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Growth, Volatility And Stabilisation Policy In A DSGE Model With Nominal Rigidities And Learning-By-Doing

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GROWTH, VOLATILITY AND STABILISATION POLICY IN A DSGE MODEL WITH NOMINAL RIGIDITIES AND LEARNING-BY-DOING[♣]

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Abstract

The paper aims to analyse the question of how cyclical fluctuations might affect long run growth. The analysis is based on a dynamic stochastic general equilibrium model for an imperfectly competitive economy with fully optimising agents. The model is characterized with nominal rigidities, an endogenous technology, and multiple shocks. It predicts either a negative or positive relationship between short run volatility and long run growth depending on the source of shocks and the reaction of the central bank. The model also shows that, even when the negative relationship exits the policy that is designed to stabilise short run volatility may either increase or decrease growth depending on the source of shocks.

JEL codes: E31; E37; E52; O42.

Keywords: Imperfect Competition; Nominal Rigidities; Growth; Volatility; Stabilisation Policy

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1. Introduction

The interaction between short run fluctuations and long run growth has been ignored, even in the real business cycle models, until recently. However, stochastic endogenous growth models suggest that any temporary disturbance can have a permanent effect on output as long as it re-allocates the amount of resources used for productivity improving activities. Therefore, it is possible that the amplitude and frequency of temporary of shocks may have effects on long run growth.

Recently, much effort has been put on this issue. There is a consensus that any temporary shocks may have an impact on long run growth however, the sign of the impact is ambiguous in both theoretical and empirical analyses. The conclusions reached vary differently depending not only on each type of models employed but also on the values of parameters assumed (see Blackburn and Pelloni, 2005 for a survey).

In the present paper we develop a stochastic dynamic general equilibrium model with optimising agents and sticky price setting to investigate the relationship between short run fluctuations and long run growth. Most existing analyses in this literature so far are based on purely real models of the economy without exploring the role of monetary factors. The exceptions are allocated to Dotsey and Sarte (2000) and Blackburn and Pelloni (2004). However, neither of these analyses studies the policy implications of a link between growth and volatility, which is another important issue to which we attend. In this respect, the present paper is most closely related to the contributions of Martin and Rogers (1997), Blackburn (1999) and Blackburn and Pelloni (2005). The first of these analyses is concerned with the effects of fiscal stabilisation policy on growth in a purely real model based on explicit optimising behaviour. The second considers the growth effects of monetary stabilisation policy in a more stylised model with nominal shocks and nominal rigidities. And the third also focuses on the growth implication of monetary stabilisation policy but in a model of wage stickiness based on clearer optimising principles. The present paper may be seen as a complement to this last contribution, focusing on price stickiness which is embedded in an optimising framework, where the economy is subject to multiple types of shocks. The determination of prices in our model is derived from the profit maximising behaviour of firms facing the lack of updated information about the current state of the economy and is shown to be crucial for the sign of the relationship between long run growth and short run volatility. This feature allows monetary policy shocks to have a real impact on short run fluctuations and hence long run growth of the economy.

We also take a further step in analysing the issue of formulating and evaluating monetary policy. In the model, the monetary policy rule is set as a response function to real as well as nominal stochastic shocks. The existence of either positive or negative relationship between growth and volatility has potential for policies that designed to stabilise short run fluctuations to have an influence on the long run performance of the economy.

Our main results are as follows. First, we find that an increase in the variance of nominal shocks causes a rise in the variance and a fall in the mean of output growth and that, an increase in the variance of real shocks - preference or technology - causes a rise in the variance but either a fall or a rise in the mean of output growth crucially depending on the reaction of the central bank. The negative effect of the variance of nominal shocks occurs because of the increase in uncertainty about the state of the economy. Greater volatility implies greater uncertainty that induces firms to set higher prices, reducing the real impact of the shocks and lowering the average growth rate of output. In this way, the model predicts a negative relationship between short run volatility and long run growth. However, the higher volatility of real shocks, on the one hand, causes firms more uncertain about future and then set higher prices but, on the other hand, if the central bank accommodates money it may help reduce prices since the money accommodation plays as a stabilizer which helps firms not loss (or gain) too much profits at pre-determined prices when the shocks happen. We show that whether the former effect dominates the latter effect depends on the reaction parameter values of the central bank. In this way, therefore, the model may generate either a negative or positive relationship between short run volatility and long run growth.

The second main result of the paper concerns with the stabilization policy issue. We find that, even when there is an existence of a negative relationship between volatility and growth, a policy that reduces volatility may either increase or decrease growth depending on the type of shocks. For example, in the case of a real (preference or technology) shock, output growth can be maximised by an accommodating monetary policy but the ensuing fluctuations in the money supply cause greater volatility. As such, there is a conflict between the optimal policy that maximises growth and the optimal policy that minimises volatility. In contrast, in the case of a nominal shock, there is no conflict in achieving both of these targets. By implementing a counter cyclical monetary policy, the central bank can mitigate volatility, lowering prices, and therefore maximizing long run output growth. The policy designed to maximise growth is consistent with the policy designed to stabilise fluctuations.

The remainder of the paper is organized as follows. Section 2 describes the structure of the model. Section 3 presents the solutions of the model. Section 4 analyses growth, volatility and stabilization policy. Section 5 calibrates the model and generates simulations. Section 6 concludes.

2. The Model

The economy consists of a constant population (normalised to one) of identical, infinitely-lived agents who consume a fixed number (normalised to one) of differentiated goods produced by monopolistically competitive firms. Agents also supply labour to firms, and sustainable growth occurs through endogenous technological change via learning-by-doing. Stochastic fluctuations arise from three different types of shocks - a preference shock, a technology shock and a monetary shock - the last of which has real effects on the economy due to nominal rigidities as firms set prices one period in advance.

2.1 Households

The differentiated goods are aggregated to produce a single composite good and the representative household derives utility from its consumption of this composite good, real balances, and leisure. Real money balances enter the utility function because of the liquidity services that money provides. The representative household takes prices as given and maximises its expected utility subject to its intertemporal budget constraint. The total wealth of the representative household at time *t* includes money balances carried over from *t – 1*, income from supplying labour services to firms and profits which are distributed equally among households. The household derives life-time utility, *U,* according to

$$U = \sum_{t=0}^{\infty} \beta^t E_t \left[\log(C_t) + \alpha \log \left(\frac{M_t}{P_t} \right) - \Lambda_t L_t^{\eta} \right], \tag{1}$$

$$\Lambda_t = \Lambda_{t-1}^{\rho_\lambda} e^{\lambda_t} \tag{2}$$

$(\beta, \rho_\lambda \in (0,1), \alpha > 0, \eta > 1)$ where C_t denotes the consumption index, M_t denotes nominal money balances, P_t denotes the price index, and L_t denotes labour supply. To generate a demand function for money we introduce money directly into the utility function, rather than specifying explicitly a separate transactions technology. The quantity M_t represents cash balances at the end of period *t.* Λ_t is a random preference parameter, and the logarithm of Λ_t is assumed to follow an AR(1) process with the autoregressive coefficient ρ_λ. The quantity λ_t represents a preference (or taste) shock, being an identically, independently and normally distributed random variable with mean zero and variance σ_λ^2.

