

## **Global Financial Crisis, Extreme Interdependences, and Contagion Effects: The Role of Economic Structure**

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# Global Financial Crisis, Extreme Interdependences, and Contagion Effects: The Role of Economic Structure

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## Abstract

The paper examines the extent of the current global crisis and the contagion effects it induces by conducting an empirical investigation of the extreme financial interdependences of some selected emerging markets with the US. Several copula functions that provide the necessary flexibility to capture the dynamic patterns of fat tail as well as linear and nonlinear interdependences are used to model the degree of cross-market linkages. Using daily return data from Brazil, Russia, India, China (BRIC markets) and the US, our empirical results show strong evidence of time-varying dependence between each of the BRIC markets and the US markets, but the dependency is stronger for commodity-price dependent markets than for finished-product export-oriented markets. We also observe high levels of dependence persistence for all market pairs during both bullish and bearish markets.

*Keywords: Extreme comovements, copula approach, BRIC emerging markets, and global financial crisis*

*JEL classifications: F37; G01; G17*

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## 1 Introduction

The modern portfolio theory, relying on the seminal work of Markowitz (1958) and the underlying ideas of the Capital Asset Pricing Model (CAPM), posits that investors can improve the performance of their portfolios by allocating their investments into different classes of financial securities and industrial sectors that would not move together in the event of a valuable new information. In this scheme of things, sub-perfectly correlated assets are particularly appropriate for adding to a diversified portfolio. Subsequently, Solnik (1974), among others, extended the domestic CAPM to an international context and suggests that diversifying internationally enables investors to reach higher efficient frontier than doing so domestically.

Empirically, Grubel (1968) examined the potential benefits of international diversification and showed the superiority of portfolios that are composed of both domestic assets and assets denominated in foreign currencies from eleven developed markets. These findings are then confirmed by other earlier studies where the analysis of market interdependence evolves both developed and developing countries (see, e.g., Levy and Sarnat, 1970; Lessard, 1973; and Errunza, 1977). The recent literature on measuring stock market comovements has been greatly stimulated by the globalization of capital markets around the world (see, e.g., Eun and Shim, 1989; Hamao *et al.*, 1990; Karolyi, 1995; Forbes and Rigobon, 2002; Phylaktis and Ravazzolo, 2005; Syriopoulos, 2007; Gilmore *et al.*, 2008; Morana and Beltratti, 2008; Kizys and Pierdzioch, 2009). Based on a wide variety of methodologies, the majority of these works suggest that correlations of stock returns have increased in the recent periods as a result of increasing financial integration across national stock markets, leading to lower diversification benefits especially in the longer term. More importantly, the level of market correlations varies over time<sup>1</sup>.

However, modeling the comovement of stock market returns is a challenging task. The main argument is that the conventional measure of market interdependence, known as the Pearson correlation coefficient, might not be a good indicator. Indeed, it represents only the average of deviations from the mean without making any distinction between large and small returns, or between negative and positive returns (Poon *et al.*, 2004). Consequently, the asymmetric correlation between financial markets in bear and bull periods as documented, for example, by Longin and Solnik (2001), Ang and Bekaert (2002), and Patton (2004) cannot be explained<sup>2</sup>. The Pearson correlation estimate

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<sup>1</sup> See Longin and Solnik (1995) and references therein).

<sup>2</sup> By asymmetric correlations, we mean that negative returns are more correlated than positive returns. This then suggests that financial markets tend to be more dependent in times of crisis. We also test this hypothesis within this paper.

is further constructed on the basis of the hypothesis of a linear association between the financial return series under consideration whereas their linkages may well take nonlinear causality forms (see, e.g., Li, 2006; and Beine *et al.*, 2008). Other complications refer directly to stylized facts related to the distributional characteristics of stock market returns: departure from Gaussian distribution and tail dependence (or extreme comovement). Solutions for handling these problems include either the use of multivariate GARCH models with leptokurtic distributions which allows for both asymmetry and fat tails (see, e.g., Şerban *et al.*, 2007) or the use of multivariate extreme value theory and copula functions (Longin and Solnik, 2001; Chan-Lau *et al.*, 2004; and Jondeau and Rockinger, 2006). Notice that the first modeling approach allows for capturing deviations from conditions of normality, whereas the last two approaches deal essentially with the extreme dependence structure of large (negative or positive) stock market returns, all in multivariate frameworks.

Since the investigation of dependence structure is crucial for risk management and portfolio diversification issues, this paper also focuses on the issue of interactions between financial markets. For this purpose, we combine the so-called conditional multivariate copula functions with generalized autoregressive conditional heteroscedasticity process (*hereafter* copula-based GARCH or C-GARCH models)<sup>3</sup>. In this nested setting, the GARCH models with possibly skewed and fat tailed return innovations are applied to filter the stock market returns and to draw their marginal distributions, while the multivariate dependence structure between markets is modeled by parametric family of extreme value copulas which are perfectly suitable for non-normal distributions and non-linear dependencies. The model thus captures not only the tail dependence, but also the asymmetric tail dependence. With regard to the methodological choice, our work is broadly similar to that of Jondeau and Rockinger (2006) who studied the dynamics of dependency between four major stock markets<sup>4</sup>, but it is more general in terms of GARCH specifications and copula functions. In addition, we demonstrate that portfolio managers will have an interest in employing copula-based GARCH models to estimate the value at risk (i.e., expected losses in the case of the occurrence of a risk event) in their internationally diversified portfolios during widespread market panics.

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<sup>3</sup> Copulas are functions that describe the dependencies between variables, and enable modelling their joint distribution when only marginal distributions are known. These functions thus provide an useful tool for reproducing the multivariate dependence structure in cases where the normality condition does not hold. We refer to Joe (1997), and Nelsen (1999) for a comprehensive introduction to copulas and their properties. The main applications of copulas in finance can be found in Cherubini *et al.* (2004).

<sup>4</sup> This study covers the US, the UK, German and French stock markets represented respectively by the S&P500, FTSE, DAX and CAC40 indices.

Overall, we contribute to the related literature in several aspects. First, we address the question of the strength of the interdependences between emerging and US markets, and among emerging markets themselves? This is important since knowing only the degree of time-varying comovement degree is actually not sufficient to make international investment decisions because stock market returns might depart considerably from the standard normal distribution, and exhibit nonlinear linkages and extreme comovements. A number of studies have been devoted to the analysis of comovements in evolving stock markets in emerging countries (see, e.g., Gallo and Otranto, 2005 for Asian emerging markets; Johnson and Soenen, 2003 and Fujii, 2005 for Latin American emerging markets). Empirical results from these studies support the existence of significant linkages both between emerging and developed markets, and also among emerging markets. Little is known however about their extreme comovements. The sole exception refers, to the best of our knowledge, to the work of De Melo Mendes (2005) who investigates the asymmetric extreme dependence of 21 pairs of emerging market returns using copula functions. Second, given the context of the 2007-2009 global financial crisis, our study provides a general framework for detecting any evidence of extreme contagion effects from the US to major emerging markets. Finally, the fact that we focus on the most important markets in the emerging universe (Brazil, Russia, India and China) with their differing economic systems allows us to shed light on the impact of economic structure on the extreme financial dependencies. Indeed, among our BRIC markets, Brazil and Russia can be viewed as commodity-price dependent countries, whereas India and China are finished-product export-oriented countries. The comparison of comovement levels among these markets is quite interesting because both commodity and finished-products prices have experienced lengthy swings during recent times.

