996] or Lau and Park [2003]) that show Total Factor Productivity (TFP) gains actually matter in Asian NIEs growth and that future growth can be sustained. For these authors "it is possible that the potential to adopt knowledge and technology from abroad depends on a country's stage of development. Growth in the early stages may be primarily associated with physical and human capital accumulation, and significant potential for growth through catchup may only emerge once a country has crossed some development threshold" (Collins and Bosworth [1996]). These findings concerning the East Asian economies in the post-war period are also valid for developed economies in the early stages of their development (Lau and Park [2003]). They suggest that in these stages, economic growth is generally based on physical accumulation rather than technological progress. Greater gains in TFP are possible only during the second stage of development.[3]

The predominance of capital accumulation in a country's first stage of development is also compatible with cross-country empirical studies showing that development patterns differ considerably between countries in the long run (Barro&Sala-i-Martin [1995], Barro [1997]). These differences can be explained within a model of capital accumulation with convex – concave technology. In such a framework, Dechert and Nishimura [1983] prove the existence of threshold effect with poverty traps explaining alternatively "growth collapses" or taking-off. [4] However, these results are challenged by King and Rebelo [1993] who run simulations with neo-classical growth models and conclude that the transitional dynamics can only play a minor role in explaining observed growth rates. Dollar [1993] points out that divergence between countries is also due to differences in TFP. Why is technology im-

[3] More precisely, Lau and Park show there was no technical progress for Hong Kong, Korea, Singapore, Taiwan, Indonesia, Malaysia, Thailand until 1985. But suddenly, it appeared in these countries between 1986 and 1995. For Western Germany, United Kingdom, France, and Japan, technical progress always existed.

[4] For Parente and Prescott [1993], the popular thesis that countries which start below a minimum level of output will fail to grow seems not supported by the facts. Azariadis and Drazen [1990] propose an elaboration of the Diamond model that may have multiple stable steady states because the training technology has many thresholds. They provide an explanation to the existence of convergence clubs in Barro&Sala-i-Martin [1995], Barro [1997]

portant? Because it can be simultaneously employed in different uses (public good as well as productive good). Dollar [1993] wrote "there are a number of pieces of evidence indicating that succesful developing countries have borrowed technology from the more advanced economies". We share the view of Dollar and we think the so-called Solow-Krugman controversy is unfounded. Krugman's view is correct in the short and medium terms. But in the long term, TFP is the main factor of growth. In this sense, Solow is right and his 1956 model is basically a long term growth model. Even if these results seem widespread in the empirical literature on growth accounting, there is no theoretical model explaining the optimal shift of a country from the first stage (accumulation) to the second stage (assimilation) of development. The aim of our paper is to establish the formal conditions under which a country may realise (or not) this shift. More precisely, we define an endogenous threshold of development from which a country is encouraged to adopt new technologies and builds a part of its growth process on technological advances. Before reaching this threshold, the country must root its growth process into capital accumulation.

Our model is based on the existence of complementarities in the use of new technologies as it is necessary to have a minimum amount of adoption of new technologies in order for them to be efficient. This assumption may be justified by institutional structure (Atawell [1992], Castro et al. [2006]), by start-up cost effect (Ciccone and Matsuyama [1999]), set-up costs (Azariadis and Drazen [1990]) or by several kinds of technical barriers relating to the diffusion of innovation (Fichman [1992]). In order to encompass these different aspects we assume the existence of a threshold effect from which new technologies begin to have an impact on Total Factors Productivity. [5]

Capital accumulation and innovative activity take place within a two sector growth model. The first sector produces a consumption good using physical capital and non skilled labor according to a Cobb-Douglas production function. Technological progress in the consumption sector is driven by the research activity that takes place in the second sector. Research activity which produces new technologies requires technological capital and skilled labor along the line of a Cobb-Douglas production function. When new technologies produced by the research activity are used in the consumption sector they induce an endogenous increase of the Total Factor Productivity. The two kinds of capital are not substitutable while skilled and non skilled

[5] Note that threshold effect is also used by Le Van and Saglam [2004] who show that a developing country can restrain to invest in technology if the initial knowledge amount of the country and the quality of knowledge technology are low or if the level of fixed costs of the production technology is high.

labors may be substitutable.
We suppose that technological capital, used by the research activity, is not produced within the economy. The domestic economy must purchase it in the international market at a given price. Consequently, the consumption good can be consumed, invested as physical capital or exported against technological capital. The price of the consumption good is given by the international market and is used as numeraire in our economy. Finally, we assume that physical capital is less costly than technological capital.
We show that under our conditions on the adoption process of new technologies, the optimal strategy for a developing country consists in accumulating physical capital first; thus postponing the importation of technological capital to the second stage of development. All resources of the economy are devoted to consumption or investment in the physical capital sector and there is no research activity. Later, the technological progress may be generated when the country has reached a certain level of development. [6] This threshold in the level of development is endogenously determined in the model and is related to three factors: the amount of available human capital, the relative price of technological capital and the initial income of the economy. [7] For given values of these factors, we show that there is a time period after which it is optimal for the economy to import technological capital in order to produce new technologies. From that date, research activity generates an endogenous technical change and the economy follows an optimal endogenous growth path with increasing returns to scale technology. Our model exhibits an optimal pattern of economic development that is first rooted in physical capital accumulation and then in technological progress. Thus, in contrast to Dechert and Nishimura's [1983] seminal model that exhibits a convex-concave technology, our model displays first decreasing returns and then increasing returns.
Nevertheless, as a consequence of the existence of the threshold, a country may never be incited to adopt new technologies and may converge towards a traditional steady state. This explains that international convergence or divergence in income levels depends strictly on the value of the threshold.

[6] "As economic development proceeds, and per capita real output and tangible capital stock begin to rise, investment in intangible capital becomes gradually more profitable because of the complementarity between tangible and intangible capital and hence more important as a potential source of economic growth" Lau and Park [2003]

[7] Khan and Ravikumar [2002] also characterized a threshold effect in the adoption of new technologies in a model with fixed cost of adoption and linear technology. However, in their model, this threshold only depends on the value of this fixed cost and on the difference in productivity of the two technologies

This result tempers Krugman's pessimistic view on the future of East Asian NIEs and explains the various schemes of development observed among developing countries.

The initial value of human capital plays an essential role in the process we have just described. The higher this value, the sooner research activity and endogenous growth will take place. This result is in accordance with the empirical study of Benhabib and Spiegel [1994] showing that growth is related to the initial level of human capital and not to the accumulation of human capital.

In the last part of the model we relax the assumption of non substitutability between the two kinds of labor. We allow high-skilled workers to work in the production sector. We show that the optimal endogenous growth path may be compatible with an underutilization of high-skilled labor in the research activity. However, if the number of high-skilled workers is low relatively to the low-skilled workers then, after a certain time, the technological sector will fully employ high-skilled workers.

Our paper is organized as follows. We present the structure of the economy in Section 2. Section 3.1 deals with the infinitely lived optimal growth model with non substitutable labor force. In Section 3.2, we allow a shift of high skilled workers towards consumption sector but low skilled workers cannot join the innovative sector. Section 4 gathers the main results of our paper and points out weaknesses of our model. Sections 5 and 6 contain the proofs of the results stated in Section 3.1 and Section 3.2.

2 The Structure of the Economy

We consider a developing country which produces a consumption good Y_d with physical capital K_d and low-skilled labor L_d. The consumption sector may use a quantity of new technologies Y_e to increase its Total Factor Productivity. We have:

$$Y_d = \phi\left(Y_e\right) K_d^{\alpha_d} L_d^{1-\alpha_d}$$

where $\alpha_d \in (0,1)$ and ϕ is a non decreasing function which verifies $\phi\left(0\right) = x_0 > 0$.

The amount of new technologies Y_e may be produced through a Cobb-Douglas function using technological capital K_e and high-skilled labor L_e. We have:

$$Y_e = A_e K_e^{\alpha_e} L_e^{1-\alpha_e}.$$

where $\alpha_e \in (0,1)$ and A_e is the total productivity.

The economy cannot produce technological capital whereas physical capital and consumption good are homogenous. It exports consumption good in order to import technological capital. We use domestic consumption good as numeraire. Prices are respectively $\lambda > 1$ for technological capital and 1 for consumption good and physical capital.

Let c be the total consumption, u_1, u_2 be the utility functions of low-skilled and high skilled workers. Then $u(c) = \max\{u_1(c_1 L_d^*) + u_2(c_2 L_e^*) : c_1 L_d^* + c_2 L_e^* \leq c\}$.

We assume:

(H_1) The function u is strictly concave, strictly increasing, continuously differentiable, and satisfies $u(0) = 0$, and the Inada Condition $u'(0) = +\infty$.

and

(H_2) The function ϕ has the following form

$$\phi(x) = \left\{ \begin{array}{c} x_0, \forall x \leq X \\ x_0 + a(x - X), \forall x \geq X \text{ with } \ a > 0. \end{array} \right\}$$

The threshold in function ϕ may be interpreted either as a setup cost as in Azariadis and Drazen [1990], or a minimum level of adoption of new technologies which is necessary in order for them to impact the economy. In this case, the economy of the developing country must be sufficiently rich in resources or in human capital in order to take off by buying technological capital.

