DEPOCEN
Working Paper Series No. 2009/08

Total Factor Productivity, Saving Rate and Learning-by-Doing in Growth Process

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January 30, 2009

Abstract

In transitional stage saving rate play an important role in output growth rate as proposed by Krugman. Accumulationists are also right as claiming that learning-by-doing play an important role in TFP growth in NIEs. However, using a CES production technology we can show that the growth model based purely on learning-by-doing is constrained by labor growth rate. If the latter is constant in the long-run, then the growth can not be sustained.

Keywords: Optimal growth model, learning-by-doing, saving rate, Developing country.
JEL Classification: D51, E13, E21

1 Introduction

The roles of capital accumulation and technological progress in economic growth are not new stories in the literature. The Solow (1956) based on the classical assumption of diminishing returns to capital, states that without continuing improvement of technology per capita growth must eventually cease. The essential factor for economic growth, namely technological progress, is however, exogenous to the model. This shortcoming inspires scholars such as Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991), Grossman and Helpman (1991), Aghion and Howitt (1992) and many others to develop new "endogenous" growth models which provide more insight into the Solow's residual. The endogenous growth models by taking human capital accumulation, learning-by-doing, research and development (R&D), and knowledge spillover in economic

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growth into account are able to generate long-term per-capita growth endogenously.

Solow (1957) uses US data from 1909 to 1949 and shows that the capital intensity contributed for one eighth to the US economic growth. The remainder is due to increased productivity. King and Rebelo [1993] run simulations with neoclassical growth models and conclude that the transitional dynamics can only play a minor role in explaining observed growth rates. They suggest endogenous growth models such as endogenous human capital formation or technical progress as primary vehicle for research on economic growth.

Recently, the spectacularly rapid growth of many Asian economies, especially the East Asian newly industrialized economies (NIEs) gave rise to a broad and diversified literature aiming at explaining the reasons for such a long lasting period of expansion (Kim and Lau [1994, 1996], Krugman [1994], Rodrik [1995], Worldbank [1993], Young [1994, 1995]). All these economies have experienced rapid growth of their physical capital stock and very high rate of investment in human capital.

On one hand, the supporters of the accumulation view stress the importance of physical and human capital accumulation in the Asian growth process. Accordingly, the main engine of "miracle growth" in NIEs is simply, very high investment rates. Young [1994, 1995], Kim and Lau [1994, 1996] found that the postwar economic growth of the NIEs was mostly due to growth in input factors (physical capital and labor) with trivial increase in the total factor productivity. Moreover, the hypothesis of no technical progress cannot be rejected for the East Asian NIEs (Kim and Lau [1994]). Consequently, accumulation of physical and human capital seems to explain the lion's share of the NIEs' growth process. Krugman [1997] wrote that Larry Lau and Alwyn Young works suggested that Asian growth could mostly be explained by high investment rates, good education and the movement of underemployment peasants into the modern sector. Economists who take this point implicitly assumed that adoption and mastering new technology and other modern practices could be done easily by trade.

"*Accumulationists seem to believe that the state of technological knowledge at any time is largely codified in the form of blueprints and associated documents and that, for a firm to adopt a technology that is new to it but not to the world, primarily involves getting access to those blueprints" (Nelson and Pack, 1998).*

Accordingly, any economies could have experienced high rates of growth like NIEs if they could afford similar investment rates. Krugman's [1994] interpretation of these results is very pessimistic since, in his opinion, the lack of technical progress will inevitably bound the growth engine of East Asian NIEs as a result of the diminishing returns affecting capital accumulation.

On the other hand, the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. According to these authors, Asian countries have adopted technologies previously developed by more advanced economies (assimilation view) and "*the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy*" (Pack [1992]). They admit that high rates of investment into physical and human capital is necessary to achieve high economic growth rate. However, as stressed by Nelson and Pack (1998) there is nothing automatic in learning about, in risking to operate and, in coming to master technologies and other practices that are new to the economy. These processes require searching and studying, learning, and innovating to master modern technologies and new practices. Thereby, the economy enhances its stock of knowledge and efficiency. Implicitly, they suggest that technological progress exist and does play a crucial role in NIEs' economic growth.

Empirically, Collins and Bosworth [1996] or Lau and Park [2003] show Total Factor Productivity (TFP) gains actually matter in Asian NIEs growth and that future growth can be sustained. For these authors "*it is possible that the potential to adopt knowledge and technological from abroad depends on a country's stage of development. Growth in the early stages may be primarily associated with physical and human capital accumulation, and significant potential for growth through catchup may only emerge once a country has crossed some development threshold*" (Collins and Bosworth [1996]). These findings concerning the East Asian economies in the post-war period are also valid for developed economies in the early stages of their development (Lau and Park [2003]). They suggest that in these stages, economic growth is generally based on physical accumulation rather than technological progress. Greater gains in TFP are possible only during the second stage of development. More precisely, Lau and Park (2003) show there was no technical progress for Hong Kong, Korea, Singapore, Taiwan, Indonesia, Malaysia, Thailand until 1985. However, in period 1986-1995 technological progress evidently contributes to economic growth in these economies. For Western Germany, United Kingdom, France, and Japan, technical progress always existed.

In this paper we prove that the so-called Solow-Krugman controversy is not really one. Krugman's view is correct in the short and mid terms. But in the long term, TFP is the main factor of growth. In this sense, Solow is right and his 1956 model is basically a long term growth model. Specifically, in transitional stage the high saving rate induces high growth rate of output however, in the long-run the impact of saving rate on output growth rate will vanish out.

