

## **Sales Tax: Specific or Ad Valorem Tax for a Non-renewable Resource?**

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# Sales Tax: Specific or Ad Valorem Tax for a Non-renewable Resource?

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## Abstract

This paper shows that for a *time-independent* specific tax and a *time-independent* ad valorem tax that induce the same competitive equilibrium in the Hotelling model of resource extraction, the ad valorem tax yields a higher level of discounted tax revenues than the specific tax. Moreover, given the same level of discounted tax revenues, the ad valorem tax also yields a higher level of social welfare. Finally, for the time-dependent schedules of *optimal* ad valorem tax and *optimal* specific tax, we show that when appropriately set, they are equivalent in implementing the dynamic social optimum and providing the same discounted tax revenues.

Keywords: Non-renewable Resources, Ad Valorem Tax, Specific Tax, Welfare  
JEL Classification: H21, Q30

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In the literature on taxation, the form of the sales tax does not matter in static analysis: the authorities can use either a specific tax, which creates a gap  $\theta$  between producer and consumer prices, or an ad valorem tax, as a percentage  $\tau$  of the producer price, to implement the same resource allocation  $(p, q)$ , where  $p$  and  $q$  denote, respectively, the market price and quantity, with  $p = f(q)$ ,  $f$  being the inverse market demand function. The specific tax and the ad valorem tax are said to be equivalent in that they create the same distortion, and induce the same welfare loss for an equal tax receipt  $\theta q = \tau p q$ . Does this feature hold in a dynamic framework? This is the main question we ask in this paper. We first focus on the time-independent tax schedule because of its widespread use in many tax legislations (see Sarma and Naresh (2001)). In Section 1 of the paper, we show – in the simple Hotelling model of competitive resource extraction (see Hotelling (1931)) – that when a time-independent specific tax and a time-independent ad valorem tax induce the same competitive equilibrium, the ad valorem tax yields a higher level of tax revenues than the specific tax. In Section 2, we derive the welfare implications of these

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two time-independent forms of sales taxes, when they provide the same discounted tax revenues, and conclude that the ad valorem tax should be chosen as the appropriate instrument in tax design. In Section 3, we turn to the optimal time-dependent schedule of the sales tax, and argue that the dynamic ad valorem and specific taxes – although they exhibit quite different characteristics – are equivalent in implementing the same dynamic pattern of resource allocation and provide an identical level of discounted tax revenues.

## 1. Sales Tax in the Hotelling Model

Consider a competitive market for a non-renewable resource. Time is continuous and denoted by  $t, t \geq 0$ . The total resource stock of the industry at time 0 is  $S_0$ , and the resource can be extracted at a constant unit extraction cost of  $\gamma$ . Let  $p = f(q)$  be the inverse market demand curve for the resource at each instant, where  $p$  is the price paid by consumers for a unit of the resource, and  $q$  is the market demand at price  $p$ . We shall assume that  $f(q) > 0, f'(q) < 0$  for all  $q > 0$ , and  $\lim_{q \rightarrow 0} f(q) = \infty$ . The last assumption implies that the resource stock will not be depleted in finite time, but asymptotically. In what follows, we shall let  $u(q) = \int_0^q f(x)dx$  denote the instantaneous social welfare obtained when  $q$  units of the resource are consumed. The market rate of interest is  $r$ .

Suppose that the authorities levy a time-independent specific tax, say  $\theta$ , on the firms in the industry. To find the competitive equilibrium induced by the specific tax  $\theta$ , let us consider the following maximization problem:

$$(1) \quad \max \int_0^{\infty} e^{-rt} [u(q_t) - (\gamma + \theta)q_t] dt$$

subject to

$$(2) \quad q_t \geq 0, \int_0^{\infty} q_t dt \leq S_0.$$

Note that the expression under the integral sign in (1) represents the discounted value of the sum of the consumer and producer surplus at time  $t$ . Invoking the maximum principle,

we can assert that the optimal extraction rate at time  $t$  is the value of  $q$  that satisfies the following first-order condition:

$$(3) \quad e^{-rt}(u'(q) - \gamma - \theta) = \lambda.$$

In (3),  $\lambda$  is a positive constant, which represents the discounted scarcity rent (shadow price) of the resource stock. We shall let  $q_t(\theta, \lambda)$  denote the value of  $q$  that solves (3); that is,

$$(4) \quad q_t(\theta, \lambda) = f^{-1}(\gamma + \theta + \lambda e^{rt}).$$

As defined,  $q_t(\theta, \lambda)$  is the optimal extraction rate at time  $t$ . Furthermore, the optimal extraction program  $q_t(\theta, \lambda), t \geq 0$ , satisfies the following stock constraint:

$$(5) \quad \int_0^{\infty} q_t(\theta, \lambda) dt = \int_0^{\infty} f^{-1}(\gamma + \theta + \lambda e^{rt}) dt = S_0.$$