Figure 1 sketches ϕ

3 The Dynamic Model

We consider now an economy with one infinitely lived representative agent who has an intertemporal utility function. She has the possibilty to consume or to save at each period t. Savings are directly used to buy an equivalent amount of capital. This capital as before can be of two kinds, technological or physical capital. As before, we suppose that the technological capital costs more than the physical capital. There is no change in the production functions of the consumption goods and of the new technology.

We distinguish two cases.

Case 1: No Mobility of Labour

$$\forall t, L_{d,t} \leq L_d^*, \ L_{e,t} \leq h L_e^*$$

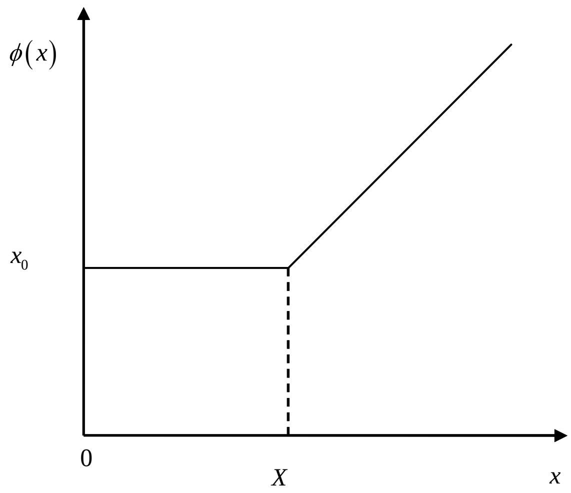

Figure 1:

These inequalities state there is no possible transfer between high-skilled and low-skilled workers. We suppose the human capital for high-skilled workers is measured by the number $h \geq 1$.

Case 2: Mobility of Labour We now assume that high-skilled people can work in the sector of consumption good if the demand for high-skilled labor is not sufficient in the research sector. But the reverse is not possible, i.e. low-skilled people cannot move in the new technology sector. We therefore replace the constraints labor demands by : for every t,

$$L_{d,t} \leq L_e^* + L_d^*$$

and

$$L_{e,t} \leq hL_e^*$$

3.1 No Mobility of Labor

The social planner will solve the following program.

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t) \text{ with } 0 < \beta < 1,$$

under the constraints: for every date t,[8]

$$c_t + S_{t+1} \leq \phi(Y_{e,t}) K_{d,t}^{\alpha_d} L_{d,t}^{(1-\alpha_d)}, \tag{1}$$

$$Y_{e,t} = A_e K_{e,t}^{\alpha_e} L_{e,t}^{1-\alpha_e}, \tag{2}$$

$$L_{d,t} \leq L_d^*, L_{e,t} \leq h L_e^*, \tag{3}$$

$$K_{d,t} + \lambda K_{e,t} = S_t. \tag{4}$$

The initial resource $S_0 > 0$ is given.

In (3), L_d^* and L_e^* are exogenous supplies of low-skilled and high-skilled workers. As we said before, there is no possible transfer between high-skilled and low-skilled workers.

Let $\theta = \frac{\lambda K_{e,t}}{S_t}$. Then (4) can be re-written

$$K_{d,t} = (1-\theta) S_t, \lambda K_{e,t} = \theta S_t \tag{5}$$

This problem is equivalent to:

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

under the constraints: for every date t,

$$c_t + S_{t+1} \leq H(r_e, S_t) \text{ with}$$

$$H(r_e, S_t) = \max_{\theta} \phi\left(r_e \theta^{\alpha_e} S_t^{\alpha_e}\right)(1-\theta)^{\alpha_d} {L_d^*}^{1-\alpha_d} S_t^{\alpha_d},$$

and $S_0 > 0$ is given, where $r_e = \frac{A_e h^{1-\alpha_e} L_e^{1-\alpha_e}}{\lambda^{\alpha_e}}$.

>From the Maximum Theorem, H is continuous. It is obviously strictly increasing with respect to S and $H(r_e, 0) = 0$.

Since the utility function is strictly increasing, at the optimum the constraints will be binding.

$$c_t = H(r_e, S_t) - S_{t+1}$$

[8] We assume the discount factor is the same for each category of workers.

A sequence $(S_t)_{t=0\ldots\infty}$ is called feasible from $S_0 \geq 0$ if we have $\forall t$, $0 \leq S_{t+1} \leq H(r_e, S_t)$. Thus the initial program is equivalent to the following problem

$$\max \sum_{t=0}^{\infty} \beta^t u\left(H\left(r_e, S_t\right) - S_{t+1}\right)$$

under the constraints,

$$0 \leq S_{t+1} \leq H\left(r_e, S_t\right), \text{ for all } t \geq 0,$$

with $S_0 > 0$ given.[9].
We denote by $K^*_{e,t}, K^*_{d,t}$ the corresponding optimal capital stocks and by θ^*_t the optimal share for technology capital.

3.1.1 Properties of the optimal path

In this subsection, we will present the main results of the model when labour is not mobile. In particular, we will show that any optimal path from $S_0 > 0$ is monotonic and does not converge to 0. Under some stronger conditions, we will show that any optimal path will grow without bound. Along this growth path, after a date T, the economy will use new technology to produce consumption goods.

Proposition 1 *(i) The function $H(r_e, S)$ is increasing in S. Hence, any optimal path from S_0 is monotonic.*
(ii) No optimal path from $S_0 > 0$ converges to 0.
(iii) There exists $S^c > 0$ such that if $S_t \leq S^c$, then $H(r_e, S_t) = x_0 L_d^{*(1-\alpha_d)} S_t^{\alpha_d}$. S^c is a decreasing function of r_e.

Proof. See Appendix 1. ■

Proposition 2 *Let S^s satisfy $x_0 L^*(1-\alpha_d)\alpha_d (S^s)^{\alpha_d - 1} = \frac{1}{\beta}$. Consider S^c in Proposition 1.*
(i) There exists $\bar{r}_e$ which depends on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L^*_d)$ such that,if $r_e > \bar{r}_e$, then any optimal path from $S_0 > 0$ will be increasing. There exists T, such that for any $t \geq T$, $K^*_{e,t} > 0$. The condition $r_e > \bar{r}_e$ is equivalent to $S^s > S^c$.
(ii) There exists $\widetilde{r}_e$ which depends only on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L^*_d)$ such that if $r_e < \widetilde{r}_e$ then for any $0 < S_0 < S^c$, there exists a unique optimal path which converges to S^s and $K^*_{e,t} = 0$ for every t.

[9] We assume that the utility function u is such that the sum $\sum_{t=0}^{\infty} \beta^t u\left(H\left(r_e, S_t\right) - S_{t+1}\right)$ is real-valued for any feasible sequence $\{S_t\}$

Proof. See Appendix 1. ■

Proposition 3 *Assume furthermore*
(H_3) $\alpha_e + \alpha_d \geq 1$.
There exists $\bar{A}_e > 0$ *(or* $\bar{h} > 0$*) such that if* $A_e > \bar{A}_e$ *(or* $h > \bar{h}$*) then any optimal path* (S_t^*) *from* $S_0 > 0$*, will converge to* $+\infty$*. Moreover,* $K_{e,t}^* \to +\infty$ *and* $\theta_t^* \to \theta^\infty = \frac{\alpha_e}{\alpha_e+\alpha_d}$*.*

Proof. See Appendix 1. ■
Comments
1. S^s is the steady-state of our economy in the case of concave technology. If the critical value from which the economy becomes to import technological capital and to produce new technologies (S^c) is higher than the steady-state value, the economy will never take off. In fact it will converge to its steady-state with a constant value of income per capita. On the contrary, if the steady-state value is higher than the critical wealth from which the economy produces new technologies, it will follow an endogenous growth path with a constant increase in income per capita. More precisely, if for any period t, the country does not invest in technology, then its economy converges to S^s. But when A_e is very large, by investing in new technology from some date T, the economy will grow without bound. Figure 2 gives a graphical interpretation of proposition 3.
2. When r_e is small, we have a poverty trap: if S_0 is smaller than S^c then any optimal path starting with a positive capital stock will converge to S^s.

3.2 Mobility of labour

We now assume that high-skilled people can work in the sector of consumption good if the demand for high-skilled labor is not sufficient in the research sector. But the reverse is not possible, i.e. low-skilled people cannot move into the new technology sector. We therefore replace the constraints labor demands (3) by

$$L_d \leq L_e^* + L_d^* \tag{6}$$

and

$$L_e \leq hL_e^* \tag{7}$$

We can write $L_e = \mu h L_e^*, L_d = L_d^* + (1-\mu)L_e^*$ with $\mu \in [0,1]$. We assume that when the high-skilled workers are in the consumption sector their human capital equals the one of this sector.