On the other hand, accumulationists are also right as claiming that learning-by-doing play an important role in TFP growth in NIEs. We also show, however,

that the growth model based purely on learning-by-doing is constrained by labor growth rate. If the latter is constant in the long-run, then the growth can not be sustained. Therefore, despite learning-by-doing generating TFP growth the long run growth essentially requires in-house capacity to generate technological progress.

The organization of the paper is as follows. The general basic neoclassical model is presented in section 2. In section 3 we use standard Solow model to prove that high investment rate improve growth rate in short-term however this effect vanish in long-term. In section 4, CES production function is used to take into account process of knowledge accumulation through learning-by-doing and the spillover in economic growth process. The last section summarize main results of the paper.

2 The Basic Neoclassical Model

In this section we set out the basic model of capital accumulation that will use in our analysis. The standard constant return to scale is defined as follows:

$$Y_t = F(A_t, K_t, L_t) \tag{1}$$

Where Y_t is output, K_t is physical capital, L_t is labour input, A_t is a parameter of technological progress. The production function, if A_t and L_t are constant, has positive and diminishing returns to the reproducible factor K_t. Mathematically, $\frac{\partial Y_t}{\partial K_T} > 0$, $\frac{\partial^2 Y_t}{\partial K_t \partial K_t} < 0$.

We follow Sollow (1956) to assume that saving (net investment) is a fixed fraction s of income; the capital stock depreciates at a fixed rate δ, and the labor growth rate is constant at n. With these assumptions the transitional dynamics of the model is given by following program:

$$\begin{aligned} C_t + S_t &= Y_t = F(A_t, K_t, L_t) && \text{(2a)} \\ S_t &= sY_t,\ s \text{ is the exogenous saving rate} \\ K_{t+1} &= K_t(1-\delta) + sY_t && \text{(2b)} \\ L_t &= L_0(1+n)^t \end{aligned}$$

$C_t, S_t, Y_t, K_t, I_t, L_t$ denote respectively the consumption, the saving, the output, the capital stock, the investment and the labour at period t. The labour force grows with an exogenous rate n. A_t denotes the technological level in the economy at time t. The growth rate of A_t is assumed to be identical with the growth rate of the Total Factor Productivity (TFP).

In the following section we use Solow model to show that in short run investment rate or physical capital accumulation do have strong effect on economic growth. However, in long-run economic growth is neutral to investment rate but positively contingent on rate of technological change.

The accumulating knowledge through learning-by-doing as mentioned in Atkinson and Stiglitz (1969) and Nelsons and Pack (1998) is modelized in a CES production function in section 4.

3 Exogenous TFP: The Solow Model

In this section we use Cobb-Douglas functional to consider a simple intertemporal growth model for a closed economy.

$$Y_t = a(1+\gamma)^t K_t^{\alpha} L_t^{1-\alpha},\ 0 < \alpha < 1 \tag{3}$$

The Total Factor Productivity (TFP) is assumed to grows at a constant rate γ. It is easy to solve the model given above. Actually, we have

$$\forall t,\ K_{t+1} = (1-\delta)K_t + saK_t^{\alpha} L_t^{1-\alpha}(1+\gamma)^t \tag{4}$$

We can easily check that there exists a Balanced Growth Path (BGP) with rate g

$$(1+g) = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$$

On the BGP, we have $K_t^* = K^s(1+g)^t,\ \forall t$, where $K^s = \left(\frac{sa}{g+\delta}\right)^{\frac{1}{1-\alpha}} L_0$. Given $K_0 > 0$, the path generated by equation (4) satisfies

$$\frac{K_t}{(1+g)^t} \to K^s$$

In other words, the path $\{K_t\}_t$ converges to the steady state $K^s(1+g)^t$ as shown in figure 1. It is interesting to notice that the rate of growth g is positively related to the rate of growth γ of the TFP.