Note that the second integral in (5) is strictly decreasing in  $\lambda$ . Furthermore, it tends to infinity when  $\lambda$  tends to 0, and tends to 0 when  $\lambda$  tends to infinity. Hence by continuity there exists a unique value of  $\lambda$ , say  $\lambda(\theta)$ , such that

$$(6) \quad \int_0^{\infty} q_t(\theta, \lambda(\theta)) dt = \int_0^{\infty} f^{-1}(\gamma + \theta + \lambda(\theta) e^{rt}) dt = S_0.$$

Next, let  $p_t(\theta, \lambda(\theta)) = f(q_t(\theta, \lambda(\theta))), t \geq 0$ , denote the resource price along the optimal trajectory of the maximization problem constituted by (1) and (2). Equation (3) now takes on the following form:

$$(7) \quad e^{-rt}(p_t(\theta, \lambda(\theta)) - \gamma - \theta) = \lambda(\theta),$$

which can also be expressed under the following form, known as the Hotelling rule:

$$(8) \quad \frac{\partial}{\partial t} p_t(\theta, \lambda(\theta)) = r[p_t(\theta, \lambda(\theta)) - \gamma - \theta].$$

Now under the competitive equilibrium the net price of one unit of the resource – net of unit extraction cost and the specific tax – must appreciate at least at the market rate of interest in order for that unit of resource to remain *in situ*. Furthermore, if the net price of the resource appreciates at a rate below the market rate of interest, then the entire resource stock will be instantaneously extracted. Differential equation (8) thus describes the evolution of the resource price under the competitive equilibrium induced by the

specific tax  $\theta$ . It follows directly from (8) that the resource price increases monotonically over time, while the corresponding extraction rate decreases, and approaches zero asymptotically.

We now turn to the ad valorem tax, which is levied as a constant percentage  $\tau$  of the resource price. Let  $p_t, t \geq 0$ , denote the resource price at time  $t$  under the competitive equilibrium induced by the time-independent ad valorem tax  $\tau$ . The equilibrium condition for the asset market along the time path of the competitive equilibrium induced by the constant ad valorem tax  $\tau$  is given by

$$(9) \quad e^{-rt}[(1-\tau)p_t - \gamma] = (1-\tau)p_0 - \gamma.$$

Dividing (9) by  $(1-\tau)$ , we obtain

$$(10) \quad e^{-rt} \left[ p_t - \gamma - \frac{\tau\gamma}{1-\tau} \right] = p_0 - \frac{\gamma}{1-\tau} - \frac{\tau\gamma}{1-\tau}.$$

Observe that (10) also describes the resource price at each instant under the competitive equilibrium induced by the specific tax rate  $\theta = \tau\gamma/(1-\tau)$ ; that is, the ad valorem tax  $\tau$  and the specific tax  $\theta$ , with  $\theta = \tau\gamma/(1-\tau)$ , induce the same competitive equilibrium. We now establish:

**PROPOSITION 1:** *If a constant specific tax and a constant ad valorem tax induce the same competitive equilibrium, then at each instant the ad valorem sale tax yields a higher level of tax revenues than the specific tax.*

**PROOF:** Let  $\theta$  be a time-independent specific tax, and  $\tau$  be the time-independent ad valorem tax, where  $\theta = \tau\gamma/(1-\tau)$ . Then  $\tau = \theta/(\gamma + \theta)$ , and the two sales taxes induce the same competitive equilibrium. Furthermore, the tax revenue collected at time  $t$  under the specific tax  $\theta$  is  $\theta q_t(\theta, \lambda(\theta))$ , while the tax revenues collected – also at time  $t$  – under the ad valorem tax  $\tau$  is

$$\tau p_t(\theta, \lambda(\theta)) q_t(\theta, \lambda(\theta)) = \frac{\theta}{\gamma + \theta} p_t(\theta, \lambda(\theta)) q_t(\theta, \lambda(\theta)) > \theta q_t(\theta, \lambda(\theta)),$$

where the strict inequality is due to the fact that  $p_t(\theta, \lambda(\theta)) > \theta + \gamma$ .