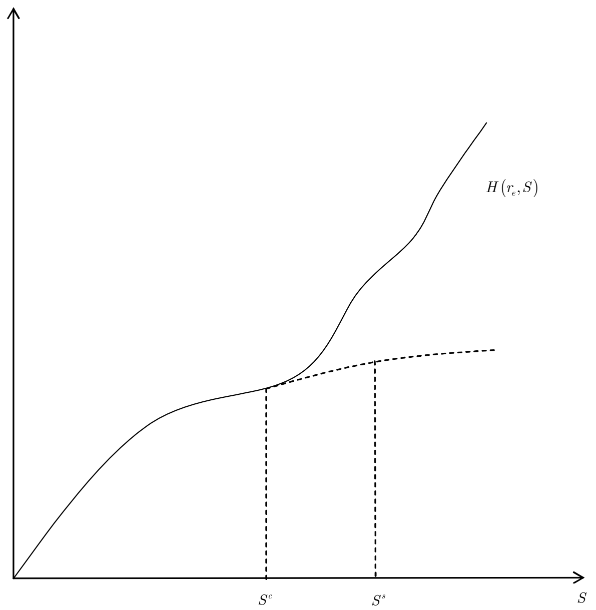

Figure 2:

The production function in the new technology sector will be:

$$Y_e = \frac{A_e}{\lambda^{\alpha_e}} \theta^{\alpha_e} S^{\alpha_e} \mu^{1-\alpha_e} h^{1-\alpha_e} {L_e^*}^{1-\alpha_e}$$

where μ represents the part of high skilled labor used in this sector. The production function in the consumption good sector now is:

$$Y_d = \phi(Y_e)(1-\theta)^{\alpha_d} S^{\alpha_d} \left(L_d^* + (1-\mu) L_e^*\right)^{1-\alpha_d}$$

The social planner will solve the following program.

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t) \text{ with } 0 < \beta < 1,$$

under the constraints: for every date t,[10]

$$c_t + S_{t+1} \leq \phi(Y_{e,t}) K_{d,t}^{\alpha_d} L_{d,t}^{(1-\alpha_d)}, \tag{8}$$

$$Y_{e,t} = A_e K_{e,t}^{\alpha_e} L_{e,t}^{1-\alpha_e}, \tag{9}$$

[10] We assume the discount factor is the same for each category of workers.

$$L_{d,t} \leq L_e^* + L_d^*, \;\; L_{e,t} \leq hL_e^*, \tag{10}$$

$$K_{d,t} + \lambda K_{e,t} = S_t. \tag{11}$$

The initial resource $S_0 > 0$ is given.

In (10), L_d^* and L_e^* are exogenous supplies of low-skilled and high-skilled workers.

Again, let $\theta = \frac{\lambda K_{e,t}}{S_t}$. Let $\mu = \frac{L_{e,t}}{hL_e^*}$. Then

$$K_{d,t} = (1-\theta)\, S_t, \lambda K_{e,t} = \theta S_t, \tag{12}$$

$$L_{e,t} = \mu h L_e^*, L_{d,t} = L_d^* + (1-\mu) L_e^*, \tag{13}$$

with $\mu \in [0,1]$.

The problem becomes:

$$\max \sum_{t=0}^{+\infty} \beta^t u\,(c_t)$$

under the constraints: for every date t,

$$c_t + S_{t+1} \leq L\,(r_e, S_t) \;\text{ with}$$

$$L\,(r_e, S_t) = \max_{\theta\in[0,1],\mu\in[0,1]} \phi \left(r_e \theta^{\alpha_e} {S_t}^{\alpha_e} \mu^{1-\alpha_e} \right) (1-\theta)^{\alpha_d} \left(L_d^* + (1-\mu)\, L_e^* \right)^{1-\alpha_d} {S_t}^{\alpha_d}$$

where $S_0 > 0$ is given and $r_e = \frac{A_e h^{1-\alpha_e} L_e^{1-\alpha_e}}{\lambda^{\alpha_e}}$.

>From the Maximum Theorem, L is continuous. It is obviously strictly increasing with respect to S and $L\,(r_e, 0) = 0$.

Since the utility function is strictly increasing, at the optimum the constraints will be binding.

$$c_t = L\,(r_e, S_t) - S_{t+1}$$

A sequence $(S_t)_{t=0\ldots\infty}$ is called feasible from $S_0 \geq 0$ if we have $\forall t$, $0 \leq S_{t+1} \leq L\,(r_e, S_t)$. Thus the initial program is equivalent to the following problem

$$\max \sum_{t=0}^{\infty} \beta^t u\,(L\,(r_e, S_t) - S_{t+1})$$

under the constraints,

$$0 \leq S_{t+1} \leq L\,(r_e, S_t)\,, \text{ for all } \;t \geq 0,$$

with $S_0 > 0$ given.[11].
We denote by $K^*_{e,t}, K^*_{d,t}$ the corresponding optimal capital stocks and by θ^*_t μ^*_t, the optimal share for technology capital and the proportion of skilled workers used in the new technology sector.

3.2.1 Properties of the optimal path with mobility of labor

In this subsection, we will present the main results of the model when labour is mobile. As before, we will show that any optimal path from $S_0 > 0$ is monotonic and does not converge to 0. Under some stronger conditions, we will show that any optimal path will grow without bound. Along this growth path, after a date T, the economy will use new technology to produce consumption goods and all the skilled workers will entirely be in the new technology sector.

Proposition 4 *(i) The function $L(r_e, S)$ is increasing in S. Hence, any optimal path from S_0 is monotonic.*
(ii) No optimal path from $S_0 > 0$ converges to 0.
(iii) There exists $S^c > 0$ such that if $S_t \leq S^c$, then $L(r_e, S_t) = x_0(L^*_d + L^*_e)^{(1-\alpha_d)} {S_t}^{\alpha_d}$. S^c is a decreasing function of r_e.

Proof. See Appendix 2. ■

Proposition 5 *Let S^s satisfy $x_0 L^*(1-\alpha_d)\alpha_d (S^s)^{\alpha_d - 1} = \frac{1}{\beta}$. Consider S^c in Proposition 4.*
(i) There exists $\bar{r}_e$ which depends on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L^*_d)$ such that,if $r_e > \bar{r}_e$, then any optimal path from $S_0 > 0$ will be increasing. There exists T, such that for any $t \geq T$, $K^*_{e,t} > 0$. The condition $r_e > \bar{r}_e$ is equivalent to $S^s > S^c$.
(ii) There exists $\widetilde{r}_e$ which depends only on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L^*_d)$ such that if $r_e < \widetilde{r}_e$ then for any $0 < S_0 < S^c$, there exists a unique optimal path which converges to S^s and $K^*_{e,t} = 0$ for every t.

Proof. See Appendix 2. ■

Proposition 6 *Assume furthermore*
(H_3) $\alpha_e + \alpha_d \geq 1$.

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[11] We assume that the utility function u is such that the sum $\sum_{t=0}^{\infty} \beta^t u\left(L\left(r_e, S_t\right) - S_{t+1}\right)$ is real-valued for any feasible sequence $\{S_t\}$

(i) There exists $\bar{A}_e > 0$ (or $\bar{h} > 0$) such that if $A_e > \bar{A}_e$ (or $h > \bar{h}$) then any optimal path (S_t^*) from $S_0 > 0$, will converge to $+\infty$ and $K_{e,t}^* \to +\infty$ and $\theta_t^* \to \theta^\infty = \frac{\alpha_e}{\alpha_e+\alpha_d}$.
(ii) If we assume furthermore $\frac{L_e^*}{L_d^*} < \frac{1-\alpha_e}{1-\alpha_d}$, then there exists $T \geq 0$; such that $\mu_t^* = 1$ for every $t \geq T$.

Proof. See Appendix 2. ■
Comment Part (ii) of Proposition 6 states that for t large enough the new technology sector may use all the skilled workers.

Remark 7 *Consider the case where $x_0 = aX$, $\alpha_e + \alpha_d \geq 1$ and $\frac{L_e^*}{L_d^*} \geq \frac{1-\alpha_e}{1-\alpha_d}$. From the first order conditions (36, 37) when $S > S^c$, we obtain that the optimal values (θ^*, μ^*) are independent of S and respectively equal*

$$\theta^\infty = \frac{\alpha_e}{\alpha_e + \alpha_d},\ \mu^\infty = \frac{(1-\alpha_e)(L_d^* + L_e^*)}{(2-\alpha_d-\alpha_e)L_e^*}.$$

The technological function of the optimal growth model becomes

$$L(r_e, S) = x_0(L_d^* + L_e^*)^{1-\alpha_d} S^{\alpha_d} \ \textit{if} \ \ S \leq S^c,$$

and

$$L(r_e, S) = ar_e{\theta^\infty}^{\alpha_e}(1-\theta^\infty)^{\alpha_d}{\mu^\infty}^{1-\alpha_e}(L_d^*+(1-\mu^\infty)L_e^*)^{1-\alpha_d}S^{\alpha_e+\alpha_d},\ \textit{if} \ \ S > S^c.$$

The function $L(r_e, \cdot)$ is concave-convex and differs from the case in Dechert and Nishimura [1983] where the technology is convex-concave.
Add the assumptions (i) $\alpha_e + \alpha_d = 1$ and (ii) $u(c) = \frac{c^\sigma}{\sigma}$ with $\sigma < 1$. Then for t large enough, we have

$$(\frac{c_{t+1}^*}{c_t^*})^{1-\sigma} = \beta a r_e(\alpha_e)^{2\alpha_e}(\alpha_d)^{2\alpha_d}(L_e^* + L_d^*)^{\alpha_e}.$$

We have a balanced growth path in the long term. The rate of growth is positively related to r_e (i.e. to A_e, h, and L_e^*). In other words, if the qualities of the new technology and/or the human capital and/or the number of skilled workers are high, the rate of growth will be high too.