Each household chooses consumption, real money balances and labour supply given the price level P_t and subject to the following budget constraint,

$$C_t + \frac{M_t}{P_t} = \frac{W_t}{P_t} L_t + \Phi_t \frac{M_{t-1}}{P_t} + \Pi_t \tag{3}$$

where we make use of that fact that $\int_0^1 p_{it} c_{it} di = P_t C_t$. W_t is the nominal wage, Φ_t is a proportional money transfer and Π_t is real profits. The left-hand-side shows the allocation of resources between consumption and further additions to money holdings, while the right hand side gives the total value of these resources as labour income, existing money balances and profits.

Following Dixit and Stiglitz (1977), the composite consumption good, C_t of the household is defined by the constant elasticity of substitution function,

$$C_t = \left(\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \tag{4}$$

($\theta > 1$) where c_{it} is consumption of differentiated good of type *i*. The general price index, P_t is given by

$$P_t = \left(\int_0^1 p_{it}^{1-\theta} di \right)^{\frac{1}{1-\theta}} \tag{5}$$

The household's problem is described in two stages. First, for any given level of C_t, the household minimises its total expenditures on consumption. Second, given this cost, the household chooses C_t, M_t and L_t in order to maximise its utility. We consider each stage in turn.

In the first stage, the household minimizes the total cost of obtaining the differentiated goods, given their nominal prices p_{it}. Solving this problem we obtain the conventional product demand function,

$$c_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\theta} C_t \tag{6}$$

This demand function states that the demand for good *i* of the representative household is proportional to the aggregate demand, C_t, decreasing in its own price, p_{it} and increasing in the average price level, P_t. In other words, the demand for each type of good is a decreasing function of its relative price.

In the second stage, the representative household chooses plans for the composite consumption good, real money balances, and labour supply so as to maximise its expected utility in (1) subject to the budget constraint in (3). The first order conditions for this problem with respect to C_t, M_t and L_t are, respectively,

$$\frac{1}{C_t} = \kappa_t \tag{7}$$

$$\frac{\kappa_t}{P_t} = \frac{\alpha}{M_t} + \beta E_t \left[\kappa_{t+1} \frac{\Phi_{t+1}}{P_{t+1}} \right] \tag{8}$$

$$\Lambda_t \eta L_t^{\eta-1} = \kappa_t \frac{W_t}{P_t} \tag{9}$$

where κ_t is the Lagrange multiplier associated with (3). Assume (as described later) that the aggregate money supply, H_t, evolves according to $H_t = \Phi_t H_{t-1}$. Money market equilibrium requires that the desired level of total money holdings of the household be equal to the total money stock supplied by the central bank: that is, $M_t = H_t$. Substitution in (8) gives

$$\kappa_t \frac{M_t}{P_t} = \alpha + \beta E_t\left[\kappa_{t+1}\frac{M_{t+1}}{P_{t+1}}\right] \tag{10}$$

This is an expectations difference equation which can be solved forwards in time. Imposing transversality condition, $\lim_{s\to\infty}\left(\kappa_{t+1+s}\frac{M_{t+1+s}}{P_{t+1+s}}\right)=0$, the solution is

$$\kappa_t \frac{M_t}{P_t} = \frac{\alpha}{1-\beta} \tag{11}$$

Substituting (7) into (11) yields

$$C_t = \frac{1-\beta}{\alpha}\frac{M_t}{P_t} \tag{12}$$

Equation (12) states that aggregate demand is proportional to aggregate money holdings. Finally, substituting this into (6) and (9) gives

$$c_{it} = \left(\frac{1-\beta}{\alpha}\right)\left(\frac{M_t}{P_t}\right)\left(\frac{p_{it}}{P_t}\right)^{-\theta}, \tag{13}$$

$$\Lambda_t \eta L_t^{\eta-1} = \left(\frac{\alpha}{1-\beta}\right)\left(\frac{W_t}{M_t}\right) \tag{14}$$

2.2 Firms

For simplicity, firms are assumed to produce differentiated goods through a constant returns to scale technology with labour only. The production function of firm i has the following form:

$$y_{it} = \Gamma_t Z_t n_{it}, \tag{15}$$

$$Z_t = A\Gamma_{t-1} Z_{t-1} N_{t-1} \tag{16}$$

$$\Gamma_t = \Gamma_{t-1}^{\rho_\gamma} e^{\gamma_t} \tag{17}$$

($A > 0, \rho_\gamma \in (0,1)$), where y_{it} denotes output, n_{it} denotes labour and Z_t is a technology factor that represents the accumulated aggregate stock of knowledge, common to all firms in the economy. The evolution of Z_t, given in (16), reflects learning-by-doing from past aggregate output. This provides the mechanism of endogenous growth in the model. The term Γ_t in (15) represents the aggregate technology index and its logarithm is assumed to follow an AR(1) process with the autoregressive coefficient ρ_γ. The term γ_t in (17) represents a technology shock which is assumed to be identically, independently, and normally distributed with mean zero and variance σ_γ^2.

We introduce nominal rigidities into the model by supposing that all prices p_{it} are set one period in advance. That is, each firm chooses a price for its commodity in period t prior to the realisation of shocks in that period. We also assume that the producer of good i is committed to supply whatever quantity consumers demand at the predetermined price, and hence to hire whatever quantity of labour is needed to meet market demand.

Given the above, the problem of a firm is given as

$$\max_{p_{it}} E_{t-1}\left[p_{it}y_{it} - W_t n_{it}\right], \tag{18}$$

subject to

$$y_{it} = c_{it} \tag{19}$$

where c_{it} is given in (13). Note that, since prices are set one period in advance, both p_{it} and P_t are known at time $t-1$. Note also that each firm treats P_t as given since it perceives no influence on aggregate variables. With these considerations in mind, the solution to the above problem implies

$$(\theta-1)p_{it}E_{t-1}\left[M_t\right] = \theta E_{t-1}\left[\frac{M_t W_t}{\Gamma_t Z_t}\right] \tag{20}$$

Observe that, in the absence of uncertainty (or if prices are chosen based on current information), (20) reduces to the standard mark up rule, $p_{it} = \left(\frac{\theta}{\theta-1}\right)\left(\frac{W_t}{\Gamma_t Z_t}\right)$.