In the empirical part of the paper we investigate the extreme dependence structure between the daily returns of stock market indices over the period from March 22, 2004 to March 20, 2009. As a preliminary step, we first proceed to choose the ARCH/GARCH models suitable for dealing the time-varying conditional heteroscedasticity and variance persistence in stock returns since many specifications are possible for the return-generating process. We then apply the generalized Pareto distribution to the filtered return innovations in order to determine the marginal distribution of univariate series in the tails. Finally, we employ different copula functions to model the multivariate dependency between stock market returns. Our findings reveal that the GARCH-in-Mean specification which allows for asymmetric effects from negative and positive shocks is the most appropriate for the data, and that stock market volatility is highly persistent over time. With regard to copula modeling, the Gumbel (1960) extreme value copula appears to fit at best the tail dependence of the markets studied. Overall, we find strong evidence of extremely negative and positive co-exceedances for all market pairs, but extreme comovement with the US is higher for commodity-price dependent markets than export-price depen-

dent markets. Within the universe of BRIC markets, the results indicate that they are more dependent in the bull markets than in the bear markets.

The remainder of this paper is organized as follows. Section 2 presents the theoretical background of the copula functions and shows how they can be applied to study the extreme comovements between the BRIC markets and the US, especially over the 2007-2009 period of the subprime crisis. In Section 3, the empirical results are reported and interpreted with reference to the economic structure of the considered emerging markets considered. We provide summary of our conclusions in Section 4.

## 2 Copula Functions and their Applications

Copulas are functions that link multivariate distributions to their univariate marginal functions. A good introduction to copula models and their fundamental properties can be found in Joe (1997) and Nelson (1999). Formally, we refer to the following definition:

**Definition 1** *A  $d$ -dimensional copula is a multivariate distribution function  $C$  with standard uniform marginal distributions.*

**Theorem 2** *Sklar's theorem*

*Let  $X_1, \dots, X_d$  be random variables with marginal distribution  $F_1, \dots, F_d$  and joint distribution  $H$ , then there exists a copula  $C: [0, 1]^d \rightarrow [0, 1]$  such that:*

$$H(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d))$$

*Conversely if  $C$  is a copula and  $F_1, \dots, F_d$  are distribution functions, then the function  $H$  defined above is a joint distribution with margins  $F_1, \dots, F_d$ .*

Therefore copulas functions provide a way to create distributions that model correlated multivariate data. Using a copula, one can construct a multivariate distribution by specifying marginal univariate distributions, and then choose a copula to provide a correlation structure between the variables. Bivariate distributions, as well as distributions in higher dimensions are possible.

If we are particularly concerned with extreme values the concept of tail dependence can be very helpful in measuring the dependence in the tails of the distribution. The coefficient of tail dependence is, in this case, a measure of the tendency of markets to crash or boom together.

Let  $X, Y$  be random variables with marginal distribution functions  $F$  and  $G$ . Then the coefficient of lower tail dependence  $\lambda_L$  is

$$\lambda_L = \lim_{t \rightarrow 0^+} \Pr[Y \leq G^{-1}(t) \mid X \leq F^{-1}(t)]$$

which quantifies the probability of observing a lower  $Y$  assuming that  $X$  is lower itself. In the same way, the coefficient of upper tail dependence  $\lambda_U$  can be defined as

$$\lambda_U = \lim_{t \rightarrow 1^-} \Pr[Y > G^{-1}(t) \mid X > F^{-1}(t)]$$

There is a symmetric tail dependence between two assets when the lower tail dependence coefficient equals the upper one, otherwise it is asymmetric otherwise. The tail dependence coefficient provides a way for ordering copulas. One would say that copula  $C_1$  is more concordant than copula  $C_2$  if  $\lambda_U$  of  $C_1$  is greater than  $\lambda_U$  of  $C_2$ .

In order to measure the time-varying degrees of interdependence among markets, we employ an empirical method based on the combination of copulas and extreme value theory. At the estimation level, we will proceed as follows:

*i)* We first test the presence of ARCH effects in raw returns using the ARCH LM test. Various GARCH specifications that allow for the leverage effect are estimated and compared using the usual information criteria such as AIC, BIC and Loglik statistics. We choose the GARCH-in-Mean model as giving the best fit. This model, initially introduced by Engle, Lilien and Robins (1987), extends the basic GARCH model by allowing the conditional mean to depend directly on the conditional variance. The conditional variance specification considered allows for a leverage effect, i.e. it may respond differently to previous negative and positive innovations. Instead of assuming normal distributions for the errors, we use the Student- $t$  distribution to capture the fat tails usually present in the model's residuals. The GARCH-M model may be expressed as:

$$\begin{aligned} y_t &= c + \lambda \sigma_t^2 + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha(|\epsilon_{t-1}| - \gamma \epsilon_{t-1})^2 + \beta \sigma_{t-1}^2 \end{aligned} \tag{1}$$

Where  $c$  is the mean of  $y_t$  and  $\epsilon_t$  is the error term which follows a Student- $t$  distribution with  $\nu$  degrees of freedom. A positive ARCH-in-mean term  $\lambda$  implies that higher return and higher risk are positively correlated. The conditional variance equation depends upon both the lagged conditional standard deviations and the lagged absolute innovations. Here the GARCH model works like a filter in order to remove any serial dependency from the returns.

ii) We consider the innovations computed in step 1 and we fit the generalized Pareto distribution (GPD) to the excess losses over a high threshold. We note that in the extreme value theory (EVT) the tail of any statistical distribution can be modeled by the GPD. The use of EVT is of great importance for emerging markets since they are significantly influenced by extreme returns (Harvey, 1995a,b). The main difference between emerging and industrial markets resides in the tail of the empirical distribution produced by extreme events. Indeed, stock returns from emerging markets have significantly fatter tails than stock returns from industrial markets.

iii) The uniform variates are obtained by plugging the GPD parameter estimates into the GPD distribution function and the following selected copula models belonging to the extreme-value copula family: the Gumbel, Galambos, and Husler-Reiss copulas are fitted.