4 Conclusion

We summarize the results we obtain in the paper.

1. When there is an adoption effect of new technologies by firms, there is a critical value S^c for the domestic resource. Below this value, it is not optimal for the country to invest in new technology. Above this value, it is optimal to invest in new technology. The critical value decreases when the human capital or/and the productivity of the new technology sector is high, or/and the price of the new technology capital is low.
2. When the human capital is high and/or the productivity of the new technology sector is high, in the dynamic setting, there is a date T_1 such that for any date beyond T_1 the country will adopt the new technology and grows without bound.
3. If we allow the high skilled workers to move to the consumption sector, then may be a date T_2 such that the new technology sector will use all the high skilled workers only after this date. This result shows that is not always optimal to have a very large number of high-skilled workers.
4. Here we want to sketch a country which faces the corruption phenomenon. There are many ways to formalize this phenomenon. One is to assume that the production function in the consumption sector exhibits fixed costs. But it will raise many mathematical complications. So, we use the way proposed by Dimaria and Le Van [2002]. We assume that at every date t, the bribers divert a fraction $\eta \in (0,1)$ of national resource, S_{t+1}, devoted to the next period investment. The amount ηS_{t+1} either goes abroad or is used for consuming imported goods. In this case the constraints at every period are:

$$c_t + S_{t+1} = L(r_e, (1-\eta)S_t)$$

and

$$K_{d,t} + \lambda K_{e,t} = (1-\eta)S_t.$$

It is obvious that the new critical value ${S'}^c$ equals $\frac{S^c}{1-\eta}$, i.e. larger than S^c. It converges to $+\infty$ when η goes to 1. The new value ${S'}^s$ (corresponding to the steady state of the optimal growth model without new technology) will be $S^s(1-\eta)^{\frac{\alpha_d}{1-\alpha_d}}$, i.e. smaller than S^s. It converges to zero when η converges to one. Assume the initial wealth S_0 satisfy $S^c < S_0 < S^s$ allowing the country to take off. In presence of high corruption (η close to 1), we have ${S'}^s < S_0 < {S'}^c$ and the country will fall down by converging to ${S'}^s$.
Observe that in this paper we obtain an optimal path which grows without bound in contrast with usual Ramsey models. Our condition is the same as in Kamihigashi and Roy [2007] for a more general setting.

Some limits of our model:

1. We are in presence of non-convex technology. Our results must be interpreted as normative prescriptions for development since the optimal paths of our model cannot be decentralized.
2. Here, technological capital is just a name for the capital that acts as engine for growth and where there can be a threshold effect. Our technology output may be knowledge. In this case, skilled workers might be researchers or qualified engineers. Technology capital will be the machines used for experiments necessary for producing knowledge.
3. In our model we introduce the relative price λ of the technology capital. We want to point out that is is not always optimal to invest in the short term in the technology if the price of the required capital is very high.
4. Our model has many weaknesses. The first is to assume that there is full depreciation. In principle, we should write the budget constraint as $I_{d,t} + \lambda I_{e,t} = S_t$ where $I_{d,t}$ and $I_{e,t}$ are respectively investments in capital for domestic good and technology capital. The capital stocks will be $K_{d,t+1} = K_{d,t}(1-\delta_d) + I_{d,t}$ and $K_{e,t+1} = K_{e,t}(1-\delta_e) + I_{e,t}$ (δ_d and δ_e are the depreciation rates). There will be a three dimension dynamics $(S_t, K_{e,t}, K_{d,t})$. The problem will be hardly tractable. One possible way is to use lattice programming. But our technology contraints do not ensure that the feasible sequences are in a sublattice.
5. Another weakness of our model is to consider Y_e as an intermediate good and does not take into account the stock of all technologies. Suppose, to make simple, that the TFP depends just on the stock of technologies over two periods. In other words, suppose TFP is $\phi(Y_{e,t} + Y_{e,t-1})$. In this case, the constraints become $0 \leq S_{t+1} \leq H(r_e, S_t, S_{t-1})$. The dynamics will become more complicated (see e.g. Mitra and Nishimura [2005]). We will show that the critical value will decrease. Indeed, consider first the static model. Assume the problem in the domestic sector now becomes

$$\max_{c,K_e,K_d,L_e,L_d} Y_d = \phi\left(Y_e + Y_{e,-1}\right) K_d^{\alpha_d} L_d^{1-\alpha_d} \tag{14}$$

where $Y_{e,-1}$ is the stock of technology of the previous period. Recall that $\phi(x) = x_0$ if $x \leq X$, and $\phi(x) = x_0 + a(x - X)$ if $x \geq X$. In our case, we will have $\phi\left(Y_e + Y_{e,-1}\right) = x_0$ if $Y_e \leq X - Y_{e,-1}$ and $\phi\left(Y_e + Y_{e,-1}\right) = x_0 + a(Y_e + Y_{e,-1} - X)$ if $Y_e \geq X - Y_{e,-1}$. The fixed cost now diminishes by $Y_{e,-1}$. Hence the critical value decreases too. The country will escape more easily from the poverty trap. The dynamic problem will be:

$$\max \sum_{t=0}^{+\infty} \beta^t u\left(c_t\right) \text{ with } 0 < \beta < 1,$$

under the constraints: for every date t,

$$c_t + S_{t+1} \leq \phi\left(Y_{e,t} + Y_{e,t-1}\right) K_{d,t}^{\alpha_d} L_{d,t}^{(1-\alpha_d)}, \tag{15}$$

$$Y_{e,t} = A_e K_{e,t}^{\alpha_e} L_{e,t}^{1-\alpha_e}, \tag{16}$$

$$L_{d,t} \leq L_e^* + L_d^*,\ \ L_{e,t} \leq h L_e^*, \tag{17}$$

$$K_{d,t} + \lambda K_{e,t} = S_t. \tag{18}$$

Along the optimal path, the TFP of the domestic sector will be obviously higher than when it depends only on the the flow of technology. The optimal path in our model will be feasible for the new model and then smaller than the new optimal path. Observe that the new problem is non stationary. One can not claim that the new optimal path is monotonic.

Anyway, we recognize that our modeling of new technology in terms of flow is not realistic. However, it shows that new technology, even with this restrictive assumption, allows emergence of growth. If we take into account the stock of new technology, the sketch of the proof given above shows that growth will then be enhanced.

5 Appendix 1: Proofs of Propositions 1, 2, 3

Preliminary

Consider the static problem:

$$\max_{c,K_e,K_d,L_e,L_d} Y_d = \phi\left(Y_e\right) K_d^{\alpha_d} L_d^{1-\alpha_d} \tag{19}$$

$$Y_e = A_e K_e^{\alpha_e} L_e^{1-\alpha_e} \tag{20}$$

$$K_d + \lambda K_e = S \tag{21}$$

$$L_d \leq L_d^* \tag{22}$$

$$L_e \leq L_e^* h \tag{23}$$

Let $\theta = \frac{\lambda K_e}{S}$. Then (21) can be re-written

$$K_d = (1-\theta)\, S, \lambda K_e = \theta S \tag{24}$$

Since at the optimum, $L_e = L_e^* h$, $L_d = L_d^*$, the problem turns out to be

$$\max_{\theta \in [0,1]} \phi\left(\frac{A_e h^{1-\alpha_e} {L_e^*}^{1-\alpha_e}}{\lambda^{\alpha_e}} \theta^{\alpha_e} S^{\alpha_e}\right)(1-\theta)^{\alpha_d} S^{\alpha_d} {L_d^*}^{1-\alpha_d} \tag{25}$$

Let $r_e = \frac{A_e}{\lambda^{\alpha_e}} L_e^{*1-\alpha_e} h^{1-\alpha_e}$ and $\psi\left(r_e, S, \theta\right) = \phi\left(r_e \theta^{\alpha_e} S^{\alpha_e}\right)(1-\theta)^{\alpha_d} L_d^{*1-\alpha_d}$
Solving the previous problem becomes equivalent to solve

$$\max_{0 \leq \theta \leq 1} \psi\left(r_e, S, \theta\right) \tag{26}$$

Since the function ψ is continuous in θ, there will always be an optimal solution. Let be

$$G(r_e, S) = Argmax\{\psi(r_e, S, \theta) : \theta \in [0,1]\}$$

and $F(r_e, S) = \max\{\psi\left(r_e, S, \theta\right) : \theta \in [0,1]\}$. From the Maximum Theorem, F is continuous, and the maximum output of the consumption sector will be $H(r_e, S) = F(r_e, S) S^{\alpha_d}$.