Q.E.D.

## 2. Tax Design and Welfare Consideration

We now consider the distortion of a specific tax. Let

$$(11) \quad W(\theta) = \int_0^{\infty} e^{-rt} [u(q_t(\theta, \lambda(\theta))) - \gamma q_t(\theta, \lambda(\theta))] dt.$$

represent the discounted social welfare under the competitive equilibrium induced by the specific tax  $\theta$ . We expect that a rise in  $\theta$ , by creating more distortion in the competitive extraction of the resource, reduces  $W(\theta)$ . The proof of this result requires some more technical arguments. First, we establish some preliminary results.

CLAIM 1: *A rise in the specific tax*

- (i) *reduces the shadow price of the resource, i.e.,  $\lambda'(\theta) < 0$ , and*
- (ii) *shifts production from the present to the future. More precisely, there exists a time  $t(\theta) = -\text{Log}[-\lambda'(\theta)]/r$ , such that  $d[q_t(\theta, \lambda(\theta))]/d\theta < 0$  for  $t < t(\theta)$ ,  $d[q_t(\theta, \lambda(\theta))]/d\theta = 0$  at  $t = t(\theta)$ , and  $d[q_t(\theta, \lambda(\theta))]/d\theta > 0$  for  $t > t(\theta)$ .*

PROOF: To establish (i), note that if  $\lambda(\theta)$  rises or remains the same after a rise in  $\theta$ , then according to (4), the resource extraction at each instant will be lower after the rise in  $\theta$  than before the rise in  $\theta$ , and this means that the resource stock is not depleted after the rise in  $\theta$ : a contradiction. To establish (ii), first note that the extraction rate at time 0 must be lower after the rise in  $\theta$ . Indeed, if the extraction rate at time 0 is higher after the rise in  $\theta$ , then this result, coupled with a decline in the shadow price of the resource, imply that the extraction at each instant is higher after the rise in  $\theta$ : the cumulative resource extraction after the rise in  $\theta$  exceeds the available stock, and this is not possible. Hence  $d[q_t(\theta, \lambda(\theta))]/d\theta < 0$  for small values of  $t$ . Next, differentiate (7) with respect to  $\theta$ , we obtain

$$(12) \quad \frac{d}{d\theta} [p_t(\theta, \lambda(\theta))] = 1 + e^{rt} \lambda'(\theta).$$

We claim that the right side of (12) is positive for small values of  $t$ . Indeed, if this is not the case, then  $1 + e^{rt}\lambda'(\theta)$  is negative for all  $t > 0$ , and tends to  $-\infty$  when  $t \rightarrow \infty$ , which means that the equilibrium resource price at any time  $t > 0$  is lower after the rise in  $\theta$ , and this in turn implies that the cumulative extraction after the rise in  $\theta$  exceeds the available resource stock. Hence there exists a unique time  $t(\theta) = -\text{Log}[-\lambda'(\theta)]/r$ , such that

$$(13) \quad \begin{aligned} \frac{d}{d\theta}[p_t(\theta, \lambda(\theta))] &> 0 \text{ for } t < t(\theta), \\ &= 0 \text{ at } t = t(\theta), \\ &< 0 \text{ for } t > t(\theta). \end{aligned}$$

Finally, note that (ii) follows from (13),  $f'(q) < 0$ , and the fact that

$$d[p_t(\theta, \lambda(\theta))]/d\theta = [f'(q_t(\theta, \lambda(\theta)))] [d[q_t(\theta, \lambda(\theta))]/d\theta]. \quad \text{Q.E.D.}$$

The claims we have made are quite intuitively appealing. A rise in the specific tax is considered by producers as an increase in the cost of extracting the resource, thus rendering it less profitable. Since the resource becomes less valuable, its shadow price would naturally fall. Furthermore, the increase in the cost in term of tax payments may be postponed: by reducing the extraction in the near future (small values of  $t$ ), and increasing it in a distant future (large values of  $t$ ), resource producers will raise discounted profits by deferring tax payments. We are now ready to analyze the distortions generated by the specific tax. Let us state:

*CLAIM 2: The higher the time-independent specific tax, the lower will be the social welfare associated with the extraction of the resource.*