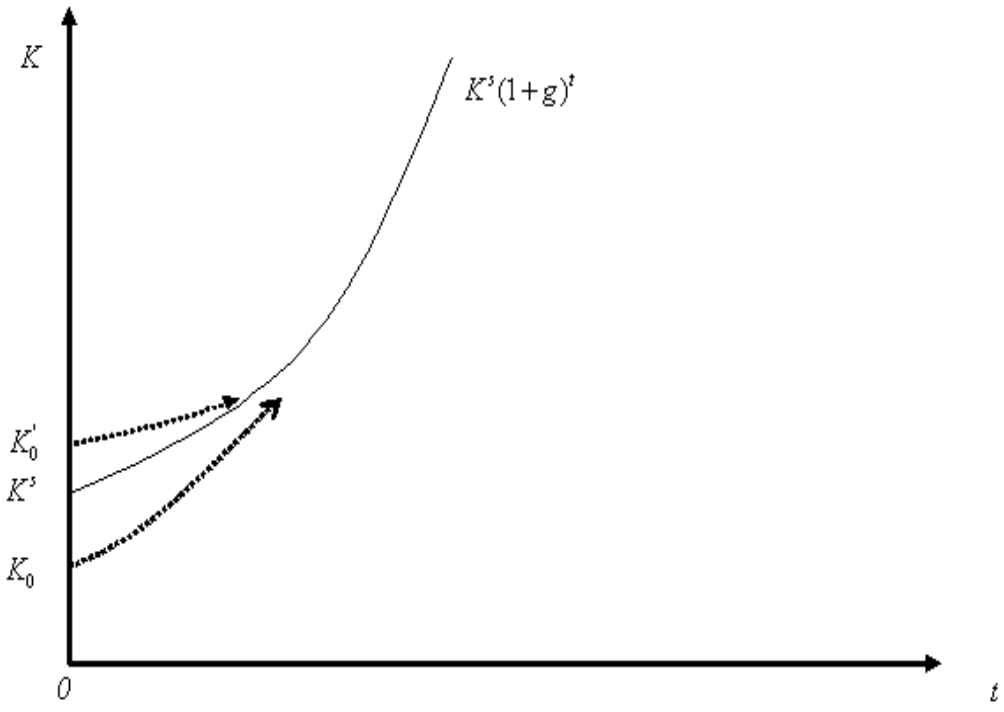

Figure 1

From (4) we have:

$$\forall t,\ K_{t+1} = (1-\delta)K_t + saK_t^{\alpha}L_0^{1-\alpha}(1+\gamma)^t(1+n)^{t(1-\alpha)} \tag{5}$$

and $\{K_t\}$ converges to $\{K^s(1+g)^t\}$ where g is growth rate of capital stock and output at steady state and $1+g = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$ and $K^s = [\frac{sa}{g+\delta}]^{\frac{1}{1-\alpha}}L_0$.

Notice that in Cobb-Douglas technology as defined in (3) the growth rate of output is identical as growth rate of capital. Let us define this growth rate as follows:

$$\nu_t = \frac{K_t}{K_{t-1}}$$

From equation (5) we have:

$$\frac{K_t}{K_{t-1}} - (1-\delta) = saL_0^{1-\alpha}(1+\gamma)^{t-1}(1+n)^{(t-1)(1-\alpha)}K_{t-1}^{\alpha-1} \tag{6}$$

$$\nu_t - (1-\delta) = (1+\gamma)(1+n)^{1-\alpha}\nu_{t-1}^{\alpha-1}[\nu_{t-1} - (1-\delta] \tag{7}$$

Lemma 1 *Let* $\varphi(\nu) = [\nu - (1-\delta)]\nu^{\alpha-1}$ *with* $\nu > 0$ *then* $\varphi(\bullet)$ *is increasing with* ν.

Proof:

$$\begin{aligned}\varphi'(\nu) &= \nu^{\alpha-1} + [\nu - (1-\delta)](\alpha-1)\nu^{\alpha-2} \\ &= \nu^{\alpha-2}[\nu + (\alpha-1)\nu + (1-\delta)(1-\alpha)] \\ &= \nu^{\alpha-2}[\alpha\nu + (1-\delta)(1-\alpha) > 0]\end{aligned}$$

■

It is easy to check that

$$K_1 = saK_0^{\alpha}L_0^{1-\alpha} + (1-\delta)K_0 \tag{8}$$

$$\nu_2 = sa(1+\gamma)K_1^{\alpha-1}L_1^{1-\alpha} + 1 - \delta \tag{9}$$

The equation (9) shows that better rate of technological progress (γ is higher) or better saving rate (s) will bring about a higher growth rate for period 2 (ν_2). Using Lemma (1) and equation (7) we can show that the growth rate is not only improved in period 2 but for all latter periods. Put it differently, an economy with higher rate of technological progress not only has higher growth rate at steady state but also has higher growth rate in transitional period.

More interestingly, as we have shown saving rate is neutral to growth rate in the long-run, however in transitional stage Lemma (1) and equation (7) show that an increase of saving rate does accelerate ν_2 then growth rate of all period afterward. In other words, saving rate contribute effectively in transitional periods, however the influece of saving rate on growth must eventually vanish in the long-run.

Now let us consider two economies which are identical in everything, except for rates of technological progress and rates of saving (investment). The rates of technological progress and rates of saving in these two economies are (γ, s) and (γ', s') respectively. We assume that $\gamma < \gamma'$ and $s > s'$. It is obvious that: $\nu_t \to 1+g$ and $\nu'_t \to 1+g'$and $g < g'$.Therefore there exists a point T in time such that $\nu_t < \nu'_t, \forall t \geq T$. In other words, in short run the impact of higher saving rate may be superior to the impact of better productivity ($\nu_t > \nu'_t$) however in the long run the better productivity always dominates in economic growth process.