Lemma 8 *(i) If $S \leq (\frac{X}{r_e})^{\frac{1}{\alpha_e}}$ then $G\left(r_e, S\right) = \{0\}$.*
(ii) there exists $\hat{S}$ such that: $S > \hat{S} \Rightarrow G\left(r_e, S\right) \subset \left]0,1\right[$.

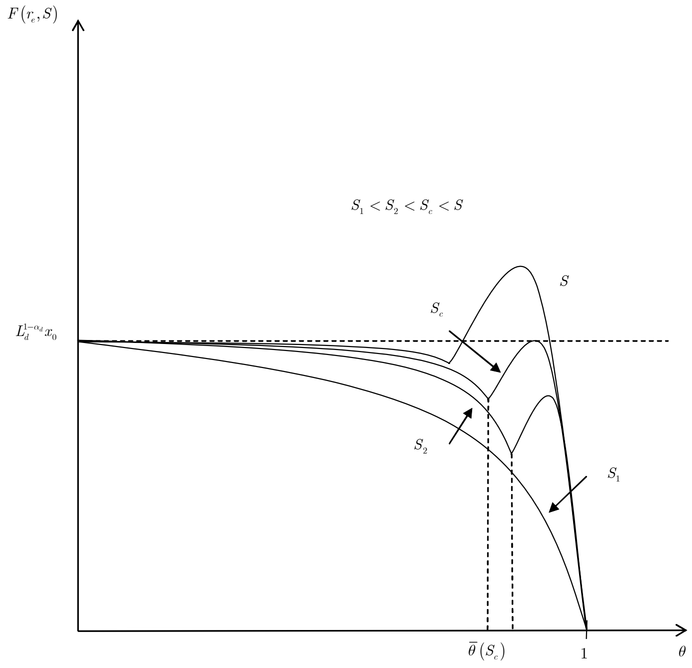
Proof. (i) If $S \leq (\frac{X}{r_e})^{\frac{1}{\alpha_e}}$, then for any $\theta \in [0,1]$, we have $r_e S^{\alpha_e} \theta^{\alpha_e} \leq X$, and hence $\psi(r, S, \theta) = x_0 (1-\theta)^{\alpha_d} L_d^{*1-\alpha_d}$. Obviously, the maximizer is unique and equals 0.
(ii) Observe that $\forall S \geq 0, F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}$. Let $S_0 > (\frac{X}{r_e})^{\frac{1}{\alpha_e}}$, and $\bar{\theta}(S_0)$ satisfies $r_e S_0^{\alpha_e} \bar{\theta}(S_0)^{\alpha_e} = X$. Let $\hat{\theta} \in]\bar{\theta}(S_0), 1[$. When S increases, $\bar{\theta}(S)$ decreases and $\hat{\theta}$ will be in $]\bar{\theta}(S), 1[$. We have $\psi(r_e, S, \hat{\theta}) \to +\infty$ when $S \to +\infty$. Hence, for S large enough, say, greater than some $\hat{S}$, then $\max_\theta\{\psi(r_e, S, \theta)\} > x_0 L_d^{*1-\alpha_d}$. That implies $0 \notin G(r_e, S)$. Since $\psi(r_e, S, 1) = 0$, we have $1 \notin G(r_e, S)$. The proof is complete. ■

We want to prove there is a critical value S^c, i.e., if $S < S^c$ then $G(r_e, S) = \{0\}$ and if $S > S^c$ then $G(r_e, S) \subset]0,1[$. In that case, the country will import technological capital and produce new technologies as soon as its wealth S is higher than the critical value S^c. Figure 3 illustrates that point.

Let $B = \{S \geq 0 : F(r_e, S) = x_0 L_d^{*1-\alpha_d}\}$.

Lemma 9 *B is non empty and compact.*

Proof. (i) B is not empty: obviously, $0 \in B$.
(ii) B is closed because the function F is continuous.



$F(r_e, S)$
$S_1 < S_2 < S_c < S$
S
S_c
$L_d^{1-\alpha_d} x_0$
S_2
S_1
$\overline{\theta}(S_c)$
1
θ

Figure 3:

(iii) To prove that B is bounded take a sequence S_n converging to $+\infty$. Fix some $\theta \in]0,1[$. For n large enough, $\bar{\theta}(S_n) < \theta < 1$. Then $\psi(r_e, S_n, \theta)$ converges to $+\infty$. This implies $F(r_e, S_n) > x_0 \ {L_d^*}^{1-\alpha_d}$ for any n sufficiently large. That contradicts $S_n \in B$. ∎

Lemma 10 *Let $S^c = \max\{S : S \in B\}$. Then if $S < S^c$ we have $G(r_e, S) = \{0\}$ and if $S > S^c$ then $G(r_e, S) \subset]0,1[$.*

Proof. >From Lemma 8(i), $S^c > 0$ since it must be greater than $(\frac{X}{r_e})^{\frac{1}{\alpha_e}}$. From Lemma 9, $S^c < +\infty$.
First, observe that if $S < S^c$ then $F(r_e, S) = x_0 {L_d^*}^{1-\alpha_d}$. Indeed, we have

$$\forall \theta \in [0,1], \psi(r_e, S, \theta) \leq \psi(r_e, S^c, \theta).$$

Hence $F(r_e, S) = \max_\theta\{\psi(r_e, S, \theta)\} \leq \max_\theta\{\psi(r_e, S^c, \theta)\} = F(r_e, S^c) = x_0 {L_d^*}^{1-\alpha_d}$. Since $\forall S \geq 0, F(r_e, S) \geq x_0 {L_d^*}^{1-\alpha_d}$, we have $F(r_e, S) = x_0 {L_d^*}^{1-\alpha_d}$
Now, (i) if $S > S^c$, then from the very definition of S^c, we have $G(r_e, S) \subset]0,1[$.
(ii) If $S < S^c$, then take some $S_0 < S^c$. Suppose for S_0 we have two solutions $\theta_M^1 = 0$ and $\theta_M^2 > 0$. There must be $\bar{\theta}_0 \in]0,1[$ which satisfies $r_e S_0^{\alpha_e} (\bar{\theta}_0)^{\alpha_e} = X$ (if not, $\forall \theta, r_e S_0^{\alpha_e} \leq X$, and $G(r_e, S_0) = \{0\}$). For $\theta \in]0, \bar{\theta}_0]$, we have $\psi(r_e, S, \theta) = (1-\theta)^{\alpha_d} x_0 {L_d^*}^{1-\alpha_d} < \psi(r_e, S, 0) = x_0 {L_d^*}^{1-\alpha_d}$. Hence $\theta_M^2 > \bar{\theta}_0$. Let $S_0 < S_1 < S^c$, and $\bar{\theta}_1$ satisfy $r_e S_1^{\alpha_e} \bar{\theta}_1^{\alpha_e} = X$. Then $\theta_M^2 > \bar{\theta}_0 > \bar{\theta}_1$. We have

$$\phi(r_e S_0^{\alpha_e} (\theta_M^2)^{\alpha_e}) = x_0 + \gamma(r_e S_0^{\alpha_e} (\theta_M^2)^{\alpha_e})$$

$$\phi(r_e S_1^{\alpha_e} (\theta_M^2)^{\alpha_e}) = x_0 + \gamma(r_e S_1^{\alpha_e} (\theta_M^2)^{\alpha_e}) > \phi(r_e S_0^{\alpha_e} (\theta_M^2)^{\alpha_e}).$$

We obtain a contradiction

$$x_0 {L_d^*}^{1-\alpha_d} = F(r_e, S_1) \geq \phi(r_e S_1^{\alpha_e} (\theta_M^2)^{\alpha_e})(1-\theta_M^2)^{\alpha_d} x_0 {L_d^*}^{1-\alpha_d}$$

$$> \phi(r_e S_0^{\alpha_e} (\theta_M^2)^{\alpha_e})(1-\theta_M^2)^{\alpha_d} x_0 {L_d^*}^{1-\alpha_d} = F(r_e, S_0) = x_0 {L_d^*}^{1-\alpha_d}.$$

∎

Lemma 11 *Every optimal path is monotonic*

Proof. Notice that we have the following Bellman equation. Let V be the value-function of the problem. We have

$$\forall S_0 \geq 0, V(S_0) = \max\{u(H(r_e, S_0) - S) + \beta V(S) : 0 \leq S \leq H(r_e, S_0)\}$$