*The Gumbel Copula* of Gumbel (1960) is probably the best-known extreme value copula. It is an asymmetric copula with higher probability concentrated in the right tail. In contrast, the Gumbel copula retains a strong relationship even for the higher values of the density function in the upper right corner. It is given by

$$C(u, v) = \exp\{-[(-\ln u)^\delta + (-\ln v)^\delta]^{1/\delta}\}, \quad \delta \geq 1$$

The parameter  $\delta$  controls the dependence between the variables. When  $\delta = 1$  there is independence and when  $\delta \rightarrow +\infty$ , there is perfect dependence. The coefficient of upper tail dependence for this copula is

$$\lambda_U = 2 - 2^{1/\delta}$$

*The Galambos copula* introduced by Galambos (1975) is

$$C(u, v) = uv \exp\{[-(-\ln u)^{-\delta} + (-\ln v)^{-\delta}]^{-1/\delta}\}, \quad 0 \leq \delta < \infty$$

*The Husler-Reiss Copula* introduced by Hüsler and Reiss (1987) has the following form:

$$C(u, v) = \exp\left\{-\tilde{u}\phi\left[\frac{1}{\delta} + \frac{1}{2}\delta \ln\left(\frac{\tilde{u}}{\tilde{v}}\right)\right] - \tilde{v}\phi\left[\frac{1}{\delta} + \frac{1}{2}\delta \ln\left(\frac{\tilde{v}}{\tilde{u}}\right)\right]\right\}, \quad 0 \leq \delta \leq \infty$$

Where  $\phi$  is a CDF of a standard Gaussian,  $\tilde{u} = -\ln(u)$  and  $\tilde{v} = -\ln(v)$ .



Figure 1 shows the contour plots of the selected copula models. These plots are very informative about the dependence properties of the copulas. For this reason, one often uses contour plots to visualize differences between various copulas and possibly to assist in choosing appropriate copula functions.

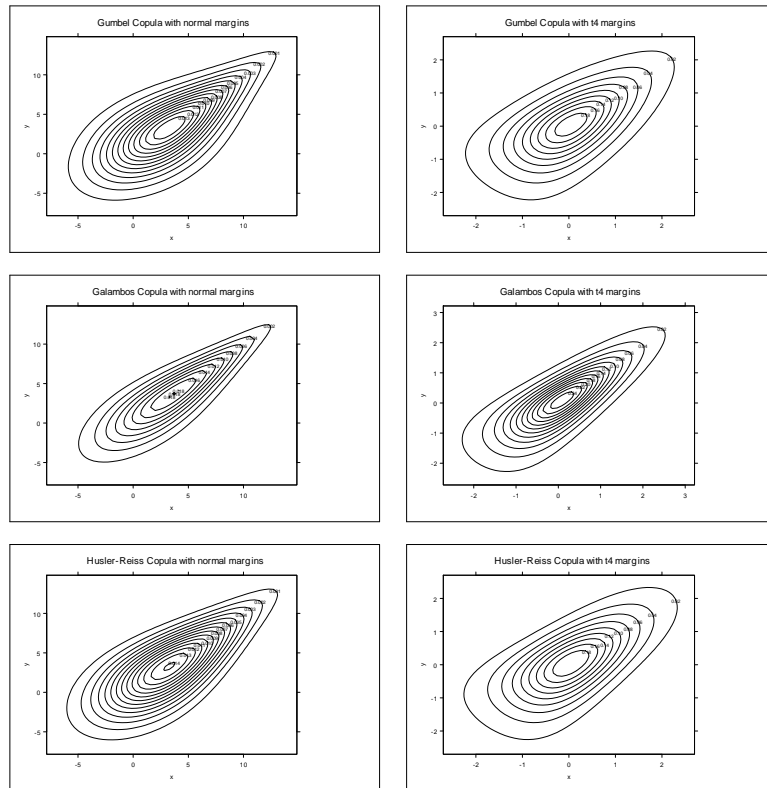


Fig. 1. Contour plots of copula models with normal margins and Student's t margins with 4 df

In order to fit copulas to our data, we use the method proposed by Joe and Xu (1996) called inference functions for margins (IFM). This method first determines first the estimate of the margins parameter by making an estimate of the univariate marginal distributions and then the parameters of the copula. The IFM method has the advantage of solving the maximization problem for cases of high dimensional distributions.

Two Goodness of fit tests are used to compare copula models. The first was proposed by Genest and Rémillard (2007) and is based on a comparison of the distance between the estimated and the empirical copulas:

$$C_n = \sqrt{n}(C_n - C_{\theta_n})$$

The test statistics considered is based on a Cramér–von Mises distances

$$S_n = \int \mathbb{C}_n(u)^2 dC_n(u)$$

High values of the statistic  $S_n$  lead to the rejection of the null hypothesis that the copula  $C$  belongs to a class  $C_0$ . In practice, we require knowledge about the limiting distribution of  $S_n$  which depends on the unknown parameter value  $\theta$ . The  $p$ -value for the test statistic is computed using a parametric bootstrap procedure and the validity of such an approach is established in Genest and Rémillard (2007). The second test developed by Genest *et al.* (2006) is based on the Kendall's process. The test procedure consists of measuring the distance between an empirical distribution  $K_n$  and a parametric estimation  $K_{\theta_n}$  of  $K$ , that is

$$\mathbb{k}_n = \sqrt{n}(K_n - K_{\theta_n})$$

The test statistics employed is based on a Cramér–von Mises distances as defined by

$$T_n = \int_0^1 \mathbb{k}_n(v)^2 dK_{\theta_n}(v)$$

The null hypothesis is rejected for high values of the computed test statistic. To find the  $p$ -values associated with the test statistic we use the parametric bootstrap procedure as described by Genest *et al.* (2007).

### 3 Empirical Results

#### 3.1 Data and stochastic properties

We empirically investigate the interaction between various stock market indices. Specifically, the data consist of five indices representing four emerging economies, namely Brazil, Russia, India, and China, together with the US market index. All data are the MSCI total return indices expressed in US dollars on a daily basis from March 22, 2004 to March 20, 2009. The returns are calculated by taking the log difference of the stock prices on two consecutive trading days, yielding a total of 1304 observations.

To assess the distributional characteristics and stochastic properties of the return data, we must first examine some descriptive statistics reported in Table 1. The reported statistics show that all the data series are negatively skewed

Table 1  
Descriptive statistics for daily stock market returns

	Brazil	Russia	India	China	US
Min	-0.183	-0.255	-0.120	-0.128	-0.095
Max	0.166	0.239	0.088	0.140	0.110
Mean	6.783e-004	-2.161e-004	2.627e-004	3.607e-004	-2.59e-004
Std. Dev	2.68e-002	2.813e-002	2.043e-002	2.168e-002	1.407e-002
Skewness	-0.431	-0.513	-0.640	-0.049	-0.348
Ex. kurtosis	7.836	17.28	4.573	6.404	13.198
Q(12)	26.346*	78.254*	55.870*	18.820***	63.609*
Q <sup>2</sup> (12)	1584.60*	688.78*	709.44*	1080.83*	1650.27*
J-B	3348.016*	16163.85*	1214.66*	2209.21*	9412.38*
ARCH(12)	505.211*	266.090*	238.288*	354.732*	454.186*

Notes: The table displays summary statistics for daily returns for the five countries. The sample period is from March 22, 2004 to March 20, 2009. Q(12) and Q<sup>2</sup>(12) are the Jarque-Bera statistics for serial correlation in returns and squared returns for order 12. ARCH is the Lagrange multiplier test for autoregressive conditional heteroskedasticity. \*, \*\* and \*\*\* indicate the rejection of the null hypotheses of no autocorrelation, normality and homoscedasticity at the 1%, 5% and 10% levels of significance respectively for statistical tests.

and exhibit excess kurtosis, which indicates evidence that the returns are not normally distributed. The Jarque-Bera statistics are highly significant for all return series and just confirm that an assumption of normality is unrealistic. Moreover, the Ljung-Box statistics (lags 12) suggest the existence of serial correlations in all the data series. Both the Ljung-Box statistics for the squared returns and the ARCH LM test are highly significant, which indicate the presence of ARCH effects in all the data series.

