DEPOCEN
Working Paper Series No. 2009/21

War and Peace: an Economic Liberalist Assessment

N.M. HUNG*

* University of Laval, Quebec, G1K 7P4 Qc, Canada, email: nmhung@ecn.ulaval.ca.

WAR AND PEACE: AN ECONOMIC LIBERALIST ASSESSMENT

N.M. HUNG *

Preliminary version, comments welcome

09-2009

Abstract:

In a simple formal model of two-country, two-good with an elementary Conflict Technology, we use a rudimentary game theoretics to study the matter of war and peace, where under peace, cooperative exchange takes place, and where, in case of war, the winner takes all through appropriation of the whole endowment left after payment of armament expenditures. We provide conditions under which war is inevitable, then go on to characterize situations where war, still probable, is not necessarily the final outcome. In this case, cooperative exchange is profitable to both countries, and they should take this welfare enhancement into account in the determination of thei armament expenditures. This problem will be cast in terms of a two-stage game, the final stage is modelled as a Nash Bargaining solution with endogenous threat-point, while the precedent stage, aimed at the determination of armament expenditure, arises as a Bayesian Nash Equilibrum in the context of incomplete information. Using backward induction to yield the perfect equilibrium of the game, this paper concurs the liberalist view according to which economic consideration would enhance not war, but peace.

Keywords: War and peace, conflict, resource appropriation, Nash bargaining, Nash Bayesian equilibrium

JEL classification: F1, F12, C70, D23

---

* Universite Laval, Quebec, G1K 7P4 Qc, Canada. E-MAIL: nmhung@ecn.ulaval.ca
Acknowledgements: This work is dedicated to Murray Kemps for discussions and constant encouragement on the subject of war and peace. I have also benefited from many exchanges on this subject with S. Skaperdas at UC Irvine and Université Laval longtime ago. Part of my work has been disseminated in seminars at University Cadi-Ayyad (Morocco), Paris I (Pantheon-Sorbonne), Paris X (Nanterre). I wish to thank the participants in these institutions for helpful comments. Also, P. Gonzalez has clarified my thinking with a discussion on the concept of Nash Bayesian equilibrium. And last but not least, I am indebted to my friend and colleague N.V. Quyen for enduring research collaboration. However, needless to say, I will bear the responsibility of all shortcomings in this work.

I INTRODUCTION

Since immemorial time the world swings between war and peace in such a dramatic way that even while living in peace, no one could ignore the prospect of war. Memories of the First and the Second World War remain vivid, and military budgets continue to represent a considerable part of national income of many countries.

Why war [1]? The economic causes of war are perhaps the most popular in public opinion. War can be seen as an outgrowth of economic competition in a chaotic international system. As with the European colonial conquest three centuries ago, wars begin as a pursuit of new markets, of natural resources, and of wealth. Unquestionably as the main cause of some wars, for instance from the empire building of Britain to the 1941 Nazi invasion of the Soviet Union in pursuit of oil, this theory has been applied to many other conflicts. Economic competion among nations becomes increasingly ferocious because of scarce resource in a world of expanding population. This source of violence was the earliest expressions of the Malthusian theory of war, in which wars are caused by expanding populations and limited resources. Thomas Malthus (1766–1834) wrote in his famous *An essay on the Principle of Population* (1803) that populations always increase until they are limited by famine, disease, or war [2].

---

[1] Their causes may apparently be religious as with the Crusades, ideological as between the Communist and the Capitalist, racial as witnessed many ethnical conflicts in Africa, etc. The root of war, as suggested by the evolutionary psychology, is simply an extension of animal behavior such as territoriality and prey-predator competition, a decision making process quite rarely rational. Digging deeper into human psyche, S. Freud mentioned the complex relationship between *Eros* and *Thanatos*, according to which human swings between two basic instints, one is grounded on the principle of pleasure (life) and the other on the principle of destruction (death). Societies live with this tension, and war occurs as a manifestation when the instinct of death takes over the instinct for life, see the exchange between Einstein and Freud in 1931 (1971). Recently, Hungtinton (1993) suggested that the fundamental sources of conflict in this new world will not be primarily ideological or primarily economic. The great divisions among humankind and the dominating source of conflict will be cultural. The fault lines between civilizations will be the battle lines of the future. Notwithstanding these spychological and/or cultural theories, the cause of war according to the rationalist school lies in the fact that some countries cannot find a mutually beneficial bargain and instead ressort to war. Bargaining solution failed because of indivisibility, of information asymmetries, and the inability to make credible commitments. These problems, quite well known by game theorists, are now recognized in Political Science with emphasis on the study of International Relation.

[2] Recently, the Malthusian view has been supplemented by Youth bulge theory. Its adherents see a combination of large male youth cohorts (as graphically represented as a "youth bulge" in a population pyramid, a cohort of fighting age between 15 and 29 years old) with a lack of regular, peaceful employment

Despite the pessimistic Malthusian theory, surprisingly somewhat, we note almost at the same period the penetrating vision of the Classics about a harmonious and free world. Since the time the French philosopher Montesquieu, in his *Lettre à un Anglais*, used the term *doux commerce* to echo that trade among nations would reduce the likelihood of war, it is widely contended that mutually gainful exchanges will somehow enhance peace. He affirmed in the famous *De l'esprit des Lois* (1758) that "the natural effect of trade is to bring about peace. Two nations which trade together, render themselves reciprocally dependent…and all unions are based upon mutual needs". This vision has been later supported by the liberalist view in Political Science, which has its root in Kant's *Essay on Perpetual Peace*(1795) where it was passionately argued that, together with free trade, democracy regime in civil societies with lawful governance would enhance peace. Since these 18th and 19th century, classical liberals were faihtfully convinced that nothing could stop the movement toward economic freedom and political democracy. They concluded that there would be no more wars under a system of economic laissez faire and popular government. Wars become obsolete because the causes for war would disappear. What was needed to make the world safe for peace, they argued, was to implement economic freedom, free trade and goodwill among the nations, and democratic government. And therefore, ever since Adam Smith, economic doctrine has essentially focused on how people interact in time of peace. Even withstanding many wars in the past that resulted from disputes over national territories, these doctrines curiously envisioned a world where the claim to property rights over resources was not a subject of conflict. The Classics took peace for granted in a world of mutual respect of each other's rights, and where people and nations are free to pursue mutually advantageous trade.

Two centuries after these optimistic visions, the whole world got into successive wars. Although living in peace, almost all nations in 20th century should have considered the eminent prospect of war. In order to keep peace, paradoxically perhaps, they prepared war as a mean of dissuasion: arm race between the two blocs - the West and the Soviet-

---

opportunities as a risk pool for violence. While Malthusian theories focus on a disparity between a growing population and available natural resources, youth bulge theory focuses on a disparity between non-inheriting, "excess" young males and available social positions within the existing social system of division of labour.

Union - after World War II ended up to accumulate a total power that could destroy more than a hundred times the whole earth. Armament is costly and diverts the use of resources from productive purpose. Thus even in peace, the prospect of war significantly affects resource allocation, in particular the trade flows between nations and consequently the resulting gains from trade.

It was Kenneth Boulding (1963) who, fifty years ago, was the first to make the analysis of conflict an integral part of our theoretical construction. Using game theory, Thomas Schelling (1960,1966) followed this path and provided many analytical concepts to study several aspects of conflict. In economics, much of the motivation behind human conflict is the desire to appropriate the wealth of others. However, until the early nineties, little was done to pursuit Boulding-Schelling's lead. Only following the insightful work of Hirshleifer (1988) on *Conflict Technology*, the profession then witnessed a series of contributions: Garfinkel (1990), Grossman (1991), Skaperdas (1992), Grossman and Kim (1995), Grossman (1998), etc… In the same vein, the present paper attempts to study a number of unresolved issues and to shed some light on new problems [3].

The main theme of this paper echoes the liberalist view by showing that, except well specified condition, enduring peace is indeed a state of equilibrium in a world of economic rationality. A simple formal model of two-country, two-good with an elementary Conflict Technology, will be presented in the next section. The wording *country* can also read a *power bloc*, with reference to East and West during the Cold War, or else *ethnical groups,* keeping in mind civil wars in Yugoslavia, Rwanda, etc… for recent historical examples.

The model we propose in this work differs from earlier studies' in at least two respects. First, we replace the partial equilibrium framework of previous studies with the simplest possible general framework. Second, we make explicit use of game theory in modeling the strategic interactions among agents. In our model, part of one of the two

[3] There is a related literature on the problem of common property in resource economics; however, the focus in this strand was on the (over) exploitation of such a resource (common paturage land, fishing in international sea etc...) and disregarded complete appropriation of the property through violent conflict resolution.

commodities can be allocated for expenditure on armament. A country's probability of winning a war is an increasing function of its own expenditure on armament, and is a decreasing function of its opponent's expenditure. Armament expenditure, as a Nash equilibrium strategy, allows the determination the expected gain for each country. In case of war, the game ends with the draw of a lottery. We shall precise the conditions that enhance the occurrence of war. Whenever these conditions unfulfilled, peace will provide to both countries higher payoffs thank to exchange gains which result from cooperation. In this case, these armament expenditures constitute the credible threat-point payoff of the Nash bargaining game, the final outcome of a two-stage game. Prior to this stage of the game, countries at stake chose their armament expenditures in a non-cooperative way. Here, the game is one of incomplete information in that the ''type'' of a country - namely, its attitude with respect to the matter of war and peace - is private information. Of course, the Spartans who taught their children at the age of nine the military art, and the Athenians who raise their offspring in the atmosphere of music, painting, philosophy…belong to different ''types''. Spartans were warriors, of course, but with which probability would they initiate a war against their neighbor? The Athenians were rather peaceful, yet this does not exclude that they could defense themselves and might as well consider probably the adage that attack is in fact the best defense. These are what we call the ''types'', and use the framework of game with Bayesian players (Harsanyi (1967-68). This approach allows us to transform the game of incomplete information into one with imperfect information and solve for the Nash equilibrium armament in pure strategies. These expenditures, which constitute Bayesian Nash equilibrium, determine the endogenous threat-point consistent with the outcome of the ensuing Nash cooperative bargaining stage so that the resulting equilibrium arises as subgame perfect.

It turns out that how the gain from making war is located with respect to the utility frontier is of critical importance. If this gain from warfare - a lottery prize- is an element in the interior of the possibility utility set, then cooperative exchange which is mutually beneficial to all is possible. Armaments undertaken, however, constitute the effective threat-point for the stage of cooperation where peace will clearly be preferable to war. The gains from trade would be a *sine qua non* condition for peace. The main point we made is that, despite the causes of war might encompass many aspects of human society, the economic reason that could be pointed out as the cause of war is far from being the most prevalent. In contrary, economic rationality enhances peace rather than war, and there are always possibilities to promote peace through the use of economic instruments.

The paper is organized as follow. The next section II is devoted to specify elements in our model, and to recapitulate the problem of cooperative exchange under the classical framework where the consideration of conflict is absent. In section III, we offer an analysis of the determination of armament in the Nash non-cooperative equilibrium framework and discuss the conditions which enhance the occurrence of war. When this condition unfulfilled, we offer in section IV the analysis of our two-stage game. Using a mixture of cooperative and non-cooperative game in a static framework, we integrate through two stages the non-cooperative Bayesian Nash equilibrium which determines the "war" effort to a cooperative outcome provided by Nash Bargaining solution whenever cooperative exchange is profitable. In the first stage of the game, we determine armament expenditure for each country given its inference of the probability about the ''type'' of its opponent, and in the second stage, we consider trade between these countries as the cooperative outcome of a Nash bargaining game with the threat-point determined in the precedent stage. Finally, in the concluding section V, we discuss a number issue of interests under incomplete, namely the possibility of deter war, the issue of disarmament, the presence of a third-party affecting the belligerent behavior of countries eventually engaged in conflict.

II. BACKGROUND OF THE PROBLEM

Consider 2-country relationship in the context of barter exchange of 2 goods denoted by x and y for country U, and X and Y for country V. The preference of each country consists of two parts, one denotes its "type" exhibiting the attitude of this country with respect to the matter of war (and peace), and the other is the conventional preference defined on the commodity space. As for the "type" of a country, it is the realization of a random variable drawn from a probability distribution, and which is private information of each country. The ''type'' taken in this paper is simply a parameter measuring the willingness of a country to initiate war against its opponent. To facilitate the transformation of a problem of incomplete information into that of imperfect information along the line suggested by Harsanyi (op.cit), we wish to explicit the following assumption:

A-0: All the mentioned '' types'' are independent of countries' preference over the commodity space.

Thus, these ''types'' will not be involved in the intrinsic evaluation of countries' payoff, but intervene simply as the probability of war (or peace) perceived by the countries at stake. This assumption, quite innocuous as it seems, merits further clarifications later in Section IV.

The preference relation over the commodity space assumed to be continuous, strictly convex, strongly monotone, is represented by the utility functions:

A-1 : $U(x,y)$ (resp. $V(X,Y)$) continuous, strictly concave, increasing in x (resp. X) and y (resp.Y) where (x,y) (resp. X,Y) denote the consumption of country U (resp.V).

For a bilateral exchange relationship, country U disposes an endowment (x_0,y_0) and country V an endowment (X_0,Y_0). Assuming away free disposal, feasible exchanges in the 2-country framework satisfy:

$$x + X = x_0 + X_0 = \sum x, \tag{1}$$

$$y + Y = y_0 + Y_0 = \sum y, \tag{2}$$

where $\sum x$ and $\sum y$ denote the total endowment for exchange of each good.

Using the classical Edgeworth box, feasible exchange of good will be located on the *contract curve,* which maximize the utility of, says country U, subject to different fixed levels of its exchange partner's utility $V(.) = \overline{V}$. It is well known that along the contract curve - the locus *C-C* of allocations where the indifference curves are tangential as depicted in Figure 1 - the marginal rates of substitution are equal for all countries, i.e. $U_x / U_y = V_X/V_Y = \pi$. These allocations are Pareto optimal, that is, the increment of the utility level of one country can only be made in detriment of other's utility. Also, translated in term of utilities, the contract curve - also called *the core* of the exchange economy – depicts the utility frontier UF in Figure 2.

There are however infinitely many Pareto optimal allocations in *the core* of our bilateral exchange framework. Which allocation among them would be chosen? We opt in this elementary exposition the well known Nash solution concept of bargaining[4] that he himself qualified as an institutional assumption[5]. Nash solution is axiomatic and would solve the following:

$$\max_{\{x,y,X,Y\}} \quad [U(x,y)- U(x_0,y_0)]\ [V(X,Y)-V(X_0,Y_0)] \qquad \text{(PC)}$$

subject to the feasible constraints (1) and (2).

The vector $\{U_0=U(x_0,y_0),\ V_0=V(X_0,Y_0)\}$ constitutes the threat-point payoff (also called the status-quo) to which one country would fall back when it decides not to cooperate. The solution to (PC) is called the Nash Bargaining Solution satisfying the 5 axioms mentioned in footnote 4. First, recall that Nash showed that this solution is unique. Second, this solution is Pareto optimal. It corresponds to a point in the core such that the corresponding utility allocation on the UF satisfies the following equality:

[4] We now take the armament expenditures as *provisory* given in this section and highlight the Nash bargaining solution with fixed disagreement point. This point, depicted by O in Figure 2, is located in the interior of the UPS. What is the outcome of a bargaining problem? To this question, Nash (1950, 1953) proposed the Nash bargaining solution which satisfies 5 axioms on feasibility, invariance, efficiency, symmetry, and the independence of irrelevant alternative (IIA). It is now quite familiar that these axioms determine *uniquely* the Nash solution as the solution of the Nash Bargaining problem (PC).

[5] Among the five axioms, the IIA is the most controversial and some alternatives such as the monotonicity (Kalai- Smorodinsky, 1975) and the consistency (Binmore, 1987) have been proposed. Nash (1953) interpreted IIA as an « institutional assumption », but lately Binmore, from the strict game-theoretic viewpoint, consider it as a consistency condition applied to conventions (rules) rather than equilibria of the bargaining game. For suppose that this game is played in two stages, with a partial agreement realized at first, then followed by a full agreement at last. If the partial agreement is not a binding, the full agreement needs not be undertaken by any of the players. If, on the other hand, a full agreement is achieved, the provisions at this final stage must be consistent with those carried out under the preceding partial agreement. In a sense, this is in accordance with the spirit of perfect equilibrium applied to rules or conventions rather than to the outcome of the game. Subscribed to the mentioned axioms, the players proceed by communicating in (expected) utility terms with each other. These terms depend on one's dispatch of resource that is proposed to the other player in return of something in exchange. If these proposals lead to an agreement, trade takes place. But otherwise, each player sticks to the threat-point and no cooperative outcome would arise.

$$- dU(.) /dV(.) = [U(x,y)-U_0] / [V(X,Y)-V_0]$$

obtained from the first-order conditions for a maximum in solving (PC). Third, this solution has a simple geometric property (that we shall use later). Depicted as N - the slope of the tangent to the UF is equal to (minus) the slope of the ray connecting it to the threat-point O (angle α) as reported in Figure 2. And finally, and interesting enough, remark that any threat-point located along ON implements the same Nash solution N, a property helping us to clarify the process of disarmament in the last part of our work.

Until now, we have abstracted from eventual conflict between countries in many real situations. Historically, the objects of conflicts in most circumstances were disputes over resource (usually land in border region) due to political and ethnical reasons. If the conflict is somehow contained, dispute might be settled through pacific negotiation and business would be carried over as usual. But when a conflict will be degenerated into a war initiated either by one opponents, the normal trading relation comes into a halt. The final outcome of a war, as we put it in drastic terms, would be the winner takes all the endowment as the prize of war, while the looser gets none in the sense that it earns only a level of reservation utility level that we arbitrarily fix at zero.

There is no war without armament. We suppose as we did in the above that for country U only good *y*, the manufactured commodity, could be at no cost transformed into weapon. The other good, a pure consumption good, may be thought of as an agricultural commodity[6]. We denote the armament by *g* and do not distinguish offensive versus defensive arms. Of course, $g \leq y_0$, i.e. the armament *g* should not exceed the endowment y_0. Moreover, $g \geq 0$, where the level 0 of armament means that country U has just the capacity to enforce internal security and will not be able to make war against country V. In similar way, we define *G*, the armament of country V.

[6] It is possible to translate the model into production terms. Let land and labor be the factors, and land is fixed. Land is used only in the agricultural sector according to a Leontief technology, so labor allocated to agricuture is fixed. For country U, this would determine the initial endowment x_0 . The remaining of labor force in a country is devoted either the manufacturing production or to military effort, all in a one-to- one transformation. This would determine y_0 and *g*.

The conflict technology is given by the function $h(g,G)$ which assigns the probability of winning the war of country U. As for country V, the probability of winning the war is of course $[1- h(g,G)]$. It is assumed that[7]:

A-2: $0 \leq h(g,G) \leq 1$ for $0 \leq g \leq y_0$ and $0 \leq G \leq Y_0$ is continuously differentiable, with[8] $h_g > 0$, $h_G < 0$, $h_{gg} < 0$, $h_{GG} > 0$ and $h_{gG} \leq 0$.

The meaning of A-2 is straightforward. Given the armament expenditure G of its opponent, an increase in a country U's armament g increases its probability of winning the war. This marginal rate (h_g) rises with further armament, but at a decreasing rate ($h_{gg} < 0$) , and does not increase with an increase in country V's armament ($h_{gG} \leq 0$).

When country U and V have access to the same conflict technology the probability of a win is $h(.)= ½$ for $g=G$. Country U is relatively at least as efficient as V if $h_g \geq H_G$ where $H(g, G) = 1- h(g,G)$. This means $h_g + h_G \geq 0$ for any given level of armament g and G, which stipulates that the marginal increase in the win probability of country U through augmenting its armament g by one unity cannot be offset by its opponent's similar action with G. This requirement, exhibiting an asymmetry in Conflict Technology, plays only a role in assessing the uniqueness of Nash equilibrium (see footnote 10).

As we assume that in case of war, the winner takes all the endowment as the prize while the looser gets none, the expected prize (payoff) of country U is readily:

$$w(g,G) = h(g,G)\, U\left(\sum x, \sum y - g - G\right). \tag{3}$$

while that of country V is:

$$W(g,G) = [1- h(g,G)]\, V\left(\sum x, \sum y - g - G\right). \tag{4}$$

[7] Here, beside A-2, the assumption usually made in the literature on contests, (see for instance Dixit (1987)) is that $h\,(\,.\,)$ is a concave with respect to their arguments. Skapersdas (1996) provided a careful analysis of various forms of this function. Hirshleifer (1989) used the logistic convex-concave form relevant to military conception while Skaperdas and Syropoulos (2001), among many others, used a specific concave-convex form that we shall adopt for purpose of numerical computation.

[8] We adopt the convention that h_g means the partial derivative of h(.) w.r.p to g, h_{gG} the second order partial derivative w.r.p to g and G, etc.This convention will be hold throughout the paper.

This work aims at setting up a framework for the determination of armament expenditures *(g,G)*. At the outset, the requirement no country to make war against the other can be put as $w(g,G) \leq U(x_0,y_0)$ and $W(g,G) \leq V(X_0, Y_0)$. These conditions assess that the payoff of making war of a country is less than doing nothing and enjoying the status-quo payoff level. Assumed away this, and before tackling the question how theses expenditures are determined, mention that their cost in term of utilities, as a first order approximation, amounts to $-U_y\, g$ for country U and. $-V_Y\, G$ for country *V*. Moreover, when the armament expenditures - or war efforts - are the decisions of the countries involved, the size of the Edgeworth box are *endogenous* to the problem at hand. In Figure 1, the total endowment on the horizontal axis is reduced by *g* and *G*. This feature could also be captured in terms of utilities. For example, the origin O which denotes the levels of utility valued at the initial endowments of the two countries in Figure 2 is shifted to O^* with armament expenditures. The utility costs due to these expenses would bring the utility frontier UF inward and downward to UF^* as depicted.

We now have sufficient ingredients to carry out our analysis. Naturally, the first question to ask is under which condition war would be necessarily the sole outcome in a bilateral relationship. In the next section, we discuss the condition under which both countries would realize that going to war is inevitable: the bilateral relationship would end with the war prize occurred as a lottery to the take-all winner.

III. WAR OR PEACE, WHAT MATTERS?

What should be the armaments *g* and *G when* both countries believe that, because of some reason, war would clash regardless what is its own willingness? In fact, war necessitates just one country to initiate arm aggression, and for other country either self-defense or capitulation will be henceforth the inevitable consequence. In this section, we attempt to provide a sufficient condition for warfare.

Assume that each country in our model is risk-neutral. Naturally, these countries maximize their expected utility, and the Nash equilibrium in this non-cooperative context

- quite natural here - will be worked out. Thus, given the strategy G of its opponent, country U would:

$$\textit{Maximize}_{\{g\}} \; w(g,G) \qquad \text{(PWU)}$$
$$\text{subject to: } 0 \le g \le y_0.$$

In similar way, given the strategy g, country V would:

$$\textit{Maximize}_{\{G\}} \; W(g,G) \qquad \text{(PWV)}$$
$$\text{subject to: } 0 \le G \le Y_0.$$

Recalling the definition of $w(.)$ by (3) and $W(.)$ by (4). We now explicitly make the assumption[9] :

A-3: *w(.) (resp. W(.))* is strictly concave in the arguments *g (resp. G)*

It is easy to see that the payoff $w(.)$ does not increase with an augmentation in the armament of the rival country in war time, i.e. $w_{gG} \le 0$. Similarly with country V, $W_{gG} \le 0$. Any pair (g, G) that solves the problems PWU and PWV constitutes a Nash equilibrium in pure strategies. Assume that the solution to the maximization problem of country U is an interior solution, the best reply of country U (BRU) is given by the first-order condition of PWU:

$$w_g(g,G) = 0 = h_g(g,G)\, U\left(\sum x, \sum y - g - G\right) - h(g,G)\, U_y\left(\sum x, \sum y - g - G\right) \qquad (5)$$

Similarly, the best reply of country V (BRV) is:

$$W_g(g,G) = 0 = -\, h_G(g,G) V\left(\sum x, \sum y - g - G\right) - [1 - h(g,G)] V_Y\left(\sum x, \sum y - g - G\right) \quad (6)$$

As usual, solve the system (5) and (6) to obtain the Nash equilibrium armament strategies g^w and G^w. It is not surprising that this equilibrium exists and is unique[10].

---

[9] We can make use of A-1, A-2 and find conditions to guarantee A3, but this is rather cumbensome and not quite necessary for the purpose at hand.

[10] Our problem existence and uniqueness of Nash equilibrium in pure strategies here is the same in the literature on rent-seeking and on contest (see Tullock (1980), Dixit (1987). In our present study, g and G is contained in a compact and convex set, thus warranting the existence of the solution to the system (5) and (6). Given A-2 and with the assumption that one country, says U, is *at least as efficient* as V's (i.e. $h_{g+} h_G \ge 0$), the slope of the BRU is negative in the plan $(g,G) \in R^{+2}$. For, differentiate totally (5) and rearrange to get $dg/dG\big|_{BRU} = -[\, h_{gG}U - (h_g + h_G)\, U_y + hU_{yy} \,] \,/\, [\, h_{gg}U - 2h_g U_y + hU_{yy} \le 0$. Likewise, differentiate totally (6)

At the outset, note that the allocation (U^0,V^0) corresponds to $g = G = 0$ where no offensive is possible, henceforth peace is necessarily the outcome. Let us now turn to an important question, namely under which conditions war would inevitably occur. First, war occurs when one country initiates it, the other has no choice but defense itself. Second, since making war needs costly armaments, the Utility Frontier shifts inward and downward. Third, war will be envisaged only if either $w(g^w,G^w) \geq U^0$ or if $W(g^w,G^w) \geq V^0$, i.e. war is profitable for at least one country when compared to the status quo. Fourth, the core defined by utility levels equal to $w(g^w,G^w)$ and $W(g^w,G^w)$ must be empty. In this case, quite obviously, no room for cooperative trading that are Pareto improving for each country exists. In Figure 1, the level of utility of country U, armed with g^w, is $w^w = U^2$. If the level of utility of country V, armed with G^w is $W^w = V^1$, the core is empty. In this case, the corresponding utility frontier is plotted as UF^w in Figure 2, with the utility allocation $O^w(g^w, G^w)$ lies outside UF^w. Since this allocation lies inside UF, the frontier plotted with g = G = 0, is it still possible to gain from cooperative exchange by deviating g (and G) from g^w (and G^w)? The answer is obviously no if war certainly occurs and the belligerent countries already committed to war strategies. But since O^w lies inside UF, both countries might as well recognize that potential gains from cooperative exchange are possible. As a result, these countries may refrain from making war. For instance, they may reduce armaments g and G so as to induce the shifting of the Utility Frontier outward, creating the opportunity to gain from cooperation. This renders the "no trade but war" here not an equilibrium. War in this case is only probable, not sure.

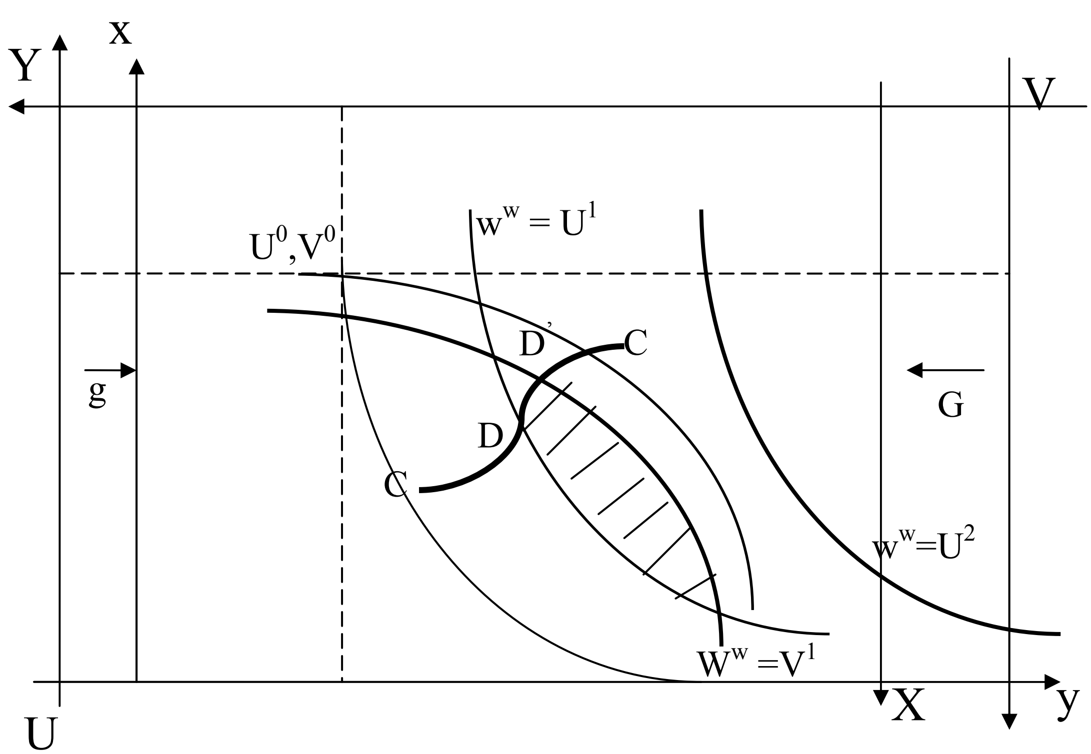
We now present the case where war is inevitably a sure outcome. In Figure 2, we depict the allocation O^w lying outside UF. To be precise, let us define U_{max}= max U (.,.) subject to V= V(X_0, Y_0), and similarly V_{max} = max V (.,.) subject to U = U(x_0, y_0). This ''no trade but war'' situation occurs if either one or the other country has a dominant payoff, i.e. either $w(g^w, G^w) \geq U_{max}$, or $W(g^w, G^w) \geq V_{max}$. Nash strategies (g^w, G^w) solving our system (5) and (6) constitute the equilibrium strategy. The utility allocation

---

and rearrange, we obtain the slope of BRV as $dG/dg\,|_{BRV} = [\, h_{gG}V-(h_g+h_G)\, V_Y+(1-h)V_{YY}\,] \,/\, [-\, h_{GG}V+2h_GV_Y +(1-h)V_{YY}]$. If V_{YY} is negligible in the denominator, the slope of BRV is also negative. Since all the best replies BRU and BRV have different negative slopes in the plan (g,G)$\in$ R^{+2}, they should intersect once, warranting therefore the uniqueness of Nash equilibrium when it exists. This fact is illustrated by a numerical illustration given in the Appendix.

Figure 1

Edgeworth box with armament



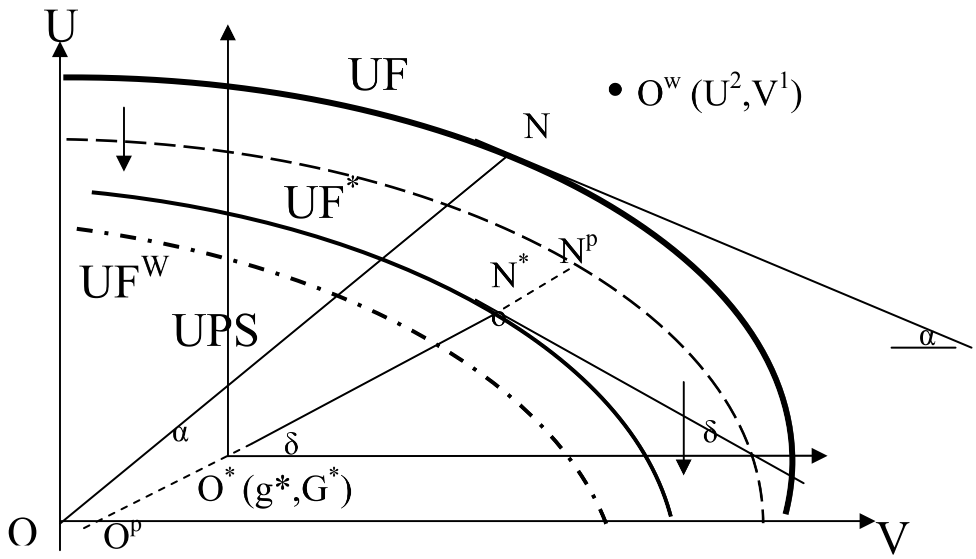
Armament expenditure: g and G.

DD': the core is non empty with $W^w = V^1$ and $w^w = U^1$, giving rise to gains from trade.

The core is empty with $W^w=V^1$ and $w^w = U^2$: no trade but war.

Figure 2

Nash bargaining solution with endogenous threat



O^w (U^2,V^1) lies outside the Utility possibility set (UPS): war is inevitable.

Utility frontier UF with zero armament expenditures;
Nash bargaining solution N lying on UF as the first best utility allocation.

UF^* with G^* and g* giving rise to the endogenous threat-point O^*.

N^*, lying on UF^*, is the equilibrium outcome with armaments G* and g*.

N^p is the outcome achieved by a proportional disarmament

$O^w(U^2,V^1\ )$ depicted in Figure 2 lies outside the Utility Frontier UF, thus *a fortiori* outside UF^w plotted for g^w and G^w. Remark that the payoff $w(\ g^w\ ,\ G^w\ ) \geq U_{max}$ is dominant among all feasible payoffs for country U, and the resulting Nash equilibrium in this case is said *dominant* with respect to at least one country. Also, when a utility allocation is on UF, it belongs to the *first-best* optimal allocation. Otherwise, to use the familiar terminoly in Welfare Economics, this allocation is of the *second-best* optimum type. Note finally that the threat-point to which any country would fall back if cooperative exchange denied is respectively the utility level w^w *and* W^w. This is not the initial utility obtained with initial endowments U_0 and V_0 specified in Section II.

What would be the reasons which delimit war versus peace in the taxonomy just mentioned? Imagine that countries U and V are identical in all respects, i.e. the initial endowment, the utility function and the conflict technology. Obviously, $g^w = G^w$ and the probability of winning the war is ½ for both countries. Assume that the utility function is non-increasing return to scale, it follows that $w(g^w,G^w)= ½\ U(2x_0\ ,2\ y_0 - 2\ g^w) \leq U(x_0\ ,\ y_0 - g^w)$ which stipulates that both countries are worse of in attempting to grasp the war prize. Note immediately that the core in this case is not empty. Thus, the perfect symmetry among the involved countries precludes warfare. From this observation, the conditions that induce war among nations necessitate some kind of basic asymmetries among the characteristics of the countries.

Which kinds of asymmetries would likely induce warfare? For country U, the expected gain from making war is $h(g^w,G^w)\ U(\ X_0\ ,Y_0 - G^w\ )$ while the expected loss is $(1-h(g^w,\ G^w)\ )\ U(x_0\ ,\ \ y_0\ -g^{\ w}\ )$. War is profitable for country U when the former exceeds the latter. The first element favorable to country U in case of war is obviously the efficiency of its conflict technology relative to its opponent's, a concept that we have mentioned earlier. The second element consists of its capability to produce arms, precisely higher its initial endowment of y_0, higher would be g^w as compared to G^w, and this enhances the probability of winning. The third element, which amounts to the same effect, is the intensity of preference of good *y* (i.e. its relative share in terms of utility): lower this intensity, higher would be the incentive of country U to intensify its armament expenditure g^w. In this case, if the intensity of preference with respect to good x is relatively high, and if country U initial endowment of this good is small, this would inflate the expected gain from warfare.

A combination of these four above mentioned elements is determinant to the matter of war or peace in a bilateral relationship. Imagine that U endows with a highly efficient Conflict Technology, a large amount of good *y* capable to be transformed into guns, a little amount of good *x*, say land, which allows the production of corn and butter, and which is much needed (or preferred intensively), it is hard to believe that this country will promote peaceful exchange rather than war! In the appendix to this paper, we report the determination of g^w and G^w with Cobb-Douglas preference and a quite familiar Conflict Technology used in earlier literature, and some computation to highlight the discussion above.

Let us now summarize our discussion:

1- For a two-country bilateral relationship, war is inevitable only if the utility allocation O^w evaluated at the maximal Nash armament expenditures g^w and G^w as solutions to (5) and (6) lies outside the Utility Possibility Set UPS. This requires that either $w(g^w, G^w) \geq U_{max}$, or $W(g^w, G^w) \geq V_{max}$*, or both. Nash equilibrium in this case exists, is unique and is dominant for at least one country.*

2- War outburst requires drastic asymmetries between the countries at stake with respect to their conflict technology, their preference characteristics, and their resource initial endowments

At this point, it is worthwhile to discuss the possibility of war deterrence. The likehood of war and peace will be affected by introducing the dimension of **war casualties** into the Conflict Technology. The simplest way consists of assuming that war destroys utilities of both belligerents, thus rising the costs of warfare and discouraging consequently the willingness to undertake conflict. We will not elaborate further on the issue, except to mention that if the sacrifice of lives is taken into account as an important casualty, the delicate problem of valuation of a human life will be at stake.

Now, when conditions for warfare are fulfilled, what actions one party could take to deter an eminent conflict? Assume that country V is disadvantageous in war. In Figure 1, the utility level of U is U^2 while that of V is V^1, thus the core is empty and the utility allocation is located at O^w in Figure 2. How country V would do to affect the incentive of country U to initiate war resides in making gift, or in destroying its own endowment. Assume for simplicity that gift or resource destruction concern only good X which does not allow making guns. Consider the example of Cobb Douglas Utility that we report in the Appendix. Consider **a gift** δX that V gives to U. The probability of winning the war h(g, G) is unchanged with this example; therefore, for country U, by accepting the mentioned gift the increase in the expected loss is (1-h(.)) [δX U_1 $(\sum x, \sum y - g - G)$] and the decrease in the expected gain is h(.) [δX U_1 $(\sum x, \sum y - g - G)$] . Overall, the gift of country V would shift the utility level U^2 in Figure 1 in the direction South - East, henceforth enhances the possibility to reestablish the gain from cooperative exchange. Alternatively, consider the **endowment destruction** as a mean to discourage warfare. Assume that country V simply destroys its endowment X_0 in the amount δX. The decrease in the expected gain of country U is h(.) [δX U_1 $(\sum x, \sum y - g - G)$] while its expected loss is left unchanged. Again, the utility level U^2 shifts in the same direction South-East, but at a relatively smaller extent, thus not as effective to deter war as the use of gift. Also, another reason to prefer gift to endowment destruction is that under cooperative exchange that follows the action of deterring war, the gain of both countries might be larger than definitely forgo scarce resources.

Let us now consider the case that war is not inevitable, that is the allocation O^w is in the interior of the UPS. In Figure 1, if the level of utility of country U is rather $w^w = U^1$, the core in this case is DD': gains from trade for both countries are possible. In this case, war will not necessarily prevail. This is once again the possible "not war but trade" situation. In this case, the utility allocation O^w (U^1, V^1), lies inside the Utility Frontier UF^w drawn with the armament expenditures g^w and G^w. Once cooperative exchange takes place as a Nash Bargaining solution, the utility allocation will be able to reach at the least a point on the UF^w contour. In fact, taking the cooperative exchange opportunity into account, it will be demonstrated that the countries at stake can even do much better by lowering the armament expenditures from the level g^w and G^w.

IV NOT WAR BUT TRADE

Whenever conditions rendering war inevitable is not fulfilled, that is when the core defined for the utility levels for $w(g^w,G^w) \geq U^0$ and $W(g^w,G^w) \geq V^0$ *is non empty*, there are paces for trade. Such cooperative behavior does not, however, exclude *ipso facto* armament expenditure beyond the minimum level that warrants only internal law and order in a country. Although war is evitable, peace is nevertheless not a sure outcome; therefore the prospect for violence outburst could not safely be ignored. *Si vis pacem, para bellum.* The adage according to which prepare war is the best way to safeguard peace seems to be the rule, not exception. Learned from multiple lessons of history where many wars in the past clashed under sometime random shocks, each country will still, conditional to its ‘’type’’, attach some probability that war might occur and acts consequently in building up their armaments.

In this section, even country U and country V devote their maximum efforts in armament g^w and G^w, the corresponding threat-point O^w lies inside the UPS. Clearly, a cooperative arrangement might lead to an allocation that is a Pareto improvement for both countries. But if war is not a certain event, yet it is still probable. Because of cultural characteristics which laid down the “type” of a country and which is private information, each country might perceive the ‘’type’’ of its opponent with some probability: uncertainty regarding the eventuality of war (and peace) still persists. Within this context, the analysis should be carried out under the condition of *incomplete information.*

We assume that country U (resp. V) does not know the ‘’type’’ - a preference characteristic- of country V (resp. U). The ‘’type’’ t_i is a discrete random variable defined over the closed interval [0,1], where 0 means the warrior ‘’type’’, and 1 the peaceful ‘’type’’. In between, for example $t_i = 0.3$ means that war would occur with probability equal to 0.3, and complementarily, peace with probability $1- t_i = 0.7$. Following Harsanyi (op.cit), assume that “nature”, a new player, who chooses the ‘’type’’ t_U of country U (resp. t_V of V) and inform this ‘’type’’ only to U (resp.V), not to its opponent country V (resp.U). The ''type'' t_i is a *a priori* drawn from a joint distribution $F(t_1, t_2,...,t_i ,...,t_N)$ which is a common knowledge for all countries. Country U (resp. V) knows its ''type'' t_i, thus obtain the inference of the conditional probability $F(t_{-i} \mid t_i)$ on the ''type'' of country V (resp. U), where $t_{-i} \equiv (t_1, t_2,...,t_{i-1} , t_{i+1} ...,t_N)$ [11]. This would inform country U (resp. V) the probability of war (and peace) initiated by country V (resp, U). It is worth recalling assumption A-0: the ''types'' at stake do not affect intrinsically countries' payoff. They intervene as uncertainty parameters in a simple problem of imperfect information.

We assume that for country U (resp.V) this conditional probability on the ''type'' of its opponent takes on the value μ (resp. η), meaning to U that war would occur with probability μ (resp. η) , and peace with probability (1-μ) (resp. 1- η). Using Bayes' Rule, we get:

$$\mu = F(t_{-i} \mid t_U) = F(t_1, t_2,...,t_i ,...,t_N)/ t_U.$$

Similarly, for V the probability of war is:

$$\eta = F(t_{-i} \mid t_V) = F(t_1, t_2,...,t_i ,...,t_N)/ t_V .$$

Given both μ and η, the decision making of the two countries might be casted as a problem under imperfect information where the approach commonly adopted is the Morgenstern-Von Neumann's . The two extreme cases are the following. If both μ and η are one, war must be the outcome in the bilateral relationship, the analysis is already reported in the previous section. If both μ and η are zero, peace prevails and the resource allocation will be that of the classical exchange model.

We now look for the solution in the case where $0 < \mu < 1$ and $0 < \eta < 1$. The game considered in this section encompasses two stages. The final stage, one where cooperative exchange may arise with probability 1-μ and 1-η, would be preceded by a non-cooperative first stage where the involved countries determine their armament. Prepare war in order to enforce peace seems to be well rooted.. In this context, for

[11] Although Harsanyi (1967-68) was the pionner in exploring this game under incomplete information, Krep and Wilson (1982) went further with their concept of sequential equilibium, allowing the study of many interesting subjects in Industrial Organization such as Signaling, Reputation,etc. On these subjects, see J. Tirole (1988), or A. Mas-Colell, M D. Whinston and J R. Green (1995) for more advanced materials.

country U (resp.V) the armament expenditure *g* (resp. *G*) should be determined so as to take into account the potential cooperative allocation arising with positive probability in the ensuing final stage. In this way, these armament expenditures constitute the threat-point of the ensuing cooperative stage captured as a Nash bargaining solution. Conceive a two-stage game by mixing the cooperative to a non-cooperative framework and solve the problem by backward induction, the equilibrium strategies that stem from our model are *subgame perfect*. Note that the threat-point in the cooperative Nash bargaining game - being determined on the sole basis of self interest of each country - is endogenous to the whole game. Also, note that decisions of the countries at stake are *simultaneous*: there is no possibility for any one to observe the decision made by the other and profit from a gain in information in making its own decision[12].

4.1 The cooperative Nash bargaining solution

In this stage of cooperation, we continue to use the Nash bargaining solution. Formally, given the armament g and *G* determined in the first stage, we have to solve the following problem:

$$\max_{\{x,y,X,Y\}} \quad [U(x,y)- w(g,G)]\,[V(X,Y)-W(g,G)] \qquad \text{(C)}$$

subject to:

$$x + X \;=\; x_0 + X_0 \;=\; \sum x\,,$$

$$y + g + Y + G \;=\; y_0 \;+\; Y_0 = \sum y\,,$$

where the constraints are those of feasibility, and according to the definition given by (3) and (4), we recall that $w(g,G) = h(g,G)\, U(\sum x, \sum y - g - G)$, and $W(g,G) = [1- h(g,G)]\, V(\sum x, \sum y - g - G)$. Note that $w(g,G)$ and $W(g,G)$ are the fall back level of utility which are secured when peace would not prevail. These threat-point payoffs are credible in that they require $w(g,G) \geq U_0$ and $W(g,G) \geq V_0$ where *g* and *G* are determined at the first stage of the game, and where U_0 (resp. V_0) denotes the utility level of country U (resp.

[12] When updating information is framed in a dynamic setting, the appropriate concept to use is the sequential equilibrium (Krep and Wilson (op.cit)).

country V) obtained with its initial endowment. Solving problem C for $\{x,y,X,Y\}$ as the functions of g and G, we write the reduced form $U(x(g,G), y(g,G))$ as $U(g,G)$ for country U (resp. $V(g,G)$ for country V).

Given μ and η, we first solve the problem of Nash bargaining (C). Substitute $X = \sum x - x$, and $Y = \sum y - y - g - G$, the necessary conditions for an interior solution are:

$$U_x [\, V(\sum x - x\,,\ \sum y - y - g - G) - W(g,G)] - V_X [\, U(x,y) - w(g,G)\,] = 0, \qquad (7)$$

and

$$U_y [\, V(\sum x - x\,,\ \sum y - y - g - G) - W(g,G)] - V_Y [\, U(x,y) - w(g,G)\,] = 0. \qquad (8)$$

These two equations allow us to determine the net demand x (and resp. y) as a function of g and G, both of which are taken as given in solving the problem (C). From (7) and (8), it is immediate that $U_x/U_y = V_X/V_Y = p$. This confirms that the marginal rate of substitution of good x for good y is the same in two countries, a well known Pareto optimality condition for a barter exchange characterizing the Nash bargaining solution. Goods are normal, and with positive income effect, it follows that $\partial x(g,G)/\partial g \leq 0$ for the demand $x(g,G)$[13]. Likewise, $\partial X(g,G)/\partial G \leq 0$ for the demand $X(g,G)$. Using the condition that net demand equals its supply in barter exchange (the feasibility constraints in problem (C) are binding), it follows readily that $\partial x(g,G)/\partial G \geq 0$ and $\partial X(g,G)/\partial g \geq 0$. Repeat *in verbatim* the above, we obtain also $\partial y(g,G)/\partial g \leq 0$, $\partial y(g,G)/\partial G \geq 0$, and $\partial Y(g,G)/\partial G \leq 0$, $\partial Y(g,G)/\partial g \geq 0$. It follows readily that the payoff function $U(g,G)$ obtained through cooperative exchange is decreasing in g and increasing in G. Respectively, the function $V(g,G)$ is decreasing in G and increasing in g. We can write the reduced form of the expected payoff in the cooperative game at the final stage as:

$(1-\mu)U(x(g,G), y(g,G)) = (1-\mu)U(g,G)$, with $U_g \leq 0$ and $U_G \geq 0$,

and

$(1-\eta)V(X(g,G), Y(g,G)) = (1-\eta)V(g,G)$, with $V_g \geq 0$ and $V_G \leq 0$,

[13] Heuristically, the marginal rate of substitution plays the role of relative price p with good y taken to be the numeraire, the total income is $m - g = px + y$ for country U (resp. $M - G = pX + Y$ for country V). An increase in g (resp. G) is a decrease in disposable income, thus induces a negative effect on the demand for x (resp. for X), a normal good.

where μ (resp. η) is the probability of war perceived by country U (resp. country V), and where g and G are the armament expenditures determined at the first stage of the game. If these expenditures are g^* and G^*, the corresponding threat-point payoffs O* lying inside the feasible utility frontier is UF*. In this case, the Nash Bargaining solution is N*. Since UF* lies inside the utility contour UF, the allocation N* is a *Second Best* allocation. But how g^* and G^* are determined ? This task will be undertaken next.

4.2 Non-cooperative gaming over the armament strategies

We make use in this stage of the game the concept of Bayesian Nash Equilibrium, which gives rise to the set of ''type''-contingent choice such that each player maximizes his expected utility of his own ''type'', taking the other player's ''type''- contingent action as given. The problem for country U is therefore to find the value g^* so as to get his maximum expected payoff, given G. To recall, we have assumed that country U attaches the probability μ, and country V attaches η, to the ''type'' of their opponent. These would dictate the probability of occurrence of war as perceived by these countries. We wish to recall assumption A-0 according to which the ''type'' is independent of the utility function U(x,y) and V(X,Y). It is therefore also independent of $U(g,G)$, $V(g,G)$, $w(g,G)$, $W(g,G)$. However, the ''type'' would intervene as uncertainty parameter concerning the matter of war and peace. For country U, the probability of war is μ and peace $(1-\mu)$. The expected payoff of country U amounts to $\mu\, w(g,G) + (1-\mu)\, U(g,G)$. Given G, this country should find g^* in order to:

$$\text{Maximize}_{\{g\}}\ [\mu\, w(g,G) + (1-\mu)\, U(g,G)]$$
$$\text{subject to : } 0 \le g \le y_0 . \qquad \text{(NU)}$$

Similarly, given g, the problem of country V is to find G^* so as to

$$\text{Maximize}_{\{G\}}\ [\eta W(g,G) + (1-\eta) V(g,G)] \qquad \text{(NV)}$$
$$\text{subject to } 0 \le G \le Y_0 .$$

To warrant that problems NU and NV have solution, we make the following assumption:

A-4. The payoff function $U(g,G)$ is twice differentiable and concave in g, and $V(g,G)$, also twice differentiable and concave in G. Moreover, $U_{gG} \leq 0$ and $V_{gG} \leq 0$.[14]

We now turn to solve the problem (NU). Since the expected payoff of country U is a linear combination of $U(g,G)$ and $w(g,G)$, A-3 and A-4 assures that this function is concave in g. Given the strategy G of its opponent, country U must find therefore the strategy g^* in view of maximizing its expected payoff. For an interior solution, the necessary condition for this maximum is:

$$\mu\, w_g(g^*,G) + (1-\mu)\, U_g(g^*, G) = 0, \qquad (9)$$

where w_g satisfies equation (5). This is the best reply for country U.

Similarly, for country V, it must find G^* so as to maximize $\{\eta\ V(g,G) + (1-\eta)W(g,G)\}$. The necessary condition for an interior solution to this problem reads:

$$\eta\ W_G(g,G^*) + (1-\eta)V_G(g,G^*) = 0, \qquad (10)$$

where W_G satisfies equation (6). This is the best reply for country V.

The pair $\{g^*,G^*)$ is the Nash equilibrium strategy of the non-cooperative game in this subsection if it satisfies the best replies (9) and (10). Under A-4, this equilibrium exists and is unique [15]. Note also that since the expected payoff functions here are bounded, (strictly) concave in terms of the strategies defined over a compact and convex subset, existence of Nash equilibrium strategies is not, as usual, particularly problematical.
From (9), we get $w_g(g^*,G) = (-(1-\mu)/\mu)\ U_g(g^*,G) > 0$. Recalling that $w(g,G)$, defined by (3), attains a maximum at g^w where $w_g(g^w,G) = 0$. It follows immediately that $g^* < g^w$.

[14] For purpose of simple exposition, we assume such concavity instead of working out conditions that involve second-order derivatives of the functions $x(g,G)$, $y(g,G)$, etc...This is also the reason that we also assume the cross-derivative U_{gG} (resp. V_{Gg}) is non positive. Note that (U_g) is the marginal cost of armament in country U. The latter assumption means that this marginal cost does not decrease when the armament of the rival country increases, a rather reasonably natural property.

[15] To the slope of the best reply $dg/dG\big|_{BRU}$ given in footnote 10, we now add $(1-\mu)/\mu\ [U_{gg}/U_{gG}]$ whish is negative thanks to A-4 to obtain the slope of the best reply (9). Thus, this slope is still negative. Repeat this operation to obtain the slope of the best reply (10), we add $(1-\eta)/\eta\ [V_{GG}/V_{gG}]$ which is also negative to $dG/dg\big|_{BRV}$ given in footnote 10. Since the slope of these best replies is all negative, we can immediately conclude on the uniqueness of Nash equilibrium if it exists.

It is easy to see that if μ = 1, then $g^* = g^w$, and if η = 1, $G^* = G^w$. On the contrary, if μ = 0, then $g^* = 0$, and if η = 0, $G^* = 0$.

We wish to turn back to Figure 2. The solution to the two-stage game in this section is depicted as N^* which represents the final stage Nash Bargaining utility allocation consistent with the choice of armament g^* and G^*, choice made in the first stage as a Nash Bayesian equilibrium under incomplete information. The threat-point O^* to which the involved countries might fall back would cooperation be denied is in fact endogenous. Using backward induction in solving the problem, the equilibrium trategies are subgame perfect.

Note that g^* and G^* are functions of μ and η, written in a short- hand notation as $g^*(\mu, \eta)$ and $G^*(\mu, \eta)$. It is not hard to see the effect on g^*(resp. G^*) when there are some change in the probability assessment μ and η: higher the perceived probability of the occurrence of war (resp. peace), higher (resp. lower) would be the armament expenditures[16]. And conversely.

To recapitulate our discussion, let us sum up:

1- In the framework a bilateral relationship which might lead to a warfare, when there are paces for trade between countries, and under incomplete information, the determination of armament expenditure and exchange result from a two-stage game: at the first stage, the involved countries determine their armament expenditure as a Nash Bayesian equilibrium, then at the second stage, they engage in a cooperative arrangement according to the Nash bargaining solution. The solution for the whole game constitutes an unique perfect equilibrium.

---

[16] First, differentiate totally (9) and obtain, after some manipulations,
$\frac{\partial g^*}{\partial \mu} = -\frac{w_g - U_g}{\mu U_{gg} + (1-\mu) w_{gg}} = [\ \frac{1}{1-\mu}\ ] \frac{U_g}{\mu U_{gg} + (1-\mu) w_{gg}}$. Recalling that $U_g < 0$, and $U_{gg} \le 0$, $w_{gg} \le 0$ according to A4, we get $\frac{\partial g^*}{\partial \mu} > 0$. Now, proceed similarly to get, for any G,
$\frac{\partial G}{\partial \mu} = -\frac{w_g - U_g}{\mu U_{gG} + (1-\mu) w_{gG}}$. Thanks to A-4, we immediately yield $\frac{\partial G}{\partial \mu} > 0$ and therefore, a fortiori, $\frac{\partial G^*}{\partial \mu} > 0$. We proceed similarly with (10) to obtain $\frac{\partial g^*}{\partial \eta} > 0$ and $\frac{\partial G^*}{\partial \eta} > 0$

2- The armament expenditures depend on the probability of the occurrence of war and peace perceived by all countries. The higher the probability μ (resp. η) with regard to he occurrence of war made by country U (resp. V), higher will be the armament expenditures in both countries g^* (resp. G^*) and G^* (resp. g^*). And conversely.

At this point, we wish to mention an earlier study (Skaperdas, op.cit) quite reminiscent of the present, but focused on the one-dimensional case and complete information. Consider two agents, each possesses one unit of resource. Under full cooperation, these resources are put together in the production of a joint output which will be equally shared by the agent. However, these agents may use their resource – the armament expenditures - to appropriate the joint product with a Conflict Technology given by A-2. The effectiveness of this technology is proportional to the marginal effect of armament on the probability of winning. Considering the Nash pure strategies simultaneously chosen by risk neutral agent, it has been shown that the outcome – called *conflict equilibrium* - where all resources are devoted to conflict is unique either if the conflict technologies effectiveness is high enough, or if the marginal contribution of the agents in the joint production is almost the same. Otherwise, the equilibrium outcome is one of *partial cooperation*, with only one agent who commits a part of its resource to armament, while the other agent does not. And when the conflict technologies are sufficiently ineffective, *full cooperation* is shown to be also Nash equilibrium. In contrast to the one-dimensional case, our two-good framework with incomplete information showed that a full cooperation – a first best optimality - could not be achieved. On the other hand, our partial cooperation - a second best optimality - is likely the outcome in most circumstances. This forcefully stipulates that, on contrary to the popular belief according to which economic reasons are the main cause of warfare, the economic rationality might save the world from violence and destruction. For the event of war which in some extreme case is inevitable as shown in our section III, our framework enables to highlight the possibility of war deterrence. Also, we can use our framework to discuss the problem of disarmament and other issues of interest in the sequel.

V TO CONCLUDE: SOME DIGRESSIONS

In concluding, we would like to discuss first the **problem of disarmament.** At this point, we wish to recall the equilibrium strategies g^* and G^* obtained above as a *partial cooperation* outcome to follow Skaperdas's terminology (1992). The corresponding utility allocation, depicted as N^*, lies inside the utility frontier NF which is of *first-best* kind and sustained by zero armaments that corresponds to a full cooperation. The question we address in the sequel is how, and to what extent, the full cooperation would arise as equilibrium game strategies in our framework.

Full cooperation outcome, arising when the belief about the other that one country forms amounts to $\mu = \eta = 0$, is exactly the Nash bargaining solution N that we have depicted in Figure 2. This corresponds to the classical cooperative solution in Edgeworth bilateral barter exchange framework where the consideration of conflict is assumed away. However, there is no *a priori* reason to support the mentioned belief of a peaceful world. If it might well be that to $\mu \neq \eta \neq 0$, the equilibrium outcome is N^*, with armament expenditures $g*$ and G^*. Nevertheless, since war is not inevitably the outcome, and armament unproductive and costly, each involved countries over a long period might find profitable to revise their probability belief about the ''type'' of its opponent and proceed to disarmament. How this can be done? Is it possible to reach the classical cooperative solution N?

The revision of belief points to a decrease of the value of μ and η. This process gradually discards the occurrence of war, therefore the armament expenditures should also decrease. As a result, the Utility Frontier shifts outward and upward, for instance from UF^* to UF^p as depicted in Figure 2. This process of disarmament, as we shall see, poses some problems, however. Let O^* depict the payoff at the threat-point with the armament g^* and $G*$ from which disarmament takes place. Now, we may use one interesting property of a Nash Bargaining solution, namely any allocation on the ray connecting the threat-point to a particular solution is also a Nash Bargaining solution. Assume that this process is **proportional** in that it is carried out along the ray O^*N^* which maintains the ratio of utility gain from the threat-point payoff $(U- w^*) / (V-W^*)$ constant. The utility allocation would increase from N^* to N^p located on the frontier UF^p, while the threat-point payoff moves from O^* to O^p, showing how disarmament process is tied to the bargaining protocol. When O^p is reached with the payoff $w^p = U_0$ and $W^p < V_0$ as depicted in Figure 2, it follows that $g^P = 0$ while $G^P > 0$. The possibility to have also $G^P = 0$ is incidental, so to speak. In sticking to proportional rule about disarmament with utility transferability in bargaining process, the credible threat-point is now O^p, and the corresponding Nash bargaining solution, depicted as N^p, lies on the utility frontier UF^p represented by the dashed line. Clearly, this is still not the *first-best* solution lying on the utility frontier UF, such as N. Any move from N^p to the fist best solution N require a unilateral disarmament of country V; and this is conceivable only when such move is a Pareto Improvement for both countries, thus not being always possible. Then how to implement the classical first best solution in the presence of conflict? Disarmament, being part of the bargaining protocol may be, in this case, worked out though mechanism designs, a subject far beyond the reach of this paper.

Take now a glance at what would happen if our two countries – small and open – could benefit from exchange with **a third party**, says the world where the term of trade is determined by free market mechanism. Of course, each of these countries would enjoy gains from trade, thus diverting their resources to make guns is more costly, and conflict in this case will be therefore discouraged. This is of importance, but not the only feature we must take into account in regard of war and peace in the real world where considerations other than just economic rationality should get involved. If globalization nowadays means capital is moving without frontier, capital owners might compete to acquire scarce (natural) resources through processes other than properly economic, for examples through corrupting the political process in the 3rd world, through exploiting ethnical conflicts, etc…All these might lead to violence, but only at the local level, and essentially because of motives lying outside the rationality embedded in economic premises.

The possibility of forming **alliances** affects naturally the matter of war and peace. Consider 3 countries identical in all respects. At the start, there is neither gain from trading nor the incentive to initiate war for any country. If two among them decide to pool their resources and efforts against the third in case of unrest, this alliance amounts to get back to the 2-country framework with one is having now a double size. Moreover, there might also be some scale effect with regard to the Conflict Technology: synergy created by alliance may, for instance, allow a higher efficiency in warfare. The asymmetries occasioned by the possibility of making alliance, therefore, enhances the eventuality of war. When the number of countries increases but is still finite, alliance - or coalition - is multiple. Consider the case of 4 countries and assume again that they are identical. If two countries form an alliance, and if the other 2 countries do the same, we end up with the framework of 2 countries of double size. Perfect symmetry excludes the possibility of war making. So, to get the asymmetries enhancing warfare, we have several scenarios. First, we may have an alliance of 3 countries against one country. After war bursts, either the alliance wins and countries at stake share equally the war prize, or the country in isolation wins, and the game ends. Second, we may also have an alliance of two countries which decides to initiate war with only one country, leaving the other country in peace. The war game, as before, ends up with one winner. This latter is having different endowment than that of the country left in peace. Trading is now feasible for some cooperative gain if the asymmetry of endowment allows for it. Otherwise, when the asymmetry so far created is sufficiently stringent that the condition for war outburst in Section 3 is satisfied, these countries would have no choice but making war . Of course, the number of all possible alliances and feasible scenarios increases when we introduce more and more countries. The peaceful - or war free - environment envisaged by the classical liberalist will be restored only when each country is atomistic and their number large enough. In this case, the core exists and shrinks into the competitive equilibrium under free trade.

The work in this paper is only a commencement of a difficult, but important subject. Our model of war and peace here is simple. It may be extended to a more dynamic setting where information, though incomplete, may be updated in comprehensive ways. Diplomatic efforts, military spy activities, international mechanism of reconciliation, the complex game of making alliances, and the setting of protocol for cooperation[17], etc …are all important aspects of further researches. A better understanding of war and peace, as well as the efforts aimed at finding ways to sustain an enduring peace to preserve mankind, require surely an integrated approach of various fields in social sciences where economic science has its role. This modest first step here has shown, if any, that economic rationality is but one of the best guards against the blind violence that, to quote W. Goethe, even ‘’ the Gods themselves contain in vain’’.

APPENDIX

Section III

For purpose of illustration, let us provide an example by adopting the functional forms for countries' utility: $U(x,y) = Ax^a y^b$ with $a+b \leq 1$; $V(X,Y) = BX^\alpha Y^\beta$ with $\alpha+\beta \leq 1$. the peaceful environment envisaged by the classical liberalist will be restored.

Also, for the conflict technology, we chose $h(g,G) = \gamma g / [\gamma g + G]$ for $g > 0$ and $G >$. This specific form been used in earlier literature, for instance Skaperdas and Syropoulos (2001), where γ is an efficiency factor. Note that $\gamma > 1$ means country U is more efficient than its opponent in conflict situation (and conversely, if $\gamma < 1$). The best replies for country U and V now are respectively the following:

$$[G/[\gamma g + G]] \left(\sum y - g - G \right) - bg = 0$$

and

$$[\gamma g / [\gamma g + G]] \left(\sum y - g - G \right) - \beta G = 0$$

Now, solve for g^w and G^w. From (5bis) and (6bis), we get:

$$G^w = [\gamma b/\beta]^{1/2} g^w$$

and

$$g^w = \frac{\sum y}{1 + b + (1+\beta)[\gamma b/\beta]^{1/2}}$$

Note also that the probability for country U to win the war prize is now $\gamma / [\gamma + [\gamma b/\beta]^{1/2}]$.

[17] It is at this point worthwhile to ask how to reach the bargaining solution. This question concerns what should be the protocol for exchange cooperation. We recall on this occasion the well known contribution of Rubinstein (1982) for a one-dimensional problem with offer and counter-offer procedure. To establish that the Nash bargaining solution is a perfect equilibrium, we need to put more structure to the problem, for instance the introduction of time that incurs the cost due to the impatience of the bargainers. Our problem here has a higher dimension, thus requires probably much more complex analysis.

We now propose some computation to show how the parameters of preferences and of the conflict technology affect the equilibrium value of armament.

1- To see how it is also related to the preference, let us keep $\gamma = 1$ for equal technology efficiency, and consider a variation of the ratio β/b consisting of an increase in β (or alternatively a decrease in b). This means the share of the manufactured good Y in term of utility in country V is more important, and so is the utility cost of armament which is the forgone consumption of Y (and in contrary, the utility cost of armament is less important in country U). Thus, this variation naturally benefits country U in enhancing its relative power. Perform now a numerical computation with $\gamma = 1$, $b = 0.2$ and $\beta = 0.8$; $y_0 = 0.6$ and $Y_0 = 0.4$. In this case, $g^w = 1 / 2.1$, $G^w = 1/2\, g^w = 1/ 4.2$. and these solutions satisfy the constraint $g^w < y_0 = 0.6$, and $G^w < Y_0 = 0.4$. The winning probability of country U in war is 2/3, that of country V is 1/3, and the relative power index of country U is 2, i.e. it gets twice as much as chance to win the prize. Now, decrease b to 0.1 and β increase β to 0.9, $g^w = 1 / 1.73$ and $G^w = 1/3\, g^w = 1/5.19$. The winning probability of country U raises to3 /4, and its relative power index reaches 3. In all these cases, war is the ultimate outcome.
2- Repeat the computation exercise with $\gamma = 4$, and the parameters $b=0.2$ and $\beta = 0.8$, we get $g^w = G^w = 1/4$. The winning probability of country U is 4/5 and its relative power index rise to 4. The asymmetry of conflict technology exacerbates the likehood of war.
3- Given $\gamma = 1$, $b = 0.2$ and $\beta = 0.8$ as in the first example above, but let us now take $\alpha = 0.1$ and vary the parameter a. For $0.5 \le a \le 0.8$, the computation indicates that war will occur, while for $a < 0.5$, the core exists, and peace would prevail. Clearly, when the preference of one country differs sharply from that of its opponent, war is likely the outcome while, conversely, when these preferences are more similar, peace will prevail.

References

BOULDING K. E., , Towards a Pure Theory of Threat Systems, *The American Economic Review*, Papers and Proceedings, 1963, vol. 53, no. 2, pp. 424-434.

BINMORE K. AND P. DASGUPTA (eds.) *the economics of Bargaining*, 1987, Basil Blackwell, Oxford, England.

BRITO D.L., INTRILIGATOR M.D., Conflict, War and Redistribution, *The American Political Science Review*, 1985, vol. 79, no.4, pp. 943-957.

CARUSO R. (2006b), Conflict and Conflict Management with Interdependent Instruments and Asymmetric stakes, (The Good-Cop and the Bad-Cop game*). Peace Economics, Peace Science and Public Policy*, vol. 12, no.1.

COPELAND, DALE C. , Economic Interdependence and War. *International Security* ,1996, 20:5-41.

CRAWFORD V. ''A Theory of Disagreement in Bargaining'', Essay 7 in BINMORE and DASGUPTA (Eds.) *The economics of Bargaining*, 1987, op.cit.

DIXIT, A, "Strategic behavior in contests," *American Economic Review*, 77(5), December 1987, 891-898.

EINSTEIN, A and FREUD, S , Why War? (An exchange of letters between Einstein and Freud in 1932. Originally in *Warum Krieg), International Journal of Group Tensions,* 1971, 1:3-25.

FRIEDMAN J. W. *Game Theory with Applications to Economics*, Oxford University Press, England, 1986.

GARFINKEL M. R,'' Arming as a Strategic Investment in a Cooperative Equilibrium '', *American Economics Review,*1990, 80, 50-68.

GROSSMAN H.I, '' A General Equilibrium Model of Insurrections'', *American Economics Review*, 1991, 81, 912-921.

GROSSMAN H.I AND M. KIM, '' Swords or Plowshares ? A Theory of the Security of Claims to Property'', *Journal of Political Economy,* 1995, 103,1275-1288.

GROSSMAN H.I, ' ' Producers and Predators'', *Pacific Economics Review,* 1998, 3, 169-189.

HARSANYI J, Games with Incomplete Information Played by Bayesian Players, *Management Science,* 1967-68*,* 14: 159-182, 320-334, 486-502.

HIRSHLEIFER J. , ‘‘ The Analytics of Continuing Conflict’’, *Synthese,* 1988, 76, 201-233.

HIRSHLEIFER J , ‘‘The Technology of Conflict as an Economic Activity’’, *American Economics Review – Papers and Proceedings*, 1991,81,130-134.

HIRSHLEIFER J , ‘‘Anarchy and Its Breakdown’’, *Journal of Political Economy,* 1995, 103, 27-52.

HIRSHLEIFER J., , The Paradox of Power, *Economics and Politics*, 1991, vol. 3, pp. 177-20. re-printed in Hirshleifer (2001), pp. 43-67.

HIRSHLEIFER J., *The Dark Side of the Force*, *Economic Foundations of Conflict Theory*, 2001, Cambridge University Press.

HUNTINGDON, SAMUEL P. The Clash of Civilizations? *Foreign Affairs,* 1993, 72:22-49.

ISARD W., SMITH C., *Conflict Analysis and Practical Management Procedures*, *An introduction to Peace Science*, 1982, Cambridge, Ballinger Publishing Company.

KREP, D and R. WILSON, Sequential Equilibrium, *Econometrica*, 1982, 50: 863-894.

MAS-COLELL A, MD. WHINSTON AND J.R GREEN, *Microeconomic Theory*, 1995, Oxford University Press, New York.

NASH J, ‘‘ The Bargaining Problem’’, *Econometrica*, 1950, 18, 286-295.

NASH J, ‘ ‘ Two-Person Cooperative Game’’, *Econometrica*, 1953, 21, 128-140.

NEARY H. M., Equilibrium Structure in an Economic Model of Conflict, *Economic Inquiry*,1997,vol. 35, no. 3, pp.480-494.

POLACHEK, S. W., Conflict and Trade. *Journal of Conflict Resolution,* 1980, *vol.* 24, no. 1, pp.55–78.

RUBINSTEIN A., Perfect Equilibrium in a Bargaining Model, *Econometrica,* 1982,50: 207-211.

SCHELLING T. C., *The Strategy of Conflict*, 1960, Harvard University Press, Cambridge.

SCHELLING T. C., *Arms and Influence*, 1966, Yale University Press, New Haven.

SKAPERDAS S, ‘‘ Conflict and Attitudes toward Risk’’, *American Economics Review – Papers and Proceedings*, 1991, 81,116-120.

SKAPERDAS S., ‘‘ Cooperation, Conflict, and Power in the Absence of Property Rights’’, *American Economics Review*, 1992, 82, 720-739.

SKAPERDAS, S AND C. SYROPOULOS, “Competitive Trade with Conflict,” in Michelle R. Garfinkel and Sergios Skaperdas, ed., *The Political Economy of Conflict and Appropriation*, 1996, Cambridge: Cambridge University Press, 73–96.

SKAPERDAS, S AND C. SYROPOULOS, “Guns Butter, and Openness: On the Relationship Between Security and Trade,” *American Economic Review, Papers and Proceedings,* 2001, 91(2), 353–357.

SYROPOULOS, C, “Trade openness and international conflict,” paper presented at the conference ‘New Dimensions in International Trade: Outsourcing,Merger, Technology Transfer”, 2004.

SKAPERDAS S., Cooperation, Conflict, and Power in the Absence of Property Rights, *The American Economic Review*, 1992, vol. 82, no. 4, pp. 720-
739.

SKAPERDAS S., Contest Success Functions, *Economic Theory*, 1996, vol. 7, pp.283-290.

TULLOCK G., Efficient Rent Seeking, in Buchanan, J. M., Tollison R., D.,Tullock G., (eds.), *Toward a Theory of the Rent-seeking Society*, 1980, Texas A&M University, College Station, pp. 97-112.

TIROLE J , *The Theory of Industrial Organization,* 1988, The MIT Press, Cambridge, Massachusetts