Let Γ denote the optimal correspondence. From Amir [1996], this correspondence is non decreasing, i.e.,
if $S_0' < S_0$ then $\forall S_1' \in \Gamma(r_e, S_0')$, and $\forall S_1 \in \Gamma(r_e, S_0), S_1' \leq S_1$.
Hence, any optimal path must be monotonic. ■

Lemma 12 *Every optimal trajectory* (S_t^*) *from* $S_0 > 0$ *cannot converge to 0.*

Proof. Suppose that $S_t^* \to 0$. Then for $t \geq T$, we have: $S_t^* < S^c$. Hence, $\forall t > T, H(r_e, S_t^*) = x_0 L_d^{*1-\alpha_d} S_t^{*\alpha_d}$ and $H_S'(r_e, S_t^*) \to \infty$, because, $S_t \to 0$. As $u'(0) = +\infty$, we have Euler equation for $t > T$,

$$u'(c_t^*) = \beta u'\left(c_{t+1}^*\right) H_S'\left(r_e, S_{t+1}^*\right).$$

There exists $T_0 \geq T$ such that for all $t \geq T_0$ we have $H_S'\left(r_e, S_{t+1}^*\right)\beta > 1$. That implies $u'(c_t^*) > u'\left(c_{t+1}^*\right)$ or equivalently, $c_{t+1}^* > c_t^* \geq c_{T_0}^* > 0$. That is contradictory with $S_t^* \to 0$ (because it would have for consequence $c_t^* \to 0$). ■

Lemma 13 *(i) The function* $F(r_e, S)$ *is continuously differentiable with respect to* S *in* $]0, S^c[\bigcup]S^c, +\infty[$. *At* S^c, *it has left derivative (equal to* 0*) and right derivative.*
(ii) For $S > S^c$, *there exists a unique* $\theta_M(S) \in G(r_e, S)$.

Proof. (i) (a) When $S < S^c$, from Lemma 10, we have $F(r_e, S) = x_0 L_d^{*1-\alpha_d}$.
(b) Consider the case where $S > S^c$. Let $\bar{\theta}(S)$ satisfy $r_e S^{\alpha_e} \bar{\theta}(S)^{\alpha_e} = X$. Since, when $\theta \leq \bar{\theta}(S)$, $\psi(r_e, S, \theta) = x_0(1-\theta)^{\alpha_d} L_d^{*1-\alpha_d} \leq x_0 L_d^{*1-\alpha_d}$, from the very definition of S^c, any solution must be larger than $\bar{\theta}(S)$. Thus, any solution θ must be interior to the interval $]\bar{\theta}(S), 1[$, because $\psi(r_e, S, 1) = 0$. The solution is unique since $\psi(r_e, S, \theta)$ is strongly concave in θ. One can check that $\frac{\partial^2 \psi}{\partial \theta^2} < 0$. It satisfies $\psi_\theta'(r_e, S, \theta) = 0$. Tedious computations give:

$$\frac{\alpha_e}{\alpha_d}\theta^{\alpha_e - 1}(1-\theta) = \frac{x_0 - aX}{a r_e S^{\alpha_e}} + \theta^{\alpha_e} \tag{27}$$

The left side member is a decreasing function in θ while the right side one is increasing in θ. The solution θ_M is unique. One can check that

$$\frac{d\theta_M}{dS} = \frac{A}{B} \tag{28}$$

with $A = \frac{aX - x_0}{a r_e} S^{\alpha_e - 1}$ and $B = \frac{\alpha_e - 1}{\alpha_d}\theta_M^{\alpha_e - 2} - (1 + \frac{\alpha_e}{\alpha_d})\theta_M^{\alpha_e - 1} < 0$. Thus $F(r_e, \cdot)$ is differentiable for $S > S^c$.

(c) When $S = S^c$, there is a solution $\theta_M^1 = 0$ and another θ_M^2 which is the unique solution to equation (27). From Clarke ([1983] , theorem 2.8.2), there is a right derivative equal to $\psi_S'(r_e, S^c, \theta_M^2)$ and a left derivative which is trivially zero.
(ii) In (i) (b), we have shown that $G(r_e, S)$ is a singleton $\{\theta_M(S)\}$ when $S > S^c$. ■

Proof of Proposition 1

Proof of (i): It follows from Lemma 11

Proof of (ii): It follows from Lemma 12

Proof of (iii): Consider S^c the critical value defined in Lemma 10. Then for $S_t \leq S^c$, from the proof of this lemma, we get $H(r_e, S_t) = x_0 L_d^{*(1-\alpha_d)} {S_t}^{\alpha_d}$.

$>$From Lemma 13, S^c and θ^c satisfy equation (27) which is

$$\frac{\alpha_e}{\alpha_d}(\theta^c)^{\alpha_e - 1}(1 - \theta^c) = \frac{x_0 - aX}{ar_e(S^c)^{\alpha_e}} + (\theta^c)^{\alpha_e}$$

and $F(r_e, S^c) = x_0 {L_d^*}^{1-\alpha_d}$ which can be rewritten as:

$$[x_0 + a\left(r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e} - X\right)](1 - \theta^c)^{\alpha_d} = x_0$$

Tedious computations show that this system is equivalent to

$$(x_0 - aX)\left[\frac{\alpha_e(1 - \theta^c)}{\alpha_e - \theta^c(\alpha_e + \alpha_d)}\right](1 - \theta^c)^{\alpha_d} = x_0, \tag{29}$$

$$\frac{\alpha_e}{\alpha_d}(\theta^c)^{-1}(1 - \theta^c) - 1 = \frac{x_0 - aX}{ar_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e}} \tag{30}$$

One can easily check that there is a unique solution $\theta^c \in (0, 1)$ to equation (29). It depends only on $(x_0, aX, \alpha_e, \alpha_d)$. Equation (30) gives $ar_e(S^c)^{\alpha_e} = (x_0 - aX)\zeta(\theta^c)$. Hence, when r_e increases, S^c decreases. The proof is complete.

Proof of Proposition 2

Let us recall that $K_{d,t}^*$ and $K_{e,t}^*$ respectively denote the optimal values of the physical and the technological capital stock and θ_t^* denotes the associated optimal capital share, i.e. $K_{d,t}^* = (1 - \theta_t^*)S_t^*$ and $\lambda K_{e,t}^* = \theta_t^* S_t^*$.

Proof of (i): Let $\bar{r}_e$ satisfy equation (30) when $S = S^s$ and $\theta = \theta^c$. Obviously, $\bar{r}_e$ only depends on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L_d^*)$. When $S_0 \geq S^c$, since the optimal path (S_t^*) is nondecreasing, we have $\forall t > 0, G(r_e, S_t^*) \subset]0, 1[$ or equivalently $K_{e,t}^* > 0$.

Consider the case $S_0 < S^c$. If for any t, $K^*_{e,t} = 0$, then the optimal path (S^*_t) will converge to S^s (see e.g. Le Van and Dana, [2005], Chapter 2). Since we assume $S^s > S^c$, there will be t with $S^*_t > S^c$. In this case $K^*_{e,t} > 0$ which is contradictory.
So, let T be the first date with $S^*_T > S^c$. Since the optimal path (S^*_t) is nondecreasing, we will have $K^*_{e,t} > 0$ for every $t > T$.
Proof of (ii): Let $\widetilde{S}$ satisfy $x_0(L^*_d)^{1-\alpha_d}\widetilde{S}^{\alpha_d} = \widetilde{S}$. Then $\widetilde{S}$ depends only on (x_0, L^*_d). Let $\widetilde{r}_e$ satisfy equation (30) with $S = \widetilde{S}$ and $\theta = \theta^c$. It depends only on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L^*_d)$. When $r_e < \widetilde{r}_e$, or equivalently, $S^c > \widetilde{S}$, any feasible path (S_t) from $S_0 < S_c$ will be less than S^c and is feasible for the convex technology $H(r_e, S) = x_0(L^*_d)^{1-\alpha_d}S^{\alpha_d}$. There exists a unique optimal path from S_0 and it converges to S^s. The proof of Proposition 2 is complete.