Figure 1 illustrates the variation of stock returns in five markets. From the graph, we can see that the stock prices were fairly stable during the period from March 2004 to the third quarter of 2008. After this date all returns series displayed more instability due in particular to the global financial crisis.

Table 2 reports the unconditional correlations for all return series. As expected, there is a positive correlation between the US and BRIC markets. The highest correlation is between the the US and Brazil (0.63) and the lowest one is between US and China (0.20). The same is true for emerging markets, although the China-India and the Russia-Brazil markets are more correlated than other BRIC markets with correlations of 0.57 and 0.53 respectively.

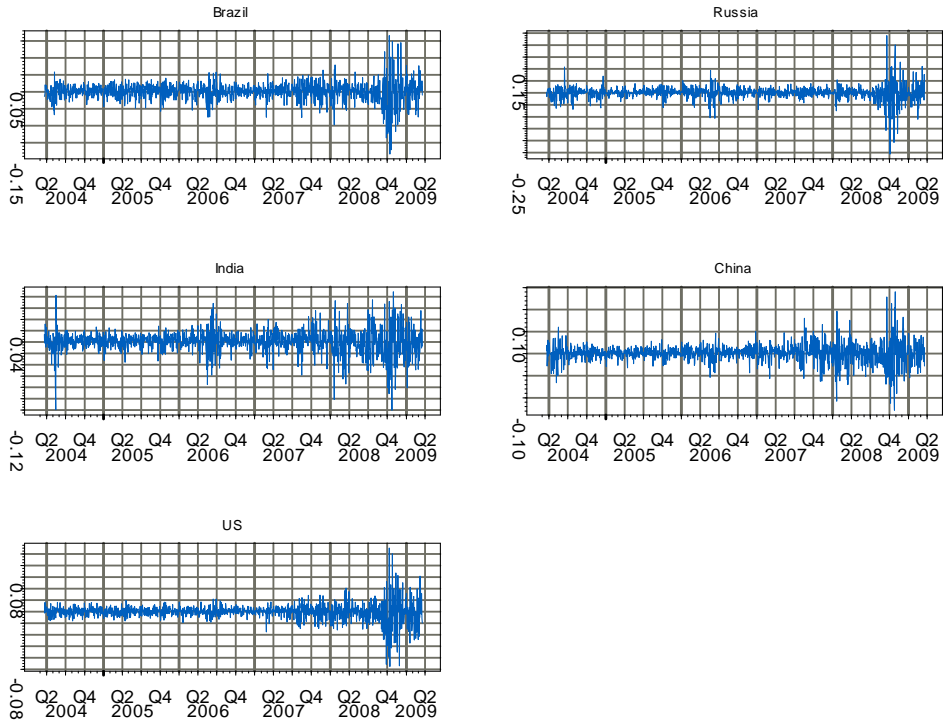


Fig. 2. Time paths of daily returns for Brazil, Russia, India, China and the US. The sample covers trading days from March, 2004 to March, 2009

Table 2  
Unconditional correlations between stock markets

	Brazil	Russia	India	China	US
Brazil	1.000	0.537	0.363	0.424	0.639
Russia		1.000	0.391	0.449	0.306
India			1.000	0.570	0.254
China				1.000	0.206
US					1.000

### 3.2 Estimation results

In the first step we fit by using the maximum likelihood method a GARCH model to the return data. Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and the log-likelihood function are used to compare various specification of the GARCH models fitted to the returns series. Regarding these statistics, the model that we considered is the GARCH-in-Mean model (GARCH-M). The results of the GARCH-M fitting are reported in Ta-

Table 3  
Estimates of the GARCH-M parameters for all returns series

	Brazil	Russia	India	China	US
$c$	$0.11e - 2$ ( $9.845e-4$ )	$0.411e - 3$ ( $1.285e-4$ )**	$0.34e - 2$ ( $9.083e-4$ )*	$2.626e - 3$ ( $3.585e-3$ )**	$0.231e - 3$ ( $3.187e-4$ )
$\lambda$	$-0.182$ ( $7.925e-1$ )	$-0.344$ ( $1.413e-1$ )**	$-3.988$ ( $2.045$ )***	$-2.173$ ( $3.359$ )***	$-1.345$ ( $2.260$ )
$\omega$	$0.17e - 4$ ( $4.507e-6$ )*	$0.145e - 4$ ( $4.087e-6$ )*	$0.10e - 4$ ( $2.495e-6$ )*	$4.025e - 6$ ( $1.452e-6$ )*	$0.147e - 5$ ( $3.830e-7$ )*
$\alpha$	$0.070$ ( $2.876e-2$ )**	$0.140$ ( $3.195e-2$ )*	$0.130$ ( $3.212e-2$ )*	$8.923e - 2$ ( $1.882e-2$ )*	$0.020$ ( $9.592e-1$ )
$\beta$	$0.868$ ( $2.377e-2$ )*	$0.837$ ( $2.493e-2$ )*	$0.817$ ( $2.616e-2$ )*	$0.897$ ( $1.638e-2$ )*	$0.928$ ( $1.531e-2$ )*
$\gamma$	$-0.540$ ( $2.60e-1$ )**	$-0.298$ ( $7.679e-2$ )*	$-0.437$ ( $1.305e-1$ )*	$-0.232$ ( $8.817e-2$ )*	$-0.992$ ( $4.693e+1$ )

Notes: The table summarizes the GARCH-M estimation results. The values between brackets represent the standard error of the parameters. \*, \*\* and \*\*\* indicate significance at the 1%, 5% and 10% levels.

ble 3<sup>5</sup>. For all returns series, we note that the parameter  $\beta$  is close to 0.9 with an extremely significant t-statistics. This result indicates that volatility is highly persistent, i.e. large changes in the conditional variance are followed by other large changes and small changes are followed by other small changes. Use of the asymmetric GARCH-M model seems to be justified. The estimated  $\gamma$  coefficients for the leverage term are significant at the 5% level except for the US returns which indicate the existence of an asymmetric response of volatility to shocks. The ARCH-in-mean terms estimated for Russia, India and China are negative with significant  $t$ -statistics which implies that high volatility leads to lower expected returns.

In Figure 3 the graph show the ACF of the standardized residuals and the squared standardized residuals obtained from the GARCH-M fit. When compared to the ACFs of the raw returns, it can be easily seen that the standardized residuals are approximately *i.i.d.*, thus far more amenable to GPD tail estimation.

In the second step we extract the filtered residuals from each returns series with an asymmetric GARCH-M model, and then we construct the marginal of each series using the empirical CDF for the interior and the GPD estimates for the upper and lower tails. The advantage of this method is that the *i.i.d.* assumption behind the extreme value theory is less likely to be violated by the filtered series. Fitting the GPD to the filtered returns requires specification of the lower and upper thresholds. We select the threshold values such that

<sup>5</sup> The estimation results for other GARCH specifications are not reported here in order to save spaces, but they can be provided upon request addressed directly to the corresponding author.