2.3 Monetary Policy

In this model shocks to the economy can arise from a variety of sources, namely preferences, technologies and money markets. As in Blackburn and Pelloni (2005), we employ a monetary feedback rule which allows the central bank to imperfectly set the monetary aggregate in response to these shocks. The central bank's imprecise control over the aggregate money supply reflects the fact that the monetary aggregate is imperfectly related to the monetary base - the instrument of the central bank. In particular, we suppose that the money supply, H_t and the proportional monetary transfer, Φ_t are governed by the following stochastic process:

$$H_t = \Phi_t H_{t-1}, \tag{21}$$

$$\Phi_t = \phi_t e^{\varepsilon_{vt}} \tag{22}$$

$$\varepsilon_{vt} = \rho_v \varepsilon_{vt-1} + v_t \tag{23}$$

$$\phi_t = \phi e^{(\varphi_v v_t + \varphi_\lambda \lambda_t + \varphi_\gamma \gamma_t)} \tag{24}$$

($\phi > 1, \rho_v \in (0,1)$) where ε_{vt} is the control error in monetary policy which is assumed to be an AR(1) process with the autoregressive coefficient ρ_v and the disturbance v_t being normally distributed with mean zero and variance σ_v^2. Equation (24) represents the reaction function of the monetary authority or the monetary feedback rule. This reaction function implies that the central bank adjusts the money stock in response to preference shocks, λ_t, technology shocks, γ_t, and monetary shocks, v_t. If any of the feedback rule parameters (φ_v, φ_λ and φ_γ) are non-zero then monetary policy is state-dependent with the central bank reacting in a systematic fashion to the realizations of the shocks. In response to positive shocks, these parameters will be positively-valued if the central bank pursues an accommodating policy, and will be negatively-valued if the bank seeks to counter fluctuations in the economy. For the case in which $\varphi_v = \varphi_\lambda = \varphi_\gamma = 0$, monetary policy is set exogenously, being completely unresponsive to changes in the state of the economy. In this case, the monetary aggregate grows at the exogenous, stochastic rate $\phi e^{\varepsilon_{vt}}$.

3. General Equilibrium

The stochastic dynamic general equilibrium of the economy describes how the exogenous shocks are propagated over time through the intrinsic dynamics that arise from the optimal decision rules of maximizing agents. To compute this equilibrium we first need to impose the clearing conditions for all markets. These include $C_t = Y_t$ for the goods market, $M_t = H_t$ for the money market, and $\int_0^1 n_{it} di = L_t$ for the labour market. We then proceed as follows, combining these conditions with the other relationships derived so far.

Equation (20) implies that $p_{it} = p_t$ for all *i,* which implies that $P_t = p_t$ from (5). Applying the goods and labour market clearing conditions, it follows from (13), (14) and (15) that

$$c_{it} = c_t = \left(\frac{1-\beta}{\alpha}\right)\left(\frac{M_t}{P_t}\right) \tag{25}$$

$$L_t = \left(\frac{\alpha}{\eta(1-\beta)}\frac{1}{\Lambda_t}\right)^{\frac{1}{\eta-1}}\left(\frac{W_t}{M_t}\right)^{\frac{1}{\eta-1}} \tag{26}$$

$$y_{it} = y_t = \left(\frac{1-\beta}{\alpha}\right)\left(\frac{M_t}{P_t}\right) = \Gamma_t Z_t L_t \tag{27}$$

In addition, since $M_t = \Phi_t M_{t-1}$ (as indicated previously), (22), (23) and (24) imply

$$M_t = \phi e^{\rho_v \varepsilon_{vt-1} + (1+\varphi_v) v_t + \varphi_\lambda \lambda_t + \varphi_\gamma \gamma_t} M_{t-1} \tag{28}$$

The above results can be used to derive the equilibrium determination of prices. The procedure is as follows. Using equations (26) and (27), we obtain

$$W_t = \eta \left(\frac{1-\beta}{\alpha} \right)^{\eta} \frac{\Lambda_t M_t^{\eta}}{\left(P_t \Gamma_t Z_t \right)^{\eta-1}} \tag{29}$$

Multiplying through by $\frac{M_t}{\Gamma_t Z_t}$ and taking expectations, E_{t-1}, we have

$$E_{t-1}\left[\frac{M_t W_t}{\Gamma_t Z_t} \right] = \eta \left(\frac{1-\beta}{\alpha} \right)^{\eta} \frac{1}{P_t^{\eta-1} Z_t^{\eta}} E_{t-1}\left[\frac{\Lambda_t M_t^{\eta+1}}{\Gamma_t^{\eta}} \right] \tag{30}$$

Substituting this equation into (20), we arrive at the following expression for p_t:

$$p_t = \left(\eta \frac{\theta}{\theta-1} \right)^{\frac{1}{\eta}} \left(\frac{1-\beta}{\alpha} \right) \frac{1}{Z_t} \frac{\left(E_{t-1}\left[\frac{\Lambda_t M_t^{\eta+1}}{\Gamma_t^{\eta}} \right] \right)^{\frac{1}{\eta}}}{\left(E_{t-1}\left[M_t \right] \right)^{\frac{1}{\eta}}} \tag{31}$$

The expectations term in the numerator of this expression is computed as follows. Combining (2) and (17) with (28) gives

$$\frac{\Lambda_t M_t^{\eta+1}}{\Gamma_t^{\eta}} = \frac{\Lambda_{t-1}^{\rho_\lambda} \left(\phi e^{\rho_v \varepsilon_{vt-1}} M_{t-1} \right)^{\eta+1}}{\Gamma_{t-1}^{\eta \rho_\gamma}} e^{(\eta+1)(\varphi_v+1)v_t + ((\eta+1)\varphi_\lambda+1)\lambda_t + ((\eta+1)\varphi_\gamma - \eta)\gamma_t} \tag{32}$$