Proof of Proposition 3

First observe that θ^*_t and S^*_t satisfy equation (27). Then when S converges to $+\infty$ then θ converges to θ^∞.
We claim that $H'_S(r_e, S) \neq \frac{1}{\beta}, \forall S$.
Case 1: $x_0 - aX < 0$. In this case $\frac{d\theta_M(S)}{dS} < 0$ when $S \geq S^c$ (see (28)). Therefore, for every $S \geq S^c$, $\theta_M(S) > \theta^\infty$.
Let $\theta_M(S^c)$ be the unique maximizer associated with $F(r_e, S^c)$ which is strictly positive. For short, write θ_c instead of $\theta_M(S^c)$. Then (θ_c, S^c) satisfy (27) and $F(r_e, S^c) = x_0 {L^*}^{1-\alpha_d}$. We obtain:

$$(x_0 - aX)\frac{\alpha_e\theta_c^{\alpha_e-1}(1-\theta_c)^{\alpha_d+1}}{(\alpha_e+\alpha_d)\theta_c^{\alpha_e} - \alpha_e\theta_c^{\alpha_e-1}} + x_0 = 0.$$

We see that θ_c is independent of r_e hence A_e and h. If A_e (or h) is large enough, then from (27) $S^c < S^s = (\beta x_0\alpha_d)^{\frac{1}{1-\alpha_d}}$ and $H'_S(r_e, S) > \frac{1}{\beta}, \forall S \leq S^c$.
Consider the case where $S > S^c$. From the envelope theorem and relation (27), we have

$$H'_S(r_e, S) = ar_e\alpha_e\theta_M(S)^{\alpha_e-1}(1-\theta_M(S))^{\alpha_d}S^{\alpha_e+\alpha_d-1}{L^*_d}^{1-\alpha_d}.$$

We can bound this derivative from below:

$$H'_S(r_e, S) > ar_e\alpha_e(1-\theta_c)^{\alpha_d}S^{\alpha_e+\alpha_d-1}{L^*_d}^{1-\alpha_d}$$

and hence, when $\alpha_e + \alpha_d > 1$,

$$S < \left(\frac{1}{\beta ar_e\alpha_e(1-\theta_c)^{\alpha_d}{L^*_d}^{1-\alpha_d}}\right)^{\frac{1}{\alpha_e+\alpha_d-1}}, \text{ if } H'_S(r_e, S) = \frac{1}{\beta}.$$

Again from (27) we can write $S^c = (\frac{\zeta(a,x_0,X)}{ar_e})^{\frac{1}{\alpha_e}}$ where the function ζ can be easily computed. One can also easily check that if A_e (hence r_e) is sufficiently large then S will be less than S^c which is a contradiction.
When $\alpha_e + \alpha_d = 1$, we have

$$H'_S(r_e, S) > ar_e\alpha_e(1-\theta_c)^{\alpha_d} S^{\alpha_e+\alpha_d-1} {L_d^*}^{1-\alpha_d} > \frac{1}{\beta}$$

if r_e is large enough. Again, a contradiction.
Case 2: $x_0 - aX > 0$. As above, θ_c is independent of r_e. If A_e (or h) is large enough, then from (27) $S^c < S^s = (\beta x_0 \alpha_d)^{\frac{1}{1-\alpha_d}} L_d^*$ and $H'_S(r_e, S) \neq \frac{1}{\beta}, \forall S \leq S^c$. When $S > S^c$, from (27), we have $\theta_M(S) < \theta^\infty$. We then have

$$H'_S(r_e, S) > ar_e\alpha_e(1-\theta^\infty)^{\alpha_d} S^{\alpha_e+\alpha_d-1} {L_d^*}^{1-\alpha_d}$$

and hence, when $\alpha_e + \alpha_d > 1$,

$$S < (\frac{1}{\beta ar_e\alpha_e(1-\theta_c)^{\alpha_d} {L_d^*}^{1-\alpha_d}})^{\frac{1}{\alpha_e+\alpha_d-1}}, \text{ if } H'_S(r_e, S) = \frac{1}{\beta}.$$

Apply the same argument as above to obtain a contradiction.
When $\alpha_e + \alpha_d = 1$, we have

$$H'_S(r_e, S) > ar_e\alpha_e(1-\theta^\infty)^{\alpha_d} {L_d^*}^{1-\alpha_d} \geq \frac{1}{\beta}$$

if r_e is large enough.
Case 3: $x_0 = aX$. From (27), we have $\theta_M(S) = \theta^\infty, \forall S \geq S^c$. It is easy to check that

$$(S^c)^{\alpha_e} ar_e(\theta^\infty)^{\alpha_e}(1-\theta^\infty)^{\alpha_d} = x_0.$$

Obviously, when A_e or h are large then $S^c < S^s$. Since we now have for $S > S^c$,

$$H(r_e, S) = ar_e(\theta^\infty)^{\alpha_e}(1-\theta^\infty)^{\alpha_d} {L_d}^{*(1-\alpha_d)} S^{\alpha_e+\alpha_d}$$

we get

$$H'_S(r_e, S) = ar_e(\theta^\infty)^{\alpha_e}(1-\theta^\infty)^{\alpha_d} {L_d}^{*(1-\alpha_d)} (\alpha_e+\alpha_d) S^{\alpha_e+\alpha_d-1}$$

and $S < S^c$ if $H'_S(r_e, S) = \frac{1}{\beta}$, and $\alpha_e + \alpha_d > 1$. That is a contradiction.
When $\alpha_e + \alpha_d = 1$, tedious computiations give

$$H'_S(r_e, S) = \frac{x_0}{(S^c)^{\alpha_e}} {L_d}^{*(1-\alpha_d)} > \frac{x_0}{(S^s)^{\alpha_e}} {L_d}^{*(1-\alpha_d)}$$

Since $S^s = (\beta x_0 \alpha_d)^{\frac{1}{(1-\alpha_d)}} L_d{}^*$ and $\alpha_e = 1-\alpha_d$, we get $H_S'(r_e, S) > (\frac{1}{\beta})(\frac{1}{\alpha_d}) > (\frac{1}{\beta})$. That is a contradiction. We have proven our claim.
Any optimal path from $S_0 > 0$ must be increasing since it cannot converge to 0. If it is bounded then it will converge to a point $\widehat{S}$ with $H_S'(r_e, \widehat{S}) = \frac{1}{\beta}$. But it is impossible from our claim. Hence any optimal path must converge to $+\infty$. Therefore, K_t^* also converges to $+\infty$. From equation (27), θ_t^* converges to $\frac{\alpha_e}{\alpha_e+\alpha_d}$. The proof is now complete.

6 Appendix 2: Proof of Propositions 4, 5, 6

Preliminary

Let $r_e = \frac{A_e}{\lambda^{\alpha_e}} h^{1-\alpha_e} {L_e^*}^{1-\alpha_e}$. The first step is:

$$\max_{\substack{0 \le \theta \le 1 \\ 0 \le \mu \le 1}} \quad Y_d = \phi\left(r_e \theta^{\alpha_e} S^{\alpha_e} \mu^{1-\alpha_e}\right) (1-\theta)^{\alpha_d} S^{\alpha_d} \left(L_d^* + (1-\mu) L_e^*\right)^{1-\alpha_d}.$$

Let

$$\varphi(r_e, S, \theta, \mu) = \phi\left(r_e \theta^{\alpha_e} S^{\alpha_e} \mu^{1-\alpha_e}\right) (1-\theta)^{\alpha_d} \left(L_d^* + (1-\mu) L_e^*\right)^{1-\alpha_d}.$$

The problem is equivalent to

$$\max_{(\theta,\mu)\in[0,1]\times[0,1]} \varphi(r_e, S, \theta, \mu).$$

Let

$$F(r_e, S) = \max_{(\theta,\mu)\in[0,1]\times[0,1]} \varphi(r_e, S, \theta, \mu).$$

Then $F(r_e, S) \ge x_0 \left(L_d^* + L_e^*\right)^{1-\alpha_d}$. As before, define $B = \{S \ge 0 : F(r_e, S) = x_0 \left(L_d^* + L_e^*\right)^{1-\alpha_d}\}$. It is easy to check that B is compact and nonempty. The critical value is

$$S^c = \max\{S : S \in B\}$$

Observe that for $S > S^c$ the function

$$Z(r_e, S, \theta, \mu) = Log(\varphi(_re, S, \theta, \mu))$$

is strongly concave in (θ, μ). Since maximizing $\varphi(r_e, S, \theta, \mu)$ is equivalent to maximize $Z(r_e, S, \theta, \mu)$ when $S > S^c$, the solution $(\theta_M(S), \mu_M(S))$ will be unique. Obviously, if $S > S^c$ then $\theta_M(S) > 0$ (if not, we will have $\mu_M(S) = 0$ and $F(r_e, S) = x_0 \left(L_d^* + L_e^*\right)^{1-\alpha_d}$).

Proof of Proposition 4

Proofs of (i) and (ii): They are similar to the ones for Proposition 1.