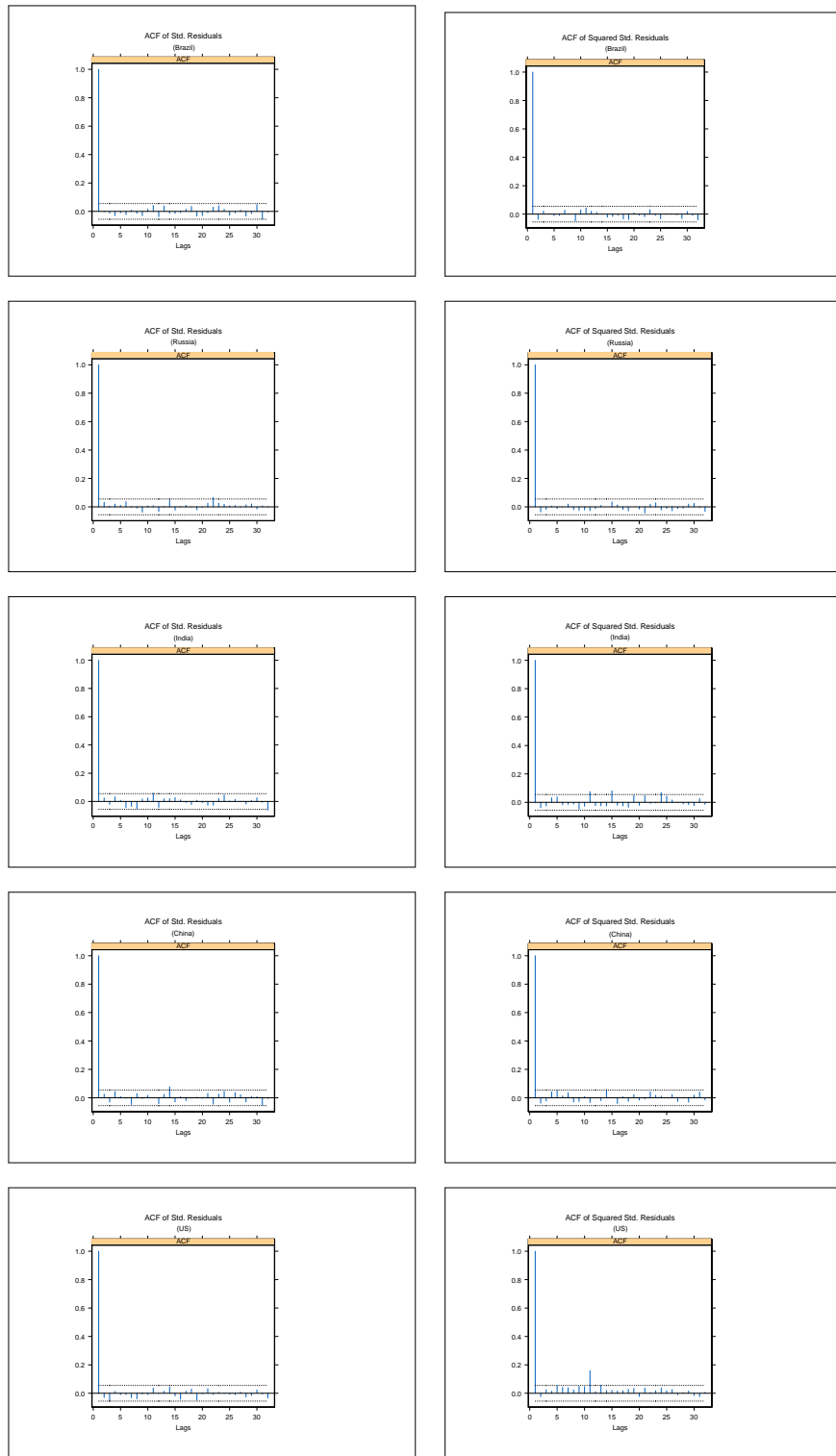


Fig. 3. ACF of the standardized residuals and the squared standardized residuals.

10% of the residuals are reserved for the upper and lower tail. To evaluate the GPD fit in the tails of the distribution, in Figure 4 we show the qq-plots of the upper and lower tail exceedances against the quantiles obtained from the GPD fit. The approximate linearity of these plots indicate that the GPD model seems to be a good choice.

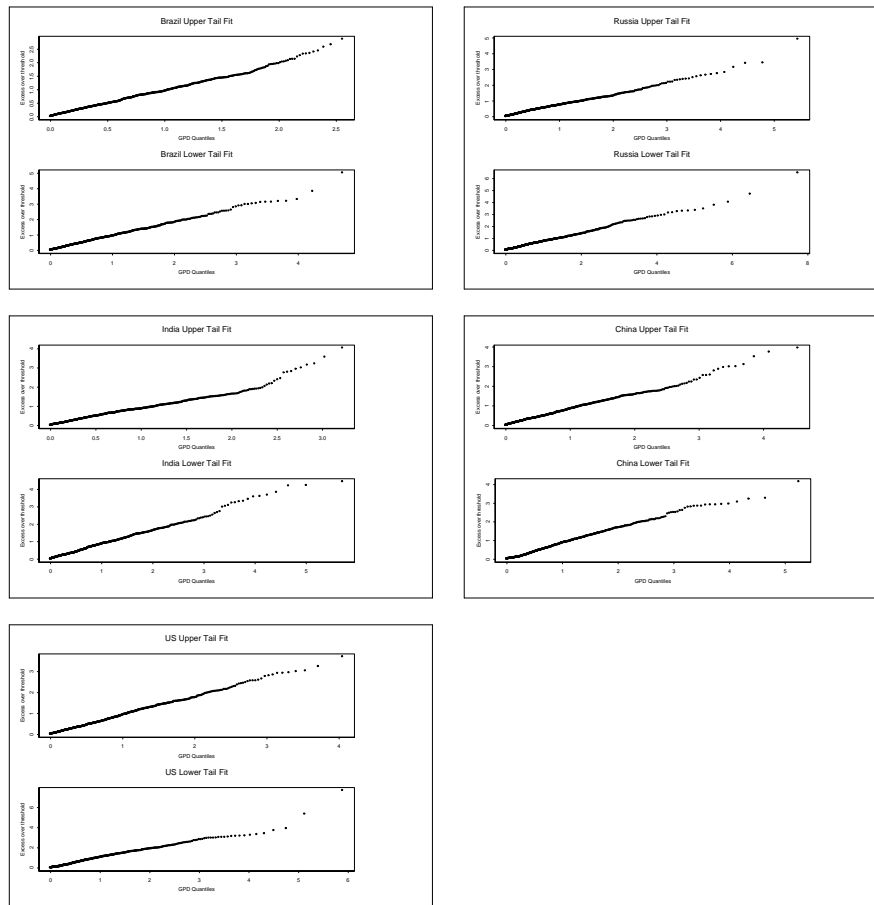


Fig. 4. Estimated tails from GPD models fit to lower and upper tails exceedances.