PROOF: Let us differentiate (11) to obtain, after some manipulation:<sup>3</sup>

$$(14) \quad W'(\theta) = \int_0^\infty e^{-rt} [u'(q_t(\theta, \lambda(\theta))) - \gamma] \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt = \theta \int_0^\infty e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt.$$

Note that the third line in (14) has been obtained with the help of (7), and the last line in (14) has been obtained by using the fact that the derivative with respect to  $\theta$  of the stock constraint (6) is equal to 0. Whether  $W'(\theta)$  is positive or negative depends on the sign of the integral on the last line of (14). Using Claim 1, we can assert that this integral satisfies the following inequality:

$$\begin{aligned} \int_0^\infty e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt &= \int_0^{t(\theta)} e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt + \int_{t(\theta)}^\infty e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt \\ &< \int_0^{t(\theta)} e^{-rt(\theta)} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt + \int_{t(\theta)}^\infty e^{-rt(\theta)} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt \\ &= e^{-rt(\theta)} \int_0^\infty \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt = 0. \end{aligned}$$

Q.E.D.

We are now able to demonstrate:

**PROPOSITION 2:** *If the authorities want to raise a given level of discounted tax revenues by imposing a sales tax, then it is more efficient to use a constant ad valorem tax than a constant specific tax because social welfare is higher under the former sales tax than under the latter sales tax although they both yield the same level of discounted tax revenues.*

PROOF: Let  $\theta$  denote the specific tax. From Proposition 1, an ad valorem tax  $\tau = \theta/(\gamma + \theta)$  would yield a higher level of tax revenues at each instant than the specific tax, and thus the discounted tax revenues collected under the ad valorem tax will exceed that collected under the specific tax, i.e.  $\int_0^\infty e^{-rt} \tau p_t q_t dt > \int_0^\infty e^{-rt} \theta q_t dt$ . Let the required

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<sup>3</sup>

$$\begin{aligned} W'(\theta) &= \int_0^\infty e^{-rt} [p_t(\theta, \lambda(\theta)) - \gamma] \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt = \int_0^\infty [\theta e^{-rt} + \lambda(\theta)] \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt \\ &= \int_0^\infty \theta e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt + \int_0^\infty \lambda(\theta) \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt \\ &= \theta \int_0^\infty e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt + \lambda(\theta) \frac{d}{d\theta} \left[ \int_0^\infty q_t(\theta, \lambda(\theta)) dt \right] = \theta \int_0^\infty e^{-rt} \frac{d}{d\theta} [q_t(\theta, \lambda(\theta))] dt. \end{aligned}$$



discounted tax revenues be the right-hand integral in the inequality. It can be achieved with a lower ad valorem tax rate, say  $\tau' < \theta/(\gamma + \theta)$ . If we let  $\theta'$  be such that  $\tau' = \theta'/(\gamma + \theta')$ , then  $\theta' < \theta$ . Furthermore, the ad valorem tax  $\tau'$  induces the same competitive equilibrium as the specific tax  $\theta'$ . Hence the required discounted tax revenues collected under the specific tax  $\theta$  could also be collected under the ad valorem tax  $\tau'$ , which – because it induces the same competitive equilibrium as the specific tax  $\theta'$  – yields a higher level of social welfare than the specific tax  $\theta$ , according to Claim 2.

Q.E.D.

### 3. Time-Dependent Optimal Sales Tax

We reserve the last part of this note to discuss the optimal sales tax in the Hotelling model. A time-independent sales tax encountered in practice is distortionary, and reduces discounted social welfare below its optimal level (see Neher (1999)). In their insightful book on resources economics, Dasgupta and Heal (1979) asserted that the time-dependent specific tax schedule  $\theta_t = \theta_0 e^{rt}, t \geq 0$ , where  $\theta_0$  is a given positive constant, induces a competitive equilibrium that is socially optimal. Indeed, under the competitive equilibrium induced by this time-dependent tax policy, the evolution of the resource price through time is governed by the following differential equation:

$$(15) \quad \dot{p}_t - r\theta_0 e^{rt} = r(p_t - \gamma - \theta_0 e^{rt}),$$

which can be simplified to

$$(16) \quad \dot{p}_t = r(p_t - \gamma).$$

The differential equation represented by (16) describes the evolution through time of the resource price under the social optimum. It is clear that the proposed tax schedule  $\theta_t = \theta_0 e^{rt}, t \geq 0$ , which acts exactly like a tax on resource rent or a tax on firms' profit, does not distort the social optimum. By varying  $\theta_0$ , we can vary the discounted tax revenues collected, which is simply  $\int_0^{\infty} e^{-rt} \theta_0 e^{rt} q_t dt = \theta_0 S_0$ . Thus there is not a unique

time-dependent schedule for the optimal specific tax. The time-dependent optimal specific tax depends on the discounted tax revenues the authority wants to collect.

Now, we derive the optimal ad valorem tax. Again, recall that a constant ad valorem tax induces a competitive equilibrium that is not socially optimal. Hence a socially optimal ad valorem tax must be time-dependent. Thus let  $\tau_t$  denote the ad valorem tax rate at time  $t$ . At any instant, the net price of the resource – net of the unit extraction cost and the ad valorem tax – is given by  $p_t - \gamma - \tau_t p_t$ . Under the competitive equilibrium induced by the ad valorem tax  $\tau_t, t \geq 0$ , the net resource price must appreciate at the market rate of interest, i.e.,

$$(17) \quad \dot{p}_t - \dot{\tau}_t p_t - \tau_t \dot{p}_t = r(p_t - \gamma - \tau_t p_t),$$

which can be rewritten as

$$(18) \quad \dot{p}_t = r(p_t - \gamma) + \frac{\dot{\tau}_t p_t - r\tau_t \gamma}{(1 - \tau_t)}.$$

In order for (18) to describe the evolution of the resource price under the social optimum, we should choose  $\tau_t$ , so that  $\dot{\tau}_t p_t - r\tau_t \gamma = 0$ , which in turn implies

$$(19) \quad \frac{\dot{\tau}_t}{\tau_t} = \frac{r\gamma}{p_t}.$$

Because  $p_t$  rises with time, and tends to infinity as  $t \rightarrow \infty$ , the growth rate of the ad valorem tax is decreasing through time and tends to 0 as  $t \rightarrow \infty$ . This result stands in contrast with the specific tax the growth rate of which is constant and given by  $\dot{\theta}_t / \theta_t = r$ . Also, as with the optimal specific tax, the optimal time-dependent ad valorem tax does not distort the social optimum. We have the following proposition:

**PROPOSITION 3:** *While the optimal specific tax is of the form  $\theta_t = \theta_0 e^{rt}, t \geq 0$ , for a given  $\theta_0 > 0$ , the optimal ad valorem tax is given by  $\tau_t = \tau_0 e^{\int_0^t \frac{r\gamma}{\gamma + e^{rs}(p_0 - \gamma)} ds}$ , where  $\tau_0$  is a constant inside the interval  $[0,1)$ . Moreover, if  $\tau_t = \theta_0 e^{rt} / p_t, t \geq 0$ , then this time-dependent ad valorem tax is optimal and yields the same discounted tax revenues as the specific tax..*

PROOF: Let  $p_t$  denote the resource price at time  $t$  under the competitive equilibrium without sales taxes. Then  $p_t$  satisfies (16), and can be rewritten as  $p_t - \gamma = e^{rt}(p_0 - \gamma)$ , which can be used in (19) to yield

$$(20) \quad \frac{d}{dt} \text{Log}[\tau_t] = \frac{r\gamma}{\gamma + e^{rt}(p_0 - \gamma)}, \quad \tau_0 \text{ is given.}$$

Integrating (20) between time 0 and time  $t$ , we obtain

$$(21) \quad \text{Log}[\tau_t] - \text{Log}[\tau_0] = \int_0^t \frac{r\gamma}{\gamma + e^{rs}(p_0 - \gamma)} ds.$$

It follows from (21) that

$$(22) \quad \tau_t = \tau_0 e^{\int_0^t \frac{r\gamma}{\gamma + e^{rs}(p_0 - \gamma)} ds}.$$

To establish the second statement of Proposition 2, first note that for a given value of  $\tau_0$ , equation (22) represents an optimal time-dependent ad valorem tax. By varying  $\tau_0$ , we obtain different optimal time-dependent ad valorem taxes, each of which allows us to collect a given discounted tax revenues. We now require that the discounted tax revenues collected under such an ad valorem tax be equal to  $\theta_0 S_0$ , the discounted tax revenues collected under the specific tax. Let  $\tau_t$  be such that  $\theta_t q_t = \tau_t p_t q_t$ , i.e.,  $\tau_t = \theta_t / p_t$ , where  $q_t$  and  $p_t$  denote, respectively, the resource price and the extraction – both at time  $t$  – under the competitive equilibrium. Clearly, the tax revenues collected at each instant are the same under the two taxes, ensuring that our requirement is satisfied. Differentiate logarithmically  $\tau_t$  with respect to time, we get  $\dot{\tau}_t / \tau_t = \dot{\theta}_t / \theta_t - \dot{p}_t / p_t$ . Note that  $\dot{\theta}_t / \theta_t = r$ , and along the competitive equilibrium price path we have  $\dot{p}_t / p_t = r(1 - \gamma / p_t)$ . It now follows that  $\dot{\tau}_t / \tau_t = r\gamma / p_t$ , which is (19), the equation that characterizes the optimal ad valorem tax. We have just shown that for any optimal time-dependent specific tax, there is an optimal time-dependent ad valorem tax that yields the exactly the same tax revenue at each instant, and a fortiori the same discounted tax revenues over the entire time horizon. The two sales taxes are linked by the relation  $\tau_t = \theta_0 e^{rt} / p_t, t \geq 0$ . Q.E.D.

## 4. Concluding Remarks

The main results we obtain with respect to time-independent sale taxes in this paper can be extended to the case of imperfect competitive resource markets. Take for example the case of resource monopoly. Again, for the same pattern of resource extraction, we may use the same arguments to assert that the ad valorem tax is superior to the specific tax: either the ad valorem tax yields a higher level of discounted tax revenues than the specific tax when both taxes induce the same competitive equilibrium, or for the same level of discounted tax revenues, the ad valorem tax induces less welfare loss than the specific tax. With respect to our discussion on time-dependent optimal tax schedules, the task is more cumbersome. Here monopoly is a source of sub-optimality, so that the optimal tax design should, together with the obligation to raise a certain level of tax revenues, also correct a market failure in order to achieve a social optimum (see Bergstrom, Cross, and Porter (1981)). This problem, although interesting, would take us too far afield.

Let us finally note that in many dynamic economic analyses, the effects of taxes are often studied as shifts of the steady-state equilibrium. No due attention is paid to the transitory paths toward a new steady state, and results of the comparative static analysis are applied without any forethought. Recall that in comparative static analysis, given a specific tax, one can always find an ad valorem tax that implements the same equilibrium outcome and yields the same tax receipt, and vice versa. This equivalence does not hold in the simple Hotelling dynamic resource model. We have shown for the same equilibrium trajectory, an ad valorem tax that is constant over time yields a higher tax revenue than a specific tax. Furthermore, for a given discounted tax revenue, the social welfare resulting from competitive equilibrium is higher with the ad valorem tax than with the specific tax. Also, finally, the equivalence of these two forms of sale tax in a dynamic setting can be restored only if they are time-dependent and – we insist – they are all optimal in the sense that they do not distort the competitive equilibrium trajectory.

The deep reason for these new and surprising results is the following. In the Hotelling model of resource extraction we refer to in this paper, no non-degenerate steady state exists because the resource stock is always depleted. Therefore, the analysis should be conducted with a sequence of transitory states, and the appropriate method to look at the effect of taxes must be the comparative dynamics, not the simple comparative static analysis usually encountered in the literature of tax analyses. In this regard, we believe, we do call attention to a set of more general problems not duly investigated.

## References

Bergstrom, T. C, J. G. Cross, and R. C. Porter (1981): “*Efficiency-Inducing Taxation for a Monopolistically Supplied Depletable Resource*,” *Journal of Public Economics*, 15(1), p. 23-32.

Dasgupta, P. and G. M. Heal (1979): *Economic Theory and Exhaustible Resources*, Cambridge University Press, Cambridge.

Hotelling, H. (1931): “*The Economics of Exhaustible Resources*,” *Journal of Political Economy*, 39 (2), p. 137-175.

Neher, P. A. (1999): “*Economics of Mining Taxation*,” in Jeroen C. J. M. van der Berg, ed., *Handbook on Environmental and Resource Economics*.

Sarma, J. V. M. and G. Naresh (2001): “*Mineral Taxation around the World: Trends and Issues*,” *Asian-Pacific Tax Bulletin*, International Bureau of Fiscal Documentation, Amsterdam.