Proof of (iii): As before θ^c, μ^c and S^c satisfy $\varphi'_\theta(r_e, S^c, \theta^c, \mu^c) = 0$, $\varphi'_\mu(r_e, S^c, \theta^c, \mu^c) = 0$, and $F(r_e, S^c) = x_0\left(L_d^* + L_e^*\right)^{1-\alpha_d}$. One can easily check that these equations are equivalent to (we write θ, μ, S instead of θ^c, μ^c, S^c, for short):

$$\frac{\alpha_e}{\alpha_d}\theta^{-1}(1-\theta) = \left[\frac{x_0 - aX}{ar_e\theta^{\alpha_e}\mu^{1-\alpha_e}S^{\alpha_e}} + 1\right] \tag{31}$$

$$\frac{1-\alpha_e}{1-\alpha_d}\mu^{-1}\left[\frac{L_d^* + (1-\mu)L_e^*}{L_d^*}\right] = \left[\frac{x_0 - aX}{ar_e\theta^{\alpha_e}\mu^{1-\alpha_e}S^{\alpha_e}} + 1\right] \tag{32}$$

$$\left[x_0 - aX + ar_e\theta^{\alpha_e}\mu^{1-\alpha_e}S^{\alpha_e}\right](1-\theta)^{\alpha_d} = x_0\left(\frac{L_d^* + L_e^*}{L_d^* + (1-\mu)L_e^*}\right)^{1-\alpha_d} \tag{33}$$

They are equivalent to equation (33) and

$$\frac{\alpha_e}{\alpha_d}\theta^{-1}(1-\theta) = \frac{1-\alpha_e}{1-\alpha_d}\mu^{-1}\left[\frac{L_d^* + (1-\mu)L_e^*}{L_d^*}\right] \tag{34}$$

$$(x_0 - aX)\left[\frac{\alpha_e(1-\theta)}{\alpha_e - \theta(\alpha_e + \alpha_d)}\right](1-\theta)^{\alpha_d} = x_0\left(\frac{L_d^* + L_e^*}{L_d^* + (1-\mu)L_e^*}\right)^{1-\alpha_d} \tag{35}$$

θ^c and μ^c are determined by equations (34) and (35) and depend only on $(x_0, aX, \alpha_e, \alpha_d, {}^*_d, L_e^*)$. From equation (33) we see that S^c is a decreasing function of r_e. We have proved Proposition 4.

Proof of Proposition 5

Proof of (i): It is similar the one of Proposition 2.

Proof of (iii): Let $\widetilde{S}$ satisfy $x_0(L_d^* + L_e^*)^{1-\alpha_d}\widetilde{S}^{\alpha_d} = \widetilde{S}$. Then $\widetilde{S}$ depends only on (x_0, L_d^*). Let $\widetilde{r}_e$ satisfy equation (33) with $S = \widetilde{S}$, $\theta = \theta^c$ and $\mu = \mu^c$. It depends only on $(x_0, a, X, \alpha_e, \alpha_d, \beta, L_d^*, L_e^*)$. When $r_e < \widetilde{r}_e$, or equivalently, $S^c > \widetilde{S}$, any feasible path (S_t) from $S_0 < S^c$ will be less than S^c and is feasible for the convex technology $L(r_e, S) = x_0(L_d^* + L_e^*)^{1-\alpha_d}S^{\alpha_d}$. There exists a unique optimal path from S_0 and it converges to S^s. We have completely proved Proposition 5.

Before proving Proposition 6 we give some lemmas.

Lemma 14 *Assume* $\frac{L_e^*}{L_d^*} < \frac{1-\alpha_e}{1-\alpha_d}$. *Then there exists* $\bar{S}$ *such that if* $S > \bar{S}$ *then* $\mu_M(S) = 1$.

Proof. Assume the statement false. Then there exists a sequence (S_n) converging to $+\infty$ with $\mu_M(S_n) < 1, \forall n$. We may assume $\mu_M(S_n) \to \bar{\mu} \leq 1$

and $\theta_M(S_n) \to \bar{\theta}$. For short, write $\mu_n = \mu_M(S_n)$, $\theta_n = \theta_M(S_n)$. For every n, we have

$$x_0 + a(r_e S_n^{\alpha_e} \theta_n^{\alpha_e} \mu_n^{1-\alpha_e} - X)(1-\theta_n)^{\alpha_d}(L_d^* + (1-\mu_n)L_e^*)^{1-\alpha_d}$$

$$\geq x_0 + a(r_e S_n^{\alpha_e} \theta^{\alpha_e} \mu^{1-\alpha_e} - X)(1-\theta)^{\alpha_d}(L_d^* + (1-\mu)L_e^*)^{1-\alpha_d}$$

for every $\theta \in [0,1]$, every $\mu \in [0,1]$. This inequality is equivalent to

$$\frac{x_0}{S_n^{\alpha_e}} + a(r_e \theta_n^{\alpha_e} \mu_n^{1-\alpha_e} - \frac{X}{S_n^{\alpha_e}})(1-\theta_n)^{\alpha_d}(L_d^* + (1-\mu_n)L_e^*)^{1-\alpha_d}$$

$$\geq \frac{x_0}{S_n^{\alpha_e}} + a(r_e \theta^{\alpha_e} \mu^{1-\alpha_e} - \frac{X}{S_n^{\alpha_e}})(1-\theta)^{\alpha_d}(L_d^* + (1-\mu)L_e^*)^{1-\alpha_d}.$$

Let S_n converge to infinity. We obtain

$$a r_e \bar{\theta}^{\alpha_e} \bar{\mu}^{1-\alpha_e}(1-\bar{\theta})^{\alpha_d}(L_d^* + (1-\bar{\mu})L_e^*)^{1-\alpha_d}$$

$$\geq a r_e \theta^{\alpha_e} \ \mu^{1-\alpha_e}(1-\theta)^{\alpha_d}(L_d^* + (1-\mu)L_e^*)^{1-\alpha_d} > 0 \text{ if } \ \theta \in]0,1[, \mu > 0.$$

That implies $\bar{\theta} \in]0,1[, \bar{\mu} > 0$. But for every n we also have:

$$x_0 + a(r_e S_n^{\alpha_e} \theta_n^{\alpha_e} \mu_n^{1-\alpha_e} - X)(1-\theta_n)^{\alpha_d}(L_d^* + (1-\mu_n)L_e^*)^{1-\alpha_d}$$

$$\geq x_0 + a(r_e S_n^{\alpha_e} \theta_n^{\alpha_e} \mu^{1-\alpha_e} - X)(1-\theta_n)^{\alpha_d}(L_d^* + (1-\mu)L_e^*)^{1-\alpha_d}.$$

Since $\mu_n \in (0,1)$ we get the first order condition for μ_n:

$$a r_e \theta_n^{\alpha_e}(1-\alpha_e)\mu_n^{-\alpha_e}(L_d^* + (1-\mu_n)L_e^*) =$$

$$L_e^*(1-\alpha_d)[\frac{x_0 - aX}{S_n} + a r_e \theta_n^{\alpha_e} \mu_n^{1-\alpha_e}].$$

Let S_n converge to infinity. We obtain $\bar{\mu} = \frac{(1-\alpha_e)(L_d^*+L_e^*)}{(2-\alpha_e-\alpha_d)L_e^*}$. And $\bar{\mu} > 1$ if $\frac{L_e^*}{L_d^*} < \frac{1-\alpha_e}{1-\alpha_d}$. That implies $\mu_n = 1$ for any n large enough. ■

Lemma 15 *Let $S > S^c$. Assume $\frac{L_e^*}{L_d^*} < \frac{1-\alpha_e}{1-\alpha_d}$ and $x_0 - aX \leq 0$. Then $\mu_M(S) = 1$.*

Proof. To make short, write (θ_M, μ_M) instead of $(\theta_M(S), \mu_M(S))$. If (θ_M, μ_M) are interior we have the following first-order conditions

$$\theta_M^{\alpha_e - 1} \mu_M^{1-\alpha_e} \alpha_e (1-\theta_M) = \left[\frac{x_0 - aX}{a r_e S^{\alpha_e}} + \theta_M^{\alpha_e} \mu_M^{1-\alpha_e}\right] \alpha_d, \tag{36}$$

$$\theta_M^{\alpha_e}\mu_M^{-\alpha_e}(1-\alpha_e)(L_d^*+(1-\mu_M)L_e^*)=\left[\frac{x_0-aX}{ar_eS^{\alpha_e}}+\theta_M^{\alpha_e}\mu_M^{1-\alpha_e}\right](1-\alpha_d)L_e^*. (37)$$

If $x_0-aX\leq 0$, then from (37) we obtain $\mu_M\geq\mu^\infty=\frac{(1-\alpha_e)(L_d^*+L_e^*)}{(2-\alpha_d-\alpha_e)L_e^*}$. But if $\frac{L_e^*}{L_d^*}<\frac{1-\alpha_e}{1-\alpha_d}$, then $\mu^\infty>1$ and we have a contradiction. Since $\mu_M>0$, we must have $\mu_M=1$. ■

Observe that along an optimal path, we cannot ensure the supply of high-skilled labor be exhausted. The following lemma gives a condition for which that will be true.

Lemma 16 *Assume that the optimal path* (S_t^*) *converges to* $+\infty$ *and* $\frac{L_e^*}{L_d^*}<\frac{1-\alpha_e}{1-\alpha_d}$. *Then there exists* T *such that* $\forall t>T,\mu_t^*=1$.

Proof. That is a corollary of Lemma 14 ■

Proof of Proposition 6

Proof of (i): The proof that K_t^* converges to $+\infty$ is the same as in Proposition 3.

Proof of (ii):

When $x_0-aX\leq 0$ then from Lemma 15, we have $\mu_t^*=1$ for every $t\geq 0$. When $x_0-aX>0$, apply Lemma 16.

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