Next, we consider the following market pairs: Brazil-US, Russia-US, India-US and China-US and we estimate the copula model parameters. Selection of the best copula fit is based on the two goodness-fit test discussed above. In Figure 3, we show the scatterplot of the markets pairs studied. The joint behavior of the returns witnesses some extreme comovements in the lower left and upper right quadrant.

For each pair, the estimated parameters of the best copula model and the values of the lower and upper tail dependence coefficients are reported in Tables 4 and 5. The results are ordered according to the value of their tail dependence coefficient. All the pairs considered are mutually dependent during bear and bull markets. Most of the symmetric fit are based on the Gumbel copula which gives the best fit in most cases. We first note that the pair

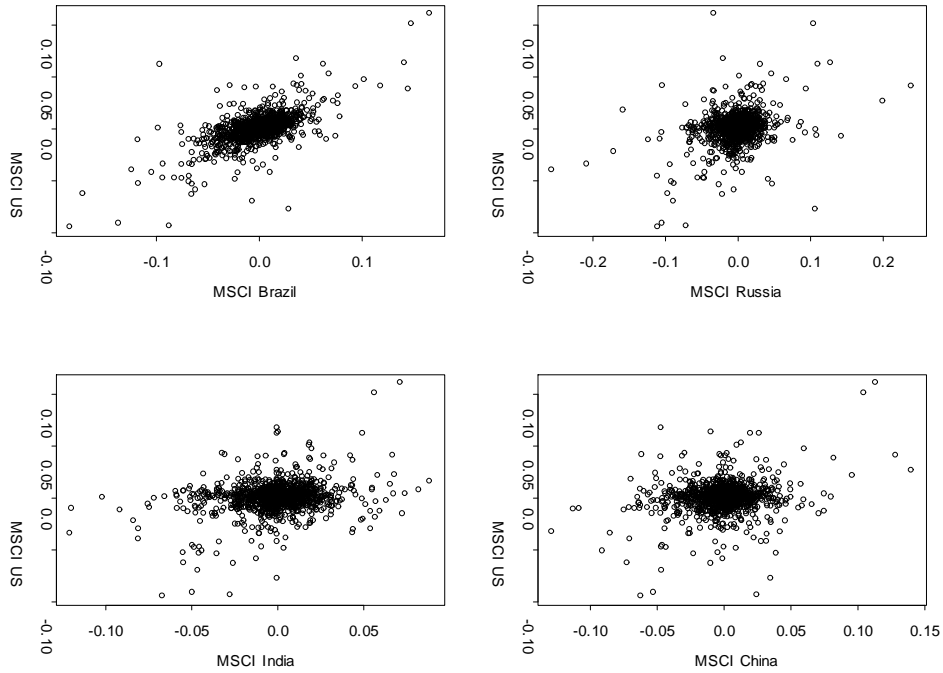


Fig. 5. The scatterplot of four pairs of markets: Brazil-US, Russia-US, India-US and China-US.

Table 4

Copula parameters and upper tail dependence coefficients for dependent positive co-exceedances

	Filtered returns		
		Copula – Estimates(S.E.)	Upper tail
Brazil-US	Gumbel	1.66 (0.045)	0.482
Russia-US	Gumbel	1.18 (0.025)	0.205
India-US	Gumbel	1.12 (0.023)	0.152
China-US	Gumbel	1.11 (0.029)	0.136

Brazil-US is the strongest tail dependent pair for both positive and negative co-exceedances. The second and the third position are occupied by the Russia-US and India-US pairs; the China-US markets show the smaller degree of tail dependence. For China-US, we note that dependence during bull markets is stronger than dependence in bear markets.

In Tables 6 and 7 we report the results for the joint losses and joint gains for the following pairs: Brazil-Russia, Brazil-India, Brazil-China, Russia-India, Russia-China and India-China. We observe that the first three positions are occupied by India-China, Brazil-Russia and Russia-China. For Brazil-Russia,



Table 5

Copula parameters and lower tail dependence coefficients for dependent negative co-exceedances

	Filtered returns		
	Copula – Estimates(S.E.)		Lower tail
Brazil-US	Gumbel	1.66 (0.046)	0.482
Russia-US	Gumbel	1.18 (0.026)	0.205
India-US	Gumbel	1.129 (0.023)	0.152
China-US	Galambos	0.342 (0.029)	0.132

Brazil-China, and Russia-China the dependence in the left lower tail is smaller than the dependence in the right tail, i.e. these markets are more dependent during bull markets than in bear markets.

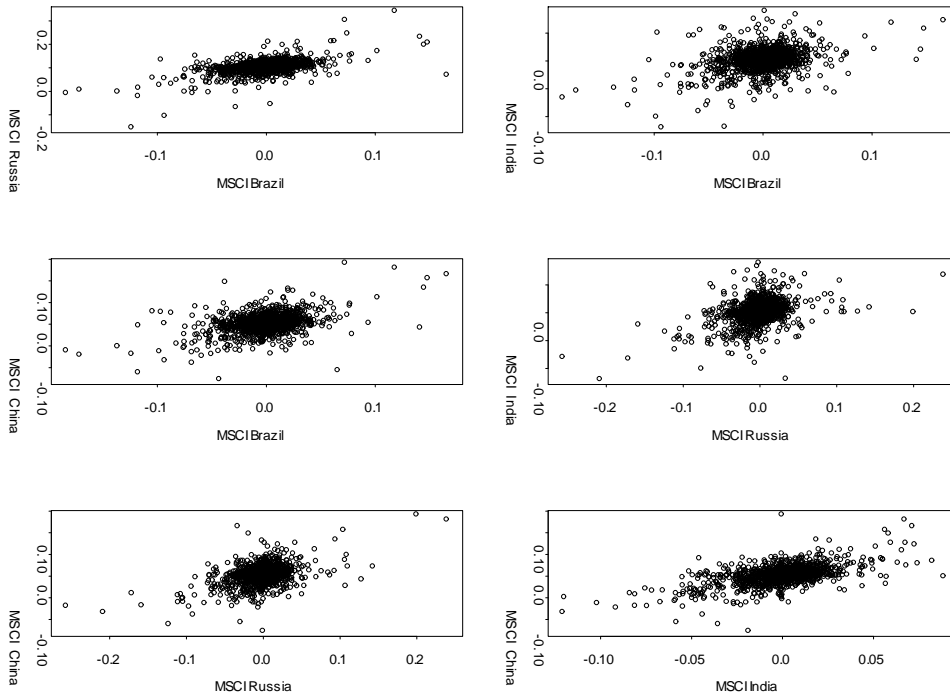


Fig. 6. The scatterplot of five pairs of markets: Brazil-Russia, Brazil-India, Brazil-China, Russia-India, Russia-China, and India-China.

Table 6

Copula parameters and upper tail dependence coefficients for dependent positive co-exceedances.