This expression involves a term of the form e^x, where x is a normally distributed random variable with mean zero and variance σ^2. The expected value of this term is $e^{\frac{1}{2}\sigma^2}$. Thus, taking expectations through (32) and (28) and substituting the results back into (31) yields

$$p_t = \left(\eta\frac{\theta}{\theta-1}\right)^{\frac{1}{\eta}}\left(\frac{1-\beta}{\alpha}\right)\frac{1}{Z_t}\frac{\left(\frac{\Lambda_{t-1}^{\rho_\lambda}\left(\phi M_{t-1}e^{\rho_v\varepsilon_{vt-1}}\right)^{\eta+1}}{\Gamma_{t-1}^{\eta\rho_\gamma}}\right)^{\frac{1}{\eta}}}{\left(\phi M_{t-1}e^{\rho_v\varepsilon_{vt-1}}\right)^{\frac{1}{\eta}}}\frac{\left[e^{\sigma_1^2}\right]^{\frac{1}{\eta}}}{\left[e^{\sigma_2^2}\right]^{\frac{1}{\eta}}}$$

$$\sigma_1^2 = \frac{1}{2}\left((\eta+1)(1+\varphi_v)\right)^2\sigma_v^2 + \frac{1}{2}\left((\eta+1)\varphi_\lambda+1\right)^2\sigma_\lambda^2 + \frac{1}{2}\left((\eta+1)\varphi_\gamma-\eta\right)^2\sigma_\gamma^2$$

$$\sigma_2^2 = \frac{(1+\varphi_v)^2}{2}\sigma_v^2 + \frac{\varphi_\lambda^2}{2}\sigma_\lambda^2 + \frac{\varphi_\gamma^2}{2}\sigma_\gamma^2$$

Or equivalently,

$$p_t = \left(\eta\frac{\theta}{\theta-1}\right)^{\frac{1}{\eta}}\left(\frac{1-\beta}{\alpha}\right)\frac{1}{Z_t}\frac{\Lambda_{t-1}^{\frac{1}{\eta}\rho_\lambda}\phi M_{t-1}e^{\rho_v\varepsilon_{vt-1}}}{\Gamma_{t-1}^{\rho_\gamma}}e^{\sigma^2} \tag{33}$$

where

$$\sigma^2 = \frac{1}{2}(\eta+2)(1+\varphi_v)^2\sigma_v^2 + \frac{1}{2\eta}\left(\eta(\eta+2)\varphi_\lambda^2 + 2(\eta+1)\varphi_\lambda+1\right)\sigma_\lambda^2 + \frac{1}{2}\left((\eta+2)\varphi_\gamma^2 - 2(\eta+1)\varphi_\gamma+\eta\right)\sigma_\gamma^2$$

Equation (31) shows that, ceteris paribus, an expected increase in M_t, an expected increase in Λ_t, or an expected decrease in Γ_t would raise the predetermined price, p_t, just as an actual increase in M_t, an actual increase in Λ_t, or an actual decrease in Γ_t would raise the expected price. The effect of M_t is due to the increase in demand for a firm's product. The effect of Λ_t is due to the increase in marginal cost (nominal wages) resulting from a decrease in labour supply. And the effect of Γ_t is due to the increase in marginal cost resulting from lower productivity.

Equation (33) gives the final expression for prices. The term M_{t-1} reflects the fact that, a higher money supply in the past implies a higher expected money supply in the future, inducing the firm to set a higher price. The term Z_t reflects the learning-by-doing effect which leads to a higher level of productivity, a lower marginal cost, and therefore a lower price. The term e^{σ^2} captures the relationship between the volatility of the shocks and prices. In the case of nominal shocks, it is always true that greater volatility implies greater uncertainty which induces the firm to set a higher price (see Benassy, 1995, Fischer, 1977, Rankin, 1998, and Sorensen, 1992 for examples). However, in the case of real shocks, this relationship crucially depends on the central bank's reaction parameters. This is because a higher variance of any of the shocks always makes firms more uncertain about the future hence set higher prices. Any response of the central bank, on the one hand, will make money more fluctuate inducing firms to increase prices further. On the other hand, however, if the central bank accommodates the shocks it helps reduce uncertainty and predetermined prices since firms will not loss or gain too much profits in case the shocks happen. The higher variance of shocks the more the central bank will respond to accommodate and the lower price level is preset by firms. There exist a range of values of the reaction parameters that make the latter effect outweigh the prior effect of the money accommodation and hence produce a negative relationship between volatility and the price level. In particular, the effect of the volatility of real shocks on prices is negative if $\varphi_\lambda \in \left(-\frac{1}{\eta}, -\frac{1}{\eta+2}\right)$ and $\varphi_\gamma \in \left(\frac{\eta}{\eta+2}, 1\right)$ and positive otherwise. This result is consistent with the findings by Kehoe (1996) and Rankin (1994) who also found a negative effect of product quality and monetary uncertainty on the price level, respectively.

4. Growth, Volatility and Stabilisation Policy

We now turn to an analysis of the relationship between the cyclical and secular fluctuations in the economy and the implications of this relationship for monetary policy. As shown in the previous section, nominal rigidities allow unexpected monetary shocks to have real effects on output. As shown below, the presence of learning-by-doing implies that these shocks (as well as others) have permanent (not just transitory) effects on output. And as also shown below, the volatility of these shocks has a permanent influence on the growth rate (not just the level) of output. The last of these results has an obvious bearing on stabilisation policy.

4.1 The Output Process

Substituting equation (33) into (27), one obtains

$$y_t = \left(\frac{1}{\eta}\frac{\theta-1}{\theta}\right)^{\frac{1}{\eta}} \frac{\Gamma_{t-1}^{\rho_\gamma}}{\Lambda_{t-1}^{\frac{1}{\eta}\rho_\lambda}} Z_t \frac{e^{\varepsilon_t}}{e^{\sigma^2}} \tag{34}$$

where $\varepsilon_t = (1+\varphi_v)v_t + \varphi_\lambda \lambda_t + \varphi_\gamma \gamma_t$.

This expression shows that, in the presence of nominal rigidities, unexpected changes in the money supply have real effects on output. This result would disappear if, instead of assuming one-period price pre-setting, we use the assumption that prices are chosen contingent on the realisation of M_t. Under such circumstances, the deviation of output from its perfectly competitive level would be proportional to the mark-up. Money would be neutral and output would depend only on the preference and the technological disturbances.

Equation (34) shows additionally that output depends not only on the realisations but also on the variances of the exogenous shocks. However, the preference shock, λ_t and the technology shock, γ_t would disappear from this expression if monetary policy was unresponsive to them (i.e. if $\varphi_\lambda = \varphi_\gamma = 0$). In any period, a positive realisation of any of the shocks leads to either a higher or a lower level of the money supply depending on whether the central bank implements an accommodating or counter-cyclical monetary policy. With prices being fixed in that period, a higher level of the money supply leads to a higher level of demand and hence output. In addition, if monetary policy is exogenous, firms will set their prices at a higher level if they are more uncertain about the future, therefore larger variances of the shocks lead to a higher level of price, lowering demand and output. However, if monetary policy is accommodating, as implied by equation (33), there exist a range of values of the central bank's feedback coefficients that lead to a negative relationship between volatility of real shocks and the price level.