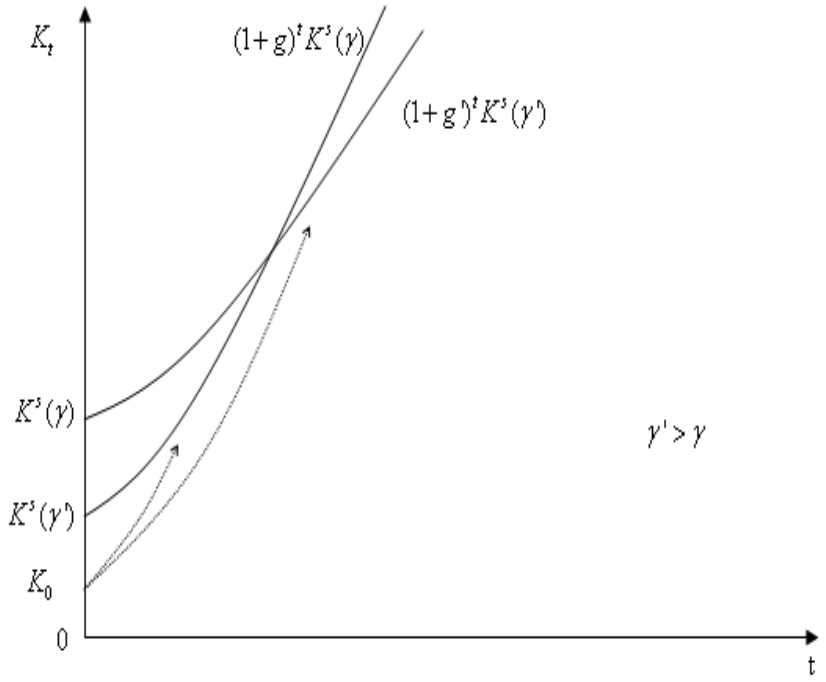

Figure 2

If the economies initially operate below the steady state level (i.e. $K_0 < K^s$) we prove that the economy with higher rate of technological progress also converges faster to its own steady state than the other.

Let us define $\zeta_t = \frac{K_t}{K^s(1+g)^t}$ as speed of convergence, then $0 < \zeta_t < 1$ and $\zeta_t \to 1$ as $t \to \infty$.

Define $\hat{K}_t = \frac{K_t}{(1+g)^t}$ from equation (5) we have:

$$\zeta_{t+1} = \frac{1}{1+g}\left[(1-\delta)\zeta_t + saL_0^{1-\alpha}\zeta_t^{\alpha}\frac{1}{(K^s)^{1-\alpha}}\left(\frac{(1+n)^{1-\alpha}(1+\gamma)}{(1+g)^{1-\alpha}}\right)^t\right]$$

Since $1+g = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$ and $K^s = [\frac{sa}{g+\delta}]^{\frac{1}{1-\alpha}}L_0$ then

$$\zeta_{t+1} = \frac{1}{1+g}\left[(1-\delta)\zeta_t + (g+\delta)\zeta_t^{\alpha}\right] \tag{10}$$

Take partial derivative equation (10) by g we get:

$$\frac{\partial\zeta_{t+1}}{\partial g} = \frac{1-\delta}{(1+g)^2}\left(\zeta_t^{\alpha-1}-1\right) + \frac{\partial\zeta_t}{\partial g}\left(\frac{1-\delta}{1+g} + \frac{g+\delta}{1+g}\alpha\zeta_t^{\alpha-1}\right) \tag{11}$$

We can see that the first part of the LHS of equation (11) is positive since $0 < \zeta < 1$ hence $\left(\zeta^{\alpha-1}-1\right) > 0$. Therefore if $\frac{\partial\zeta_t}{\partial g} > 0$ then $\frac{\partial\zeta_{t+1}}{\partial g} > 0$.Recall that $\zeta_0 = \frac{K_0}{K^s} = \frac{K_0}{\left[\frac{sa}{g+\delta}\right]^{\frac{1}{1-\alpha}}L_0}$ and then $\frac{\partial\zeta_0}{\partial g} > 0$. By induction we have $\frac{\partial\zeta_{t+1}}{\partial g} > 0, \forall t \geq$

0,which means that the economy whose rate of technological progress is higher (then higher g) will converge faster to its own steady state.

It is easy to check that ζ_1 is negatively related to s, the equation (10) implies that ζ_t is negatively related to saving rate (investment rate) s for all t. The higher saving (investment) rate helps economy grow faster but converge slower to its own steady state.

Remark 1 *1. In short and medium term (transitional stage), the saving rate (hence capital accumulation) does matter for growth rate. A permanent increase in saving rate not only raises the level of steady state but also increases the economic growth rate in transitional period.*

2. In development process, the rate of technological progress is dominant factor in long run. An economy with lower saving rate but higher growth rate of productivity than other can always overrun her contestants in long run.

3. The economy with higher rate of technological progress will converge faster to their own steady states; grow faster not only in steady state but also in transitional period. This result is consistent with findings of King and Rebelo (1993), who run simulations with neoclassical growth models and conclude that the transitional dynamics can only play a minor role in explaining observed growth rates. Furthermore, higher saving rate helps economy grow faster but converge slower to its own steady state.

4. The model also figures out the reason why there is no convergence in economic growth among developing economies (Barro and Sala-i-Martin 2004). The divergence in technological progress and investment rate among developing economies are factors which induce the divergence in development process among developing world.

4 Endogenous TFP: Learning-by-Doing

Atkinson and Stiglitz (1969) advocates that if a firm switch from one technique (say, labour-intensive) to another one (e.g. capital-intensive), it requires a technological progress. Because the switching requires new knowledge to localize the technique; new knowledge to maneuver the production process; new knowledge to reorganize the production etc.. Nelson and Pack (1998) explores further this ideas by arguing that the switching from labour-intensive economies to capital-intensive economies, as NIEs have done, can not be seen as simply "moving along production function". They admit, on the one hand, that developing economies can import technologies from developed economies. On the other hand, they argue further, only a small portion of what one needs

to know to employ a technology is codified in the form of blueprints; much of it is tacit which requires an uncertain process of, searching and studying, learning-by-doing and using, restructuring production activities. In short-run this process, as proposed by Atkinson and Stigliz (1969) may be costly, however in long-run, it indeed improves knowledge stock and technological level of the economy. The effectiveness of this process is not automatic but contingent on efforts and effectiveness of learning-by-doing, of the restructuring of production activities, of searching and studying , and many other factors. However these analyses (Collins and Bosworth [1996], Nelson and Pack (1998) or Lau and Park [2003] *etc.*,) are essentially qualitative and thus the process of learning-by-doing seems to be insufficiently deliberated. Some important questions are still open: Whether is merely learning-by-doing sufficient to sustain growth in long run? Whether the impact of saving rate on growth vanish in the presence of learning-by-doing?