	Filtered returns		
	Copula	Estimates(S.E.)	Upper tail
India-China	Gumbel	1.420 (0.035)	0.370
Brazil-Russia	Gumbel	1.370 (0.032)	0.341
Russia-China	Gumbel	1.252 (0.028)	0.260
Russia-India	Gumbel	1.224 (0.027)	0.239
Brazil-China	Gumbel	1.206 (0.027)	0.223
Brazil-India	Gumbel	1.193 (0.026)	0.212

Table 7

Copula parameters and lower tail dependence coefficients for dependent negative co-exceedances

	Filtered returns		
	Copula	Estimates(S.E.)	Lower tail
India-China	Gumbel	1.420 (0.035)	0.370
Brazil-Russia	Galambos	0.638 (0.034)	0.337
Russia-China	Galambos	0.508 (0.031)	0.256
Russia-India	Gumbel	1.224 (0.026)	0.239
Brazil-China	Galambos	0.457 (0.031)	0.219
Brazil-India	Gumbel	1.193 (0.026)	0.212

### 3.3 Estimating the value at risk

Financial institutions are exposed to risk from movements in the prices of many instruments and across many markets. To examine and measure market risk, the most commonly used technique is the Value at Risk (VaR), defined as the maximum loss in a portfolio value of given confidence level over a given time period. During currency crises and stock market crashes, traditional VaR methods fail to provide a good evaluation of the risk because they assume a multivariate normal distribution of the risk factors. Here we propose the use of copula to quantify the risk of three equally-weighted portfolios. The market pairs considered are Brazil-US, India-China, and Brazil-Russia. These pairs showed the strongest extreme dependence during both bear and bull markets. We now consider a portfolio composed of two assets; the one period log-return for this portfolio is given by

$$R = \log(\lambda_1 e^X + \lambda_2 e^Y)$$

Where  $X$  and  $Y$  denote the continuously compounded log-returns and  $\lambda_1, \lambda_2$  are the fractions of the portfolio invested in the two assets.

In order to compute risk measures such as Value at Risk and Expected Shortfall, we have to use Monte Carlo simulations because analytical methods exist only for a multivariate normal distribution (i.e. a Gaussian copula). When copula functions are used, it is relatively easy to construct and simulate random scenarios from the joint distribution of  $X$  and  $Y$  based on any choice of marginals and any type of dependence structure.

Our strategy consists of first simulating dependent uniform variates from the estimated copula model and transforming them into standardized residuals by inverting the semi-parametric marginal CDF of each index. We then consider the simulated standardized residuals and calculate the returns by reintroducing the GARCH volatilities and the conditional mean term observed in the original return series. Finally, given the simulated return series, for each pair  $(x_i, y_i)$  we compute the value of the global portfolio  $R$ . The VaR for a given level  $q$  is simply the 100q-th percentile of the loss distribution, expressed analytically as

$$VaR_q = F_{-R}^{-1}(q)$$

Consequently, the Expected Shortfall, defined as the expected loss size given that  $VaR_q$  is exceeded, is given by

$$ES = E[-R | -R > VaR_q]$$

In order to assess the accuracy of the VaR estimates a backtest for the 95%, 99%, and 99.5% VaR estimates was applied. First, at time  $t_0$  we estimate the whole model (GARCH+GPD+Copula) using data only up to this time. Then by simulating innovations from the copula we obtain an estimate of the portfolio distribution and estimate the VaR using model (2.1). This procedure can be repeated until the last observation and we compare the estimated VaR with the actual next-day value change in the portfolio. The whole process is repeated only once in every 50 observations owing to the computational cost of this procedure and because we did not expect to see large modifications in the estimated model when only a fraction of the observations is modified. However, at each new observation the VaR estimates are modified because of changes in the GARCH volatilities and the conditional mean. If the selected models are well suited for calculating the VaR then the numbers of exceedances from these models should be close to the expected numbers. Note that the expected number of exceedances at the  $(1 - \alpha)$  confidence level over a period of  $N$  days is equal to  $\alpha.N$  where  $N$  is equal to  $(1304 - 750)$ .

We started by estimating the model using a window of 750 observations. Then we simulate 5000 values of the standardized residuals, estimate the VaR and count the number of the losses that exceeds these estimated VaR values. At observations  $t = 800, 850, \dots, 1300$ , we re-estimate the model and repeat the whole procedure.

We also estimate the VaR using two other approaches: the variance-covariance (also known as analytical) and the historical simulation methods. The first approach estimates the VaR assuming that the joint distribution of the portfolio returns is normal. The VaR can be then computed as follows:

$$VaR = \mu - z_\alpha \cdot \sigma$$

Where  $\mu$  and  $\sigma$  are the mean and the standard deviation of the portfolio returns, and  $z_\alpha$  denotes the  $(1 - \alpha)$ -quantile of the standard normal distribution for our chosen confidence level. The main advantage of this method is its appealing simplicity. However, it suffers from several drawbacks. Among these, there is the fact that it gives a poor description of extreme tail events because it assumes that the risk factors are normally distributed. Also, the parametric method inadequately measures the risk of nonlinear instruments, such as options or mortgages.

The second approach considered is non-parametric, which means that it does not require any distributional assumptions for the probability distribution. The historical simulation estimates the VaR by means of ordered Loss-Profit observations. More generally, assume that we have  $N$  sorted simulated Loss-Profit observations then the VaR at the desired confidence level  $1 - \alpha$  corresponds to the  $\alpha \cdot N$ -th order statistic of the sample. Like the parametric method Historical Simulation may be very easy to implement, but it still has certain drawbacks. The main one is that it relies completely on a particular historical moving window. So when running this method immediately after a special crisis, this event will naturally be omitted from the window and the estimated VaR may change abruptly from one day to the another.

In the case of the variance-covariance and historical simulation methods, the model parameters were updated for every observation. The results for the backtesting are reported in Table 8 and Figures 5, 6 and 7. We can see that copula models outperform both the alternative models, and provide more accurate estimate of the VaR at the 95%, 99% and 99.5% confidence intervals.

### *3.4 The effects of economic structure on financial comovements*

Among emerging markets, the BRIC group of markets represents a different and dynamic set of investment opportunities. On the one hand, their economic

Table 8

Number of observations for  $t = 751$  to  $1304$ , where the portfolio loss exceeded the estimated VaR for 95%, 99% and 99.5% confidence level.

Copula			
	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.005$
Brazil-US	49	15	6
India-China	48	14	10
Brazil-Russia	42	10	6
Historical simulation			
Brazil-US	70	29	15
India-China	73	21	10
Brazil-Russia	61	17	9
Variance covariance			
Brazil-US	76	44	34
India-China	73	40	33
Brazil-Russia	58	29	24

Notes: This table reports the VaR estimation results based on the Copula-GARCH outputs. The model with the number of exceedances closest to the theoretical (or expected) number of exceedances appears to be the most appropriate for calculating the VaR. In this paper the expected number of exceedances are 27.7, 5.54, and 2.77 for the 95%, 99% and 99.5% confidence levels respectively.

rationales are straightforwardly linked to their size and their contributions to global economic growth. More precisely, a combination of large human capital, competitive work force, access to natural resources, and a sustainable revitalization of internal demand has substantially increased the role of the BRIC economies in the global economy. Today, they collectively account for nearly 30% of global output, and only China and India contribute about 1.16% and 0.41% to the global GDP growth according to the IMF's World Economic Outlook Report of April 2008<sup>6</sup>. Growth projections for these economies in the coming years are also substantially above the average growth of the world and developed economies, leading economists and experts to expect that, based on their potential of internal demand expansion and spending power, they could provide a cushion against slower growth in the global economy.