Note that with $Z_t = Ay_{t-1}$ we can derive the growth rate of output by log-linearising (34). Defining $\hat{y}_t = \log(y_t)$, we then have

$$\hat{y}_t - \hat{y}_{t-1} = \delta - \sigma^2 + \varepsilon_t - \frac{1}{\eta}\rho_\lambda \varepsilon_{\lambda t-1} + \rho_\gamma \varepsilon_{\gamma t-1} \tag{35}$$

where $\delta = \frac{1}{\eta}\log\left(\frac{\theta-1}{\eta\theta}\right) + \log(A)$, $\varepsilon_{\lambda t} = \log(\Lambda_t)$ and $\varepsilon_{\gamma t} = \log(\Gamma_t)$. Since $\varepsilon_{\lambda t}$ and $\varepsilon_{\gamma t}$ both follow an AR(1) process (that is, $\varepsilon_{\lambda t} = \rho_\lambda \varepsilon_{\lambda t-1} + \lambda_t$ and $\varepsilon_{\gamma t} = \rho_\gamma \varepsilon_{\gamma t-1} + \gamma_t$), which can always be written as a moving average - MA(∞) process of infinite order, equation (35) can be rewritten as

$$\begin{aligned} \hat{y}_t = \zeta + \hat{y}_{t-1} - \frac{1}{\eta}\left[\eta\varphi_\lambda \lambda_t + \rho_\lambda \lambda_{t-1} + \rho_\lambda^2 \lambda_{t-2} + \ldots\right] \\ + \left[\varphi_\gamma \gamma_t + \rho_\gamma \gamma_{t-1} + \rho_\gamma^2 \gamma_{t-2} + \ldots\right] + (1+\varphi_v)v_t \end{aligned} \tag{36}$$

where $\zeta = \delta - \sigma^2$ and $t = 0,1,2,\ldots$ Equation (36) follows an autoregressive moving average - ARMA($1,\infty$) process, which is a combination of a random walk, with ζ being the drift parameter, and a moving average. This process implies that the economy experiences stochastic and sustainable growth. The process can also be written as

$$\hat{y}_t = \hat{y}_0 + \zeta t + \sum\nolimits_{j=0}^{t} c_{1j}\lambda_{t-j} + \sum\nolimits_{j=0}^{t} c_{2j}\gamma_{t-j} + \sum\nolimits_{j=0}^{t} c_{3j}v_{t-j} \tag{37}$$

Equation (37) indicates that the exogenous shocks have permanent effects on the level of output. This is because of learning-by-doing which provides the mechanism of endogenous growth in the model. Under such circumstances, even purely temporary shocks can have permanent effects on output and, in the presence of nominal rigidities, even monetary shocks can have this effect. Of more interest to us is the fact that the drift term in (36), ζ , depends negatively on the variances of the shocks. This indicates a negative relationship between long run growth and volatility.

To illustrate this relationship more concretely, we compute the mean and variance of output growth as, respectively,

$$\begin{aligned}\text{Mean}(\hat{y}_t-\hat{y}_{t-1}) = \delta - \frac{1}{2\eta}\left(\eta(\eta+2)\varphi_\lambda^2 + 2(\eta+1)\varphi_\lambda + 1\right)\sigma_\lambda^2 \\ -\frac{1}{2}\left((\eta+2)\varphi_\gamma^2 - 2(\eta+1)\varphi_\gamma + \eta\right)\sigma_\gamma^2 - \frac{1}{2}(\eta+2)(1+\varphi_v)^2\sigma_v^2\end{aligned} \tag{38}$$

$$\text{Var}(\hat{y}_t-\hat{y}_{t-1}) = \left[\varphi_\lambda^2 + \frac{1}{\eta^2}\frac{\rho_\lambda^2}{1-\rho_\lambda^2}\right]\sigma_\lambda^2 + \left[\varphi_\gamma^2 + \frac{\rho_\gamma^2}{1-\rho_\gamma^2}\right]\sigma_\gamma^2 + (1+\varphi_v)^2\sigma_v^2 \tag{39}$$

These expressions show that, in general, a rise in the variance of any of the shocks causes a rise in the variance of output growth, but either a fall or a rise in the mean of output growth. As can be seen from (33), this is because firms will set their prices at a higher level if they are more uncertain about the future. A larger variance of any of the shocks results in a higher level of price, lowering demand and output. Any response of the central bank will make the monetary aggregate more fluctuate and firms more uncertain leading to a higher level of price. However, in the case of real shocks, the money accommodation which is positively related to the variance of the shocks also relieves uncertainty through stabilizing firms' loss or profits and hence reducing the price level, promoting demand and output. This general up and down in real economic performance is translated into a rise or a fall in average growth by virtue of a rise or a fall in the rate of technological progress via the process of learning-by-doing. As a result, the model may generate either a negative or a positive correlation between long run (secular) growth and short run (cyclical) volatility depending on the source of shocks and the central bank's reaction parameters.

In particular, a rise in the variance of the nominal shock always leads to a fall in output growth implying a negative correlation between long-run growth and short run volatility. However, a rise in the variance of the real, preference or technology, shocks causes a rise in the mean of output growth if $\varphi_\lambda \in \left(-\frac{1}{\eta}, -\frac{1}{\eta+2}\right)$ and $\varphi_\gamma \in \left(\frac{\eta}{\eta+2}, 1\right)$ and a fall otherwise.

The negative correlation between long run growth and short run volatility implied by the model is consistent with the predictions of a number of other theoretical models such as Martin and Rogers (1997), de Hek (1999), and Blackburn and Pelloni (2005), as well as empirical studies such as Ramey and Ramey (1995), Martin and Rogers (2000), and Kneller and Young (2001). On the contrary, the positive correlation have also been found by many theoretical models of Aghion and St. Paul (1998b), de Hek (1999), Blackburn and Galindev (2003), Dotsey and Sarte (2000) and empirical models of Kormedi and Meguire (1985), Grier and Tullock (1989), Caporale and McKiernan (1996), and Grier and Perry (2000).

4.2 Growth, Volatility and Stabilisation Policy

The existence of a relationship between long run growth and short run volatility has potentially important implications for stabilisation policy. We seek to reveal these implications in the analysis that follows.