In this section we endogenize the process of learning-by-doing in a CES model to give answers to above mentioned questions. The CES technology is used to take into account the effectiveness of structural change in the economy during growth process. The production function is presented in equation (12).

$$Y_t = [\alpha K_t^r + (1-\alpha)(K_t^\beta L_t)^r]^{\frac{1}{r}} \tag{12}$$

In this model we assume that:

(i) Stock of knowledge is accumulated through learning-by-doing and using. It works through each firm's investment. Specifically, an increase in a firm's capital stock requires firm to accumulate new knowledge of using, localizing and work organizing. As a result this increases the firm's stock of knowledge.

(ii) The knowledge accumulated can be internalized within the firm and then externalized to the whole economy through spillover effect to increase the total stock of knowledge of the economy as a whole.

(iii) The effectiveness of the knowledge accumulation through learning-by-doing and spillover determined by parameter β which we call TFP coeffiecient. We assume that, this process complies with law of diminishing return, i.e. $0 < \beta < 1$. The higher β, the more effective the knowledge accumulation is, and then the faster technological level improves. The manigtude of β depends on the concentration and linkages of industries in the economy, the effectiveness of on-job training, *etc.*,.

L_t labour used in production, is assumed to grow at constant rate n. We define $r = \frac{\rho-1}{\rho}$, where ρ is elasticity of substitution between K and L. As common in literature we assume that $1 > r > -\infty$. The higher r, then the higher ρ which implies that the structure of the economy is more flexible; it

is easier to switch from a labor-intensive technology to more capital-intensive technologies and vise versa. Where $r = 0$, the production function (12) becomes a Cobb-Douglas technology with increasing return to scale.

Lemma 2 *The production function (12) possesses all properties of a general production function.*

Proof: First, It is clear that when $K_t \to 0$, then $Y_t \to 0$; and when $K_t \to +\infty$, then $Y_t \to +\infty$.

Second we claim that $Y(K)$ is a concave function with respect to K.

Let us define

$$h(K, L_t) = \alpha + (1 - \alpha)K^{(\beta-1)r}L_t^r \tag{13}$$

It is easy to check that $r\frac{\partial h(K,L_t)}{\partial K} < 0$ and

$$Y'(K) = h(K, L_t)^{\frac{1-r}{r}}(\beta h(K, L_t) - \beta\alpha + \alpha) > 0 \tag{14}$$

$$\frac{\partial^2 Y}{\partial K^2} = \frac{1}{r}\frac{\partial h(K, L_t)}{\partial K}[h(K)^{\frac{1-r}{r}} + \alpha(1 - r)(1 - \beta)h(K)^{\frac{1-2r}{r}}] < 0$$

■

Hence, the production function (12) satisfies all properties of a general production function. Furthermore with production function (12) the economy in long run will converge to its Balance Growth Path (BGP). This property is presented in the following proposition.

Proposition 1 *Assume that* $\left[\frac{(1+n)^{\frac{1}{1-\beta}}-1+\delta}{s}\right]^r > \alpha$

(i) There exists a balanced growth path (BGP) $\{K_t^* = K^s(1 + g)^t\}$ *where* $1 + g = (1 + n)^{\frac{1}{1-\beta}}$ *and* K^s *is a steady state of capital stock which, capital path* $\{K_t\}$ *asymptotically converge to.*

(ii) In transitional stage the growth rate of capital stock and output monotonically converges to the steady state rate: $g^y = g^k = g = (1 + n)^{\frac{1}{1-\beta}} - 1$

Proof: (i) Let us define alternative path of capital $\{\hat{K}_t : \hat{K}_t = \frac{K_t}{(1+g)^t}\}$. From equation (2b) we have:

$$\hat{K}_{t+1} = \frac{1 - \delta}{1 + g}\hat{K}_t + \frac{s}{1 + g}\left[\alpha\hat{K}_t^r + (1 - \alpha)\hat{K}_t^{r\beta}(1 + g)^{rt(\beta-1)}(1 + n)^{rt}L_0^r\right]^{\frac{1}{r}} \tag{15}$$

Replace $1 + g = (1 + n)^{\frac{1}{1-\beta}}$ into equation (15) we have:

$$\hat{K}_{t+1} = \Phi(\hat{K}_t) = \frac{1 - \delta}{1 + g}\hat{K}_t + \frac{s}{1 + g}\left[\alpha\hat{K}_t^r + (1 - \alpha)\hat{K}_t^{r\beta}L_0^r\right]^{\frac{1}{r}}$$

Let define:

$$\Psi(\hat{K}_t) = \frac{\hat{K}_{t+1}}{\hat{K}_t} = \frac{1-\delta}{1+g} + \frac{s}{1+g}\left[\alpha + (1-\alpha)\hat{K}_t^{r(\beta-1)} L_0^r\right]^{\frac{1}{r}} \tag{16}$$

(a) if $r > 0$ then the assumption $\left[\frac{(1+n)^{\frac{1}{1-\beta}} - 1 + \delta}{s}\right]^r > \alpha$ is equivalent to $s\alpha^{\frac{1}{r}} < g + \delta$. We have:

$$\begin{cases} \hat{K}_t \to 0, \text{ then } \Psi(\hat{K}_t) \to +\infty \\ \hat{K}_t \to +\infty, \text{ then } \Psi(\hat{K}_t) \to \frac{1-\delta}{1+g} + \frac{s\alpha^{\frac{1}{r}}}{1+g} < 1 \end{cases}$$

(b) if $r < 0$ then the assumption $\left[\frac{(1+n)^{\frac{1}{1-\beta}} - 1 + \delta}{s}\right]^r > \alpha$ is equivalent to $s\alpha^{\frac{1}{r}} > g + \delta$.We have:

$$\begin{cases} \hat{K}_t \to 0, \text{ then } \Psi(\hat{K}_t) \to \frac{1-\delta}{1+g} + \frac{s\alpha^{\frac{1}{r}}}{1+g} > 1 \\ \hat{K}_t \to +\infty, \text{ then } \Psi(\hat{K}_t) \to \frac{1-\delta}{1+g} < 1 \end{cases}$$

Hence there always exists K^s such that $\Psi(K^s) = 1. \Longleftrightarrow \Phi(K^s) = K^s$.Notice that $\Phi(\hat{K}_t)$ is a concave function, then K^s is unique positive steady state of the capital path $\{\hat{K}_t : \hat{K}_t = \frac{K_t}{(1+g)^t}\}$.Repalce K^s in both sides of equation (15) we have

$$K^s = \left[\frac{(1-\alpha)L_0^r}{\left[\frac{(1+n)^{\frac{1}{1-\beta}} - 1 + \delta}{s}\right]^r - \alpha}\right]^{\frac{1}{r(1-\beta)}} \tag{17}$$

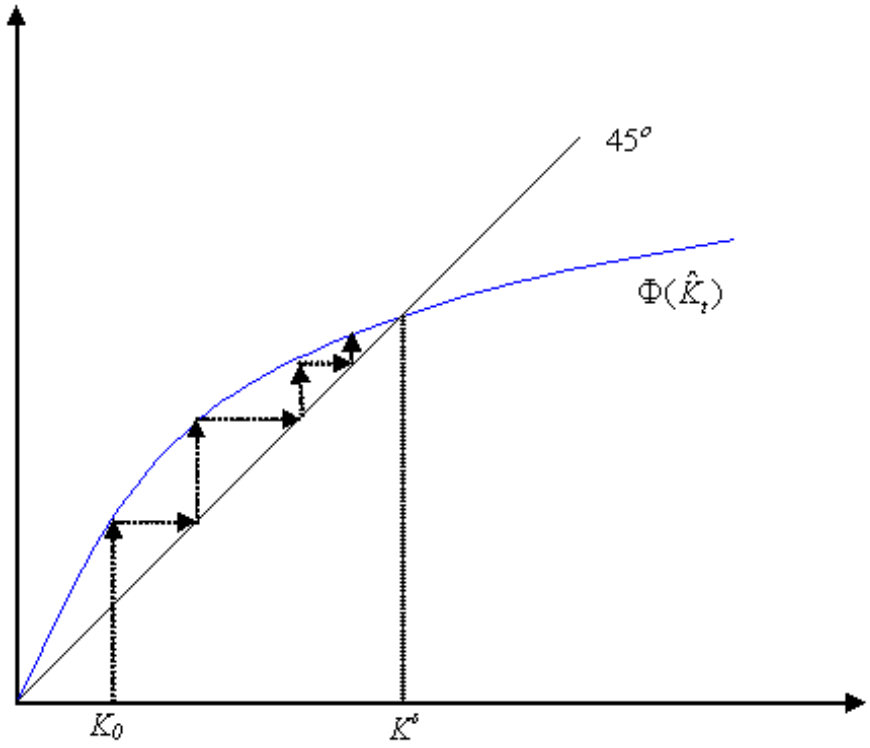

Figure 3

Hence, in long-run capital stock path $\{K_t\}$ will converge to the BGP $\{K_t^* = K^s(1+g)^t\}$. The growth rate of capital stock at steady state:

$$g = (1+n)^{\frac{1}{1-\beta}} - 1 \tag{18}$$

At the steady state the growth rate of output is neutral to saving rate while positively related to TFP coefficient, β.

Hence, in long-run capital stock path $\{K_t\}$ will converge to the BGP $\{K_t^* = K^s(1+g)^t\}$. The growth rate of capital stock at steady state:

$$g = (1+n)^{\frac{1}{1-\beta}} - 1 \tag{19}$$

At the steady state the growth rate of output is neutral to saving rate while positively related to TFP coefficient, β.Without learning-by-doing and spill-over the output growth rate conincides with the growth rate of labor, meaning output per capita is constant. It is noteworthy that if $n = 0$ then in long-run the growth will be ceased regardless how high TFP coefficient is.

(ii) Figure 3 shows that of $K_0 < K^s$, then $\hat{K}_t$ monotonically increases to $K^s$and if $K_0 > K^s$ then $\hat{K}_t$ monotonically decreases to K^s. This property implies that:

$$if\ K_0 < K^s:\ 1+g_t = \frac{K_t}{K_{t-1}} = (1+g)\frac{\hat{K}_t}{\hat{K}_{t-1}} > 1+g, \forall t > 0. \tag{20}$$

$$if\ K_0 > K^s:\ 1+g_t = \frac{K_t}{K_{t-1}} = (1+g)\frac{\hat{K}_t}{\hat{K}_{t-1}} < 1+g, \forall t > 0. \tag{21}$$

Where g_t is growth rate of capital stock in transitional stage which can be presented in another form:

$$g_t = \frac{K_t}{K_{t-1}} - 1 = s[\alpha + (1-\alpha)K_{t-1}^{(\beta-1)r} L_{t-1}^r]^{\frac{1}{r}} - \delta \tag{22}$$

Accordingly, we have:

$$\left(\frac{g_{t+1}+\delta}{s}\right)^r - \left(\frac{g_t+\delta}{s}\right)^r = (1-\alpha)K_{t-1}^{(\beta-1)r} L_{t-1}^r \left(\left(\frac{1+n}{(1+g_t)^{1-\beta}}\right)^r - 1\right) \tag{23}$$

From (20) we know that if $K_0 < K^s$, then $g_t > g, \forall t > 0 \Rightarrow \frac{1+n}{(1+g_t)^{1-\beta}} < 1, \forall t > 0$. Therefore, it can be infered from equation (23) that $g_{t+1} < g_t, \forall t > 0$. In other words the growth rate of capital stock decreases monotonically to its steady state.

By the same token, if $K_0 > K^s$, then growth rate of capital stock increase monotonically to its steady state.

Now let us define g_t^y be the growth rate of output at period t. From Equation (22) we have:

$$g_t^y + 1 = \frac{Y_t}{Y_{t-1}} = \frac{\frac{Y_t}{K_t}}{\frac{Y_{t-1}}{K_{t-1}}} \frac{K_t}{K_{t-1}} = (g_t+1)\frac{\delta + g_{t+1}}{\delta + g_t} \tag{24}$$

It can be shown from the monotony of g_t and equation (24) that:

$$\begin{cases} If\ K_0 < K^s : g_{t+1} + 1 < 1 + g_t^y < g_t + 1 \\ If\ K_0 > K^s : g_{t+1} + 1 > 1 + g_t^y > g_t + 1 \end{cases} \quad \forall t > 0$$

Then: $\begin{cases} If\ K_0 < K^s : g_t^y < g_{t-1}^y \\ If\ K_0 > K^s : g_t^y > g_{t-1} \end{cases} \quad \forall t > 0$

From part (i) we know that g_t and g_{t+1} both converge to g in long-run, therefore g_t^y also monotonically converges to its steady state $g^y = g$. ■

Lemma 3 *Let us define* $g_t(s) = \frac{K_t(s)}{K_{t-1}(s)} - 1$ *and* $\tau(s) = \frac{\alpha K_t^\gamma(s) + C}{\alpha K_{t-1}^\gamma(s) + C}$. *If* $\frac{\partial(g_t(s)+1)}{\partial s} > 0$, *then* $\gamma\tau'(s) > 0$ *where* C *is a positive and constant number.*

Proof*: Indeed,* $\frac{\partial(g_t(s)+1)}{\partial s} > 0$ *implies that*

$$K_{t-1}(s)K_t'(s) - K_t(s)K_{t-1}'(s) \text{ and } K_t'(s) > 0, \forall t > 0 \tag{25}$$

since $K_t(s) = K_0 \prod_{i=1}^{t}(1 + g_i(s))$.

If $\gamma < 0$, *from (25) we have* $(K_{t-1}(s))^\gamma$ *and* $\left(\frac{K_t(s)}{K_{t-1}(s)}\right)^\gamma$ *both deacrease with* s *which further implies that* $K_t^\gamma(s) - K_{t-1}^\gamma(s)$ *decreases too. Mathematically, we have* $\frac{\partial\left(K_t^\gamma(s) - K_{t-1}^\gamma(s)\right)}{\partial s} < 0$.

We have:

$$\tau'(s) = \frac{\alpha^2\gamma\left(K_t(s)K_{t-1}(s)\right)^{\gamma-1}\left(K_{t-1}(s)K'_t(s) - K_t(s)K'_{t-1}(s)\ \right)}{\left(\alpha K^{\gamma}_{t-1}(s) + C\right)} \quad (26)$$

$$+\alpha C\frac{\partial\left(K^{\gamma}_t(s) - K^{\gamma}_{t-1}(s)\right)}{\partial s} \quad (27)$$

Both components of the RHS of equation (26) are negative which indicates that $\tau'(s) < 0$ *if* $\gamma < 0$.

By the same token we can prove that if $\gamma > 0$ *then* $\tau'(s) > 0$.*Therefore* γ $\tau'(s) > 0$ *in general.* ■

Now let us consider two economies which are identical in everything, except for TFP coeffiecient and rates of saving. The TFP coeffiecient and rates of saving in these two economies are (β, s) and (β', s') respectively. We assume that $\beta < \beta'$ and $s > s'$.In the following proposition we show that in short run the impact of higher saving rate may be superior to the impact of better productivity ($g^y_t > g^{y'}_t$) however in the long run the better productivity always dominates in economic growth process.

Proposition 2 *In transitional stage:* $\frac{\partial g_t}{\partial s} > 0$ *and* $\frac{\partial g^y_t}{\partial s} > 0$ *and in long-run* $\frac{\partial g}{\partial \beta} > 0$

Proof: First we show that $\frac{dg_t}{ds} > 0, \forall t > 0$.