On the other hand, the rationales for equity and foreign direct investments rely particularly on the specificities of the BRIC markets, which can be considered as traditional emerging markets, compared for example to those of the most

<sup>6</sup> The contribution of the US, Euro area, Japan and other developed countries reached only 1.53%, while other developing countries accounted for 1.76%.

### Estimated VaR (Brazil-US portfolio)

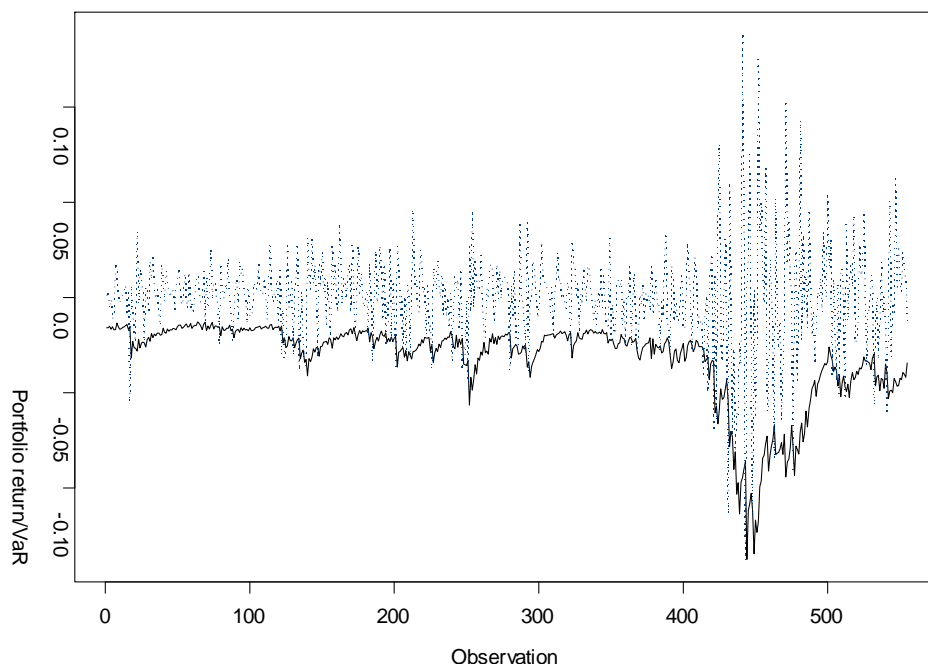


Fig. 7. Out of sample one step ahead estimated 5% portfolio VaR (Brazil-US) and observed returns.

advanced emerging markets (e.g., Mexico, South Korea, and Taiwan). The BRIC markets have lower ratios of stock market capitalization to GDP, are less correlated with developed markets, and display higher idiosyncratic risk due to the low level of their market sensitivities to global factors. As a result, this creates incentives for investors to consider dedicated strategies of asset allocations to the BRIC markets.

Although the BRIC markets have many features in common as discussed previously, the empirical evidence we report here indicates that they do not behave similarly in regard to their financial linkages with the US. If we refer to Tables 4 and 5, we observe that extreme financial dependency on the US during the 2007-2009 global financial crisis is much stronger for Brazil and Russia than for China and India; meanwhile the latter have formed important trade links with the world economy. Table 9 shows that both China and India have a high degree of economic openness with trade to GDP ratios of 71.3% and 44.9%. In particular, the shares of China's exports and imports in the world total trade activities are 9% and 7%. A careful inspection of the trade profiles of the markets studied reveals that Brazil and Russia are more dependent on the revenues from exports of fuel and mining products (20% and 73% of the respective economies' total exports), whereas the economic performance of China and India depends greatly on exports of manufactured products (93%

Estimated VaR (India-China portfolio)

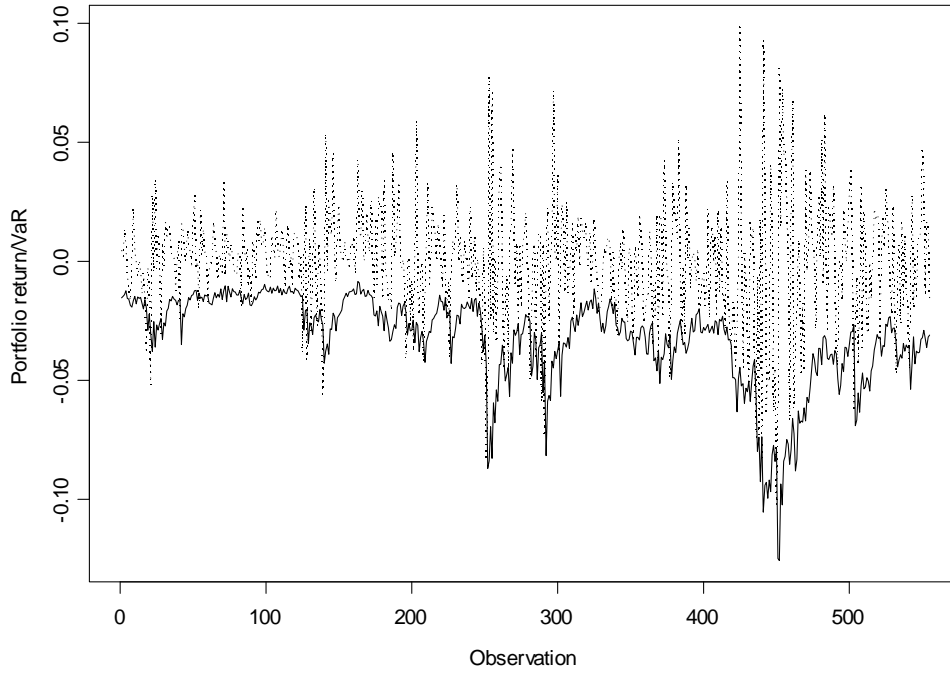


Fig. 8. Out of sample one step ahead estimated 5% portfolio VaR (India-China) and observed returns.

Table 9  
Trade profiles of BRIC markets

Trade profiles	Brazil	Russia	India	Chine	World
Exports of fuel/mining products	32208	257654	35291	41883	2658551
<i>% of total exports</i>	20%	73%	24%	3%	19%
Exports of manufactures	75818	69060	92357	1134805	9499541
<i>% of total exports</i>	47%	19%	63%	93%	68%
Total exports	160649	354403	147034	1218635	13998000
<i>% of World total exports</i>	1%	3%	1%	9%	100%
Total imports	126568	223486	216759	955950	14270000
<i>% of World total imports</i>	1%	2%	2%	7%	100%
Trade to GDP ratio (%)	25.8	54.1	44.9	71.3	-

Notes: This table presents the trade profiles of BRIC markets. The trade to GDP ratios represent the average of trade to GDP ratios over the period 2005-2007. Data are expressed in millions of US dollar and obtained from World Trade Organization statistics database.