In the present framework, stabilisation policy is modeled explicitly in (24) as a feedback rule for monetary policy through which the central bank responds endogenously and systematically to the realizations of the shocks. Suppose that the central bank cares about both minimising short term volatility, as expressed in (39), and maximising long term growth, as given in (38). In the case of the nominal shock v_t there is no conflict between these two objectives. The optimal value for φ_v is the same for either minimising $\mathrm{Var}(\hat{y}_t - \hat{y}_{t-1})$ or maximising $\mathrm{Mean}(\hat{y}_t - \hat{y}_{t-1})$. With $\varphi_v = -1$ the policy completely offsets the effect of nominal volatility which would otherwise cause greater real volatility and lower real growth. This is an example of how stabilisation monetary policy can be complementary to the promotion of growth. By contrast, in the case of the preference shock λ_t the result is different. It is optimal for the central bank to set $\varphi_\lambda = 0$ for the pursuance of minimising $\mathrm{Var}(\hat{y}_t - \hat{y}_{t-1})$ but $\varphi_\lambda = -\dfrac{\eta+1}{\eta(\eta+2)}$ for the pursuance of maximising $\mathrm{Mean}(\hat{y}_t - \hat{y}_{t-1})$. This is because (as above) prices are predetermined one period in advance and hence depend on the expectation (not the realisation) of this shock. If monetary policy does not respond to the preference shock ($\varphi_\lambda = 0$) then output remains the same. But if monetary policy responds negatively (in particular,

$\varphi_\lambda = -\frac{\eta+1}{\eta(\eta+2)}$), then output fluctuates; the price level is lower which implies higher average employment and higher average growth. Similarly, with the technology shock γ_t, minimising $\mathrm{Var}(\hat{y}_t - \hat{y}_{t-1})$ requires $\varphi_\gamma = 0$, but maximising $\mathrm{Mean}(\hat{y}_t - \hat{y}_{t-1})$ requires $\varphi_\gamma = \frac{\eta+1}{\eta+2}$. Again, due to nominal rigidities only expectations about the shock matter. In the absence of any monetary policy response ($\varphi_\gamma = 0$), output is unchanged. In the presence of a response (in particular, $\varphi_\gamma = \frac{\eta+1}{\eta+2}$) output varies but prices are lower, implying higher average employment and growth.

As shown above, the model provides a simple illustration of how different scenarios may lead to different conclusions about the extent to which there may exist a trade-off relationship between short term stabilisation and long term growth. The negative relationship between short run volatility and long term growth does not mean that the policy which maximises long run growth coincides with the policy which minimises short term fluctuations. It has been shown that, depending on the source of fluctuations a policy that decreases short run volatility may either raise (in the case of nominal shocks) or reduce (in the case of preference or technology shocks) long run growth. In terms of optimising this trade-off, assume that the policy maker has the following objective function:

$$V = \mathrm{Mean}(\hat{y}_t - \hat{y}_{t-1}) - \mu \mathrm{Var}(\hat{y}_t - \hat{y}_{t-1}) \tag{40}$$

where $\mu > 0$ is the weight the policy maker assigns to the stabilisation target relative to the growth target. Taking the first order conditions with respect to $\varphi_v, \varphi_\lambda$ and φ_γ we can obtain the optimal values of these parameters. These values are $\varphi_v = -1$, $\varphi_\lambda = -\frac{\eta+1}{\eta(\eta+2+2\mu)}$ and $\varphi_\gamma = \frac{\eta+1}{\eta+2+2\mu}$, which reduce to the previous results by setting $\mu = 0$ or $\mu = \infty$.

5. Simulations

The results obtained so far yield quantitative predictions about the relationship between growth, volatility and stabilisation based on analytical solutions for variables. In what follows, we conduct a quantitative analysis using numerical simulation of a calibrated, log-linearised version of the model. As before, we define $\hat{x} = \log(x)$. The model can be described compactly by

$$G_0 X_t = G_1 X_{t-1} + C + \Psi \xi_t \tag{41}$$

where G_o and G_1 are parameter coefficient matrices corresponding to the endogenous variables vector $X_t = (\hat{m}_t, \hat{p}_t, \hat{y}_t, \hat{z}_t, \hat{n}_t, \hat{w}_t, \varepsilon_{\lambda t}, \varepsilon_{\gamma t}, \varepsilon_{vt})'$, C is a vector of constants, and Ψ is a parameter matrix corresponding to the disturbance vector $\xi_t = (\lambda_t, \gamma_t, v_t)'$.

The model is calibrated with the parameter values given as follows. The parameter values chosen are very standard in the literature. The discount factor, β, and the structural parameter in the utility function, α, are set at 0.985 and 0.7 respectively; the inverse of the of labour supply elasticity, η, is given a value of 3; the elasticity of demand for differentiated goods, θ, is assigned a value of 10; all the shocks are assumed to follow an AR(1) process with the auto-correlated coefficients $\rho_\lambda = \rho_\gamma = \rho_v = 0.6$ and have the same standard deviations $\sigma_\lambda = \sigma_\gamma = \sigma_v = 1\%$; ϕ and A are set at 1 and 1.6 respectively; and the weight assigned to the stabilisation target relative to the growth target, μ, is set at 1/3. Using the method of solving linear difference equations by Sims (2001) we examine the impulse responses for several scenarios. These impulse response functions are plotted with respect to a 1% temporary shock in preferences, technology and monetary policy. Four scenarios of the model are considered, each summarized by a row in Table 1. The first one is the case where the central banker sets the money supply exogenously and does not respond to fluctuations in the economy. The second and the third ones are for the cases where the central banker aims at maximizing output growth and minimizing output variability, respectively. Finally, the last one is the scenario where the central banker seeks to optimize a trade-off between these objectives.

Table 1: Monetary Policy Reaction Parameters

Shocks	**Preference**	**Technology**	**Nominal**
	φ_λ	φ_γ	φ_v
(1) Exogenous Money	0	0	0
(2) Max. Mean Growth	$-(\eta+1)/\eta(\eta+2)$	$(\eta+1)/(\eta+2)$	-1
(3) Min. Volatility	0	0	-1
(4) Max. Objective Function	$\varphi_\lambda = -\dfrac{\eta+1}{\eta(\eta+2+2\mu)}$	$\varphi_\gamma = \dfrac{\eta+1}{\eta+2+2\mu}$	-1

Figures 1-3 show the impulse responses of prices and output to a 1% shock in preferences, technology and monetary policy respectively. In all cases, the way the price level responds is the same regardless of the reaction parameters set by the central bank. This is because of the nominal rigidity. Prices depend only on the expectations not the realizations of the shocks. As we can see from equation (33), changes in monetary reaction parameters lead to changes in the constant term which affects only the variable mean not the impulse responses. The impulse responses are responses to equation disturbances, not to changes in the constant terms.