From (22) we have:

$$g_1 = sY_0 + 1 - \delta \ \Rightarrow \frac{\partial g_1}{\partial s} > 0$$

and

$$K_1(s) - sK'_1(s) = (1-\delta)K_0 > 0$$

Suppose that we have $K_t(s) - sK'_t(s) > 0$ and $\frac{\partial g_t}{\partial s} > 0$ we prove that $\frac{\partial g_{t+1}}{\partial s} > 0, \forall t \geq 1$.

Indeed, from equation (22) we have:

$$\frac{\partial g_{t+1}}{\partial s} = \left(h(K_t, L_t)\right)^{\frac{1}{r}} + s\left(h(K_t, L_t)\right)^{\frac{1}{r}-1}\frac{1}{r}\frac{\partial h(K_t, L_t)}{\partial K_t}\frac{\partial K_t(s)}{\partial s}$$

$$= \left(h(K_t, L_t)\right)^{\frac{1}{r}-1}\left\{\alpha + (1-\alpha)K_t^{(\beta-1)r-1}L_t^r\left[K_t - (1-\beta)s\frac{\partial K_t(s)}{\partial s}\right]\right\} > 0$$

By the principle of induction we have $\frac{\partial g_t}{\partial s} > 0, \forall t \geq 1$.This result also implies $\frac{\partial K_t}{\partial s} > 0, \forall t > 0$.

Second, we claim that $\frac{\partial g^y_t}{\partial s} > 0, \forall t \geq 1$

Actually we have:

$$\begin{aligned}(1+g_t^y)^r &= \frac{\alpha K_t^r + (1-\alpha)K_t^{\beta r}L_t^r}{\alpha K_{t-1}^r + (1-\alpha)K_{t-1}^{\beta r}L_{t-1}^r} \\ &= (1+g)^r \frac{\alpha \hat{K}_t^r + (1-\alpha)\hat{K}_t^{\beta r}L_0^r}{\alpha \hat{K}_{t-1}^r + (1-\alpha)\hat{K}_{t-1}^{\beta r}L_0^r} \\ &= (1+g)^r \left(\frac{\hat{K}_t}{\hat{K}_{t-1}}\right)^{(1-\beta)r} \frac{\alpha \hat{K}_t^{r-\beta r} + (1-\alpha)L_0^r}{\alpha \hat{K}_{t-1}^{r-\beta r} + (1-\alpha)L_0^r} \end{aligned} \tag{28}$$

Notice that $1+g_t = \frac{K_t(s)}{K_{t-1}(s)} = (1+g)\frac{\hat{K}_t(s)}{\hat{K}_{t-1}(s)}$ .From first part of this proof we know that $\frac{\hat{K}_t(s)}{\hat{K}_{t-1}(s)}$ increases with s.

Applying lemma (3) into equation (28) we have:

$$\begin{cases} (1+g_t^y)^r \text{ increases with } s \text{ if } r>0 \\ (1+g_t^y)^r \text{ decreases with } s \text{ if } r<0 \end{cases}$$

Equivalently we have $\frac{\partial g_t^y}{\partial s} > 0, \forall t > 0$.

In addition the equation (17) also indicates that $\frac{\partial K^s}{s} > 0$, the economy with higher saving rate converges to higher steady state. ■

Hence in transitional stage the economy with higher saving rate may enjoy higher growth rate of output, $g_t^y > g_t^{y\prime}$. However, in the long run $g_t^y \to g$ and $g_t^{y\prime} \to g'$ where $g < g'$ (since $\beta < \beta'$). Therefore there exists a point T in time such that $g_t^y > g_t^{y\prime}, \forall t \geq T$.

Remark 2 *The main results of this section are:*

1. In the long-run the growth rate depends positively on: the efficiency in accumulating knowledge, β (effectiveness of learning-by-doing, of spillover of knowledge and experience,etc.,); the growth rate of labour force n.

2. If the labour force is constant the economy that based on importing technology and accumulating knowledge through learning-by-doing can not sustain its growth in long-term even though its process of knowledge accumulation is highly effective (high β).

3. Saving rate does not affect the growth rate at steady state, however, the economy with higher saving rate grows faster in transitional stage and converges to a higher level of staedy state. In other words Krugman is right when he ascribes the high growth rate in NIEs to high rate of saving.

4. Accumulationists are right when argue that TFP play a non-trivial role in growth process thanks to importing new technology and learning by doing. However in the long run the learning-by-doing growth model is contrained by growth rate of labor force. If the labor force is constant in the long run then

the economy can not sustain its growth. In this sense, learning by doing is insufficient for growth in long run. To sustain growth the in-house capacity to generate technological progress is required.

5 Conclusion

Krugman's view is correct in the sense that high saving rate play an important role in NIEs "miracle" growth rate and without TFP growth these miracles will be disappeared soon.

On the other hand, accumulationists are also right as claiming that learning-by-doing and spill-over play an important role in TFP growth in NIEs. The effectiveness of learning-by-doing and spill-over on growth depend on factors such that the concentration and linkages of industries in the economy, the effectiveness of on-job training, *etc.*, and this factors vary from economy to economy. We also show, however, that the growth model based purely on learning-by-doing is constrained by labor growth rate. If the latter is constant in the long-run, then the growth can not be sustained. Therefore, despite learning-by-doing generating TFP growth, the long run growth essentially requires in-house capacity to generate technological progress. The latter, in its turn requires investment in human capital and R&D. In short, learning-by-doing can not replace the role of human capital and technological capacity.

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