As expected, prices increase in response to a contractionary preference shock, which causes a fall in labour supply and hence output, and an expansionary nominal shock, which causes an increase in aggregate demand. However, they decease in response to a positive technology shock. The persistence of prices depends on the persistence of the shocks. With money and output being constructed as AR(1) processes and with the parameter values chosen for the model, prices converge to a new level in about ten quarters after the shocks.

The impulse response functions of output are different with different monetary policy parameters and with different sources of shocks. The solid lines represent the case where money is set exogenously ($\varphi_\lambda = \varphi_\gamma = \varphi_v = 0$). Output decreases in response to an adverse preference shock but rises in response to a positive technology and nominal shock.

In the case of real shocks, if the central bank seeks only to minimize output volatility it will not respond to these shocks so that money is exogenous. In contrast, if the central bank aims to maximise long run growth it will accommodate the shocks. Money falls (rises) in response to an adverse (favorable) real shock, causing output decrease (increase) further. As analysed in the analytical section, an accommodating monetary policy to real shocks, on the one hand, can help keep price expectations low, raising long run growth. On the other hand, this policy causes more fluctuations in output since money fluctuates. The dashed lines in Figure 1 and 2 show the impulse response functions of money and output corresponding the monetary policy feedback parameters that maximising average long run growth. The impulse response functions in the case the central bank seeking to optimize the trade-off between short run volatility and long run growth (i.e. optimising the objective function (40)) are described by the lines which are somewhere in between the solid lines (minimising volatility) and the dashed lines (maximising long run growth). The trade-offs between short run volatility and long run growth rate when monetary policy parameters changing from minimising volatility to maximising growth in response to real shocks are also depicted in Figure 4 and 5.

Finally, in the case of a nominal shock, the parameters for maximising growth policy coincide with the parameter stabilisation policy. The central bank can achieve both of these targets by implementing a counter cyclical monetary policy which fully offsets the increase in price expectations ($\varphi_v = -1$) and minimises nominal fluctuations. Money increases persistently and output jumps to a new level.

6. Conclusions

The paper aims to analyse the question of how cyclical fluctuations might affect long run growth and how this might have implications for stabilisation policy. The analysis shows that first, an increase in the variance of any type of shock – nominal or real, demand or supply – causes a rise in the variance of output growth. Second, an increase in the variance of nominal shocks causes a fall in the mean of output growth, but an increase in the variance of real shocks causes either a rise or a fall in the mean of output growth depending on the monetary reaction parameters of the central bank. In the case of nominal shocks, greater volatility implies greater uncertainty that induces firms to set higher prices, reducing the real impact of the shocks and lowering the average growth rate of output. In this way, the model predicts a negative relationship between short run volatility and long run growth. However, in the case of real shocks, greater volatility also triggers monetary accommodation which plays as a stabilizer hence reducing pre-determined prices and promoting the average growth rate of output. As a result, the effect of volatility on growth is mixed and the model generates either a negative or positive relationship between short run volatility and long run growth. We show that, the relationship depends on the source of shocks and the reaction of the central bank.

In addition, the model also shows that, despite the existence of a negative relationship between cyclical volatility and secular growth, the policy that is designed to stabilise short run volatility may either increase or decrease growth. In other words, there may be a conflict between the optimal policy that maximises growth and the optimal polity that minimises volatility. In particular, since prices are predetermined one period in advance, they depend on the expectations of the shocks. In the case of a real shock, if monetary policy does not respond then output remains the same. However if the central bank accommodates the shock, there exists a range of values of the feedback parameters that makes output increase further but also fluctuate more since it triggers fluctuations in the money supply, implying higher average growth as well as greater volatility. As a result, there is a conflict between the optimal policy that maximises growth and the optimal policy that minimises volatility.

In contrast, in the case of a nominal monetary shock, there is no conflict in achieving both of these targets. By implementing a counter cyclical monetary policy the central bank can mitigate volatility, lowering prices, and therefore maximising output growth. The policy designed to maximise growth is consistent with the policy designed to stabilise fluctuations. Analytical and numerical examinations are carried out to indicate that these results crucially depend on the source of shocks.

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LIST OF FIGURES

Figure 1: The Impulse Response Functions to a 1% Preference Shock

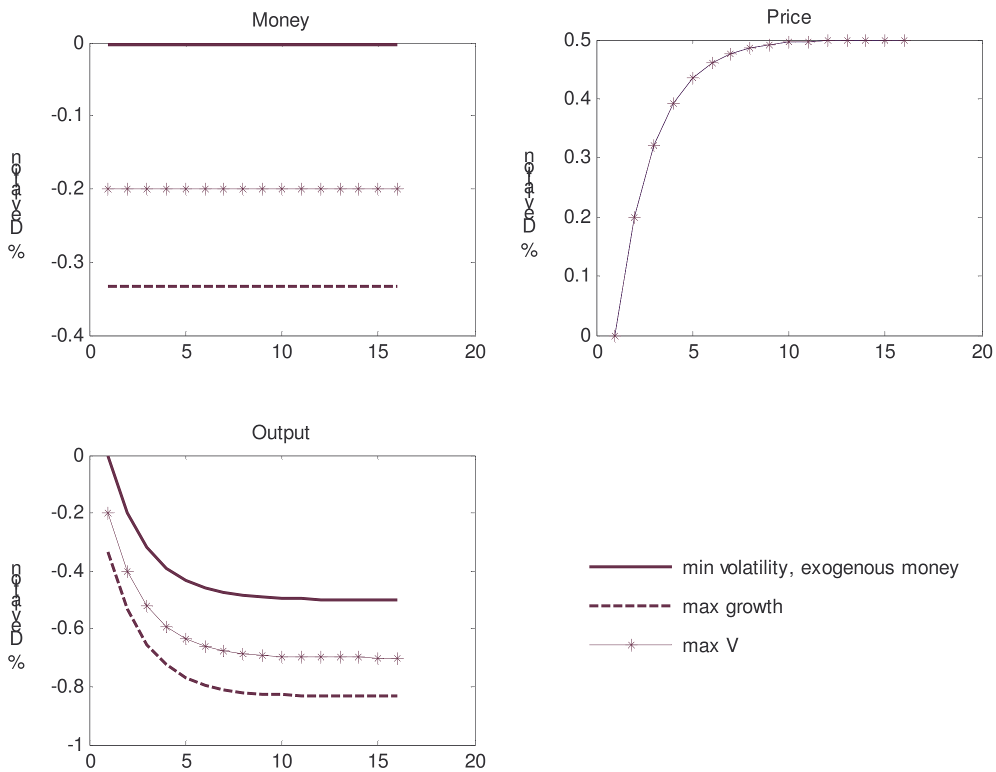

Figure 2: The Impulse Response Functions to a 1% Technology Shock

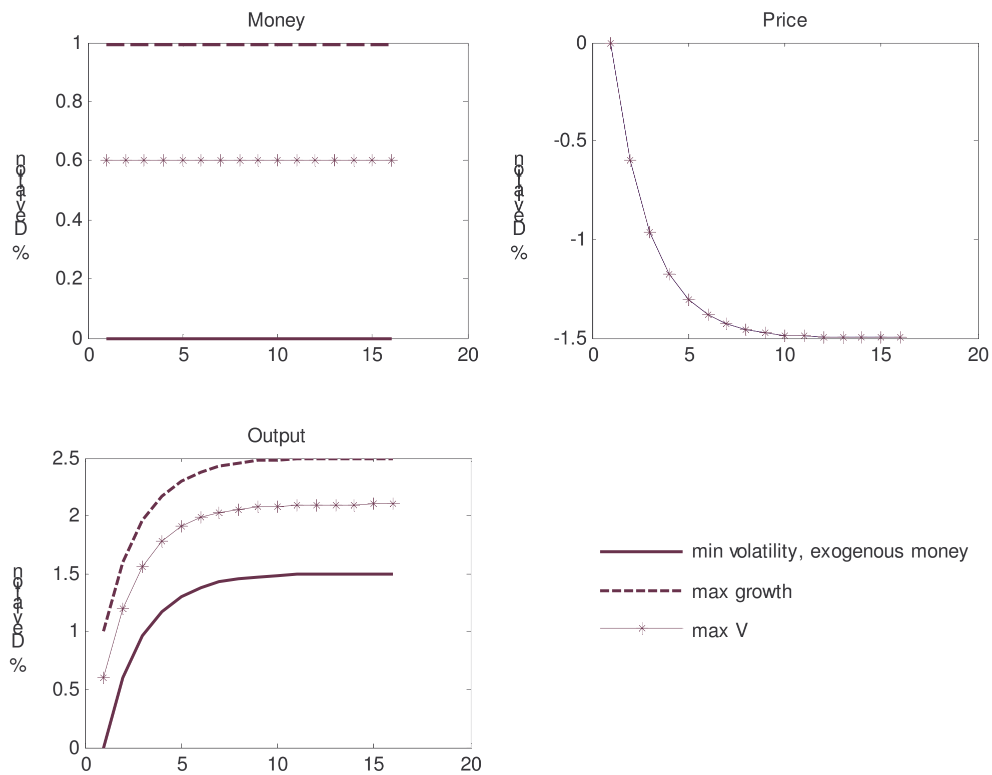

Figure 3: The Impulse Response Functions to a 1% Monetary Policy Shock

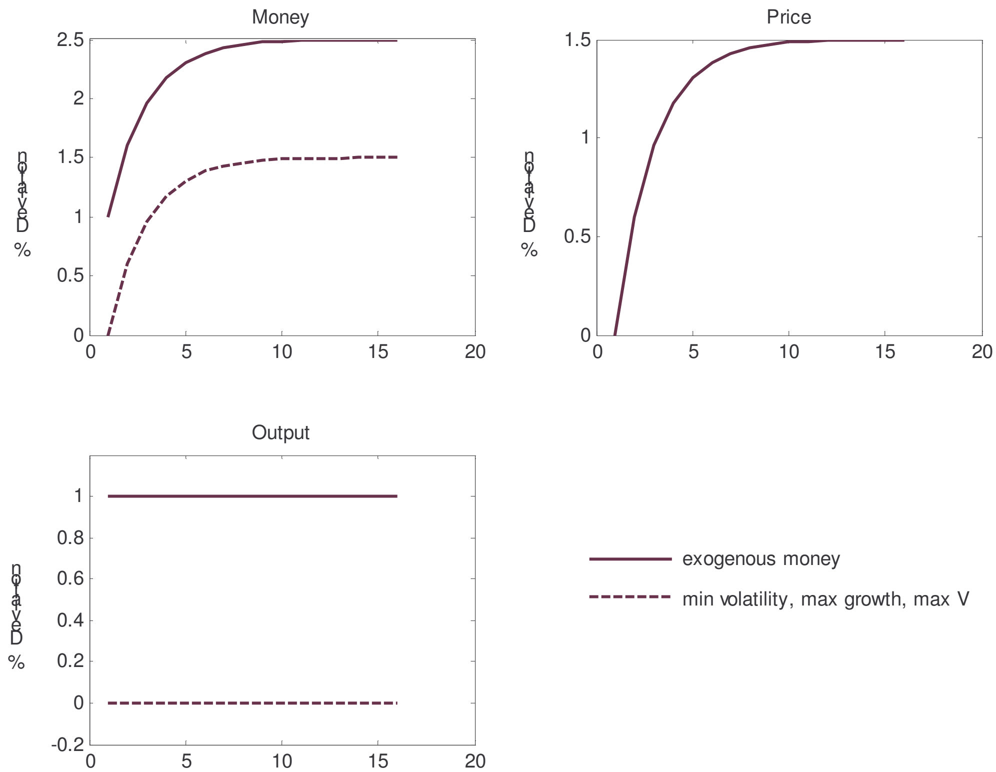

Figure 4: Growth and Volatility Trade-off: Preference Shock $\left(\varphi_\lambda = [-\frac{\eta+1}{\eta(\eta+2)}, 0]\right)$

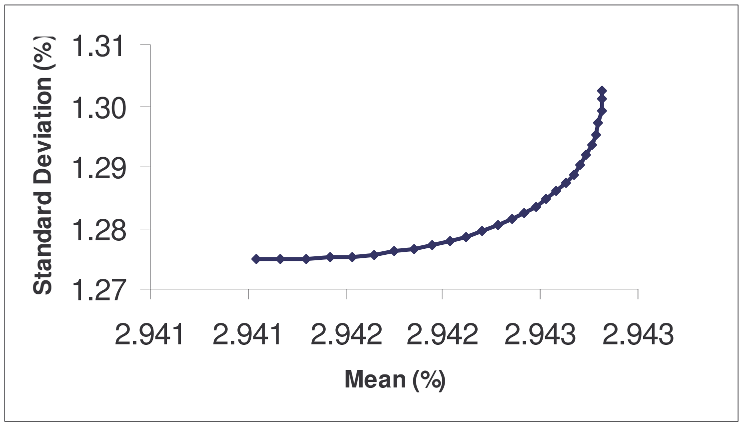

Figure 5: Growth and Volatility Trade-off: Technology Shock $\left(\varphi_\gamma = [0, \frac{\eta+1}{\eta+2}]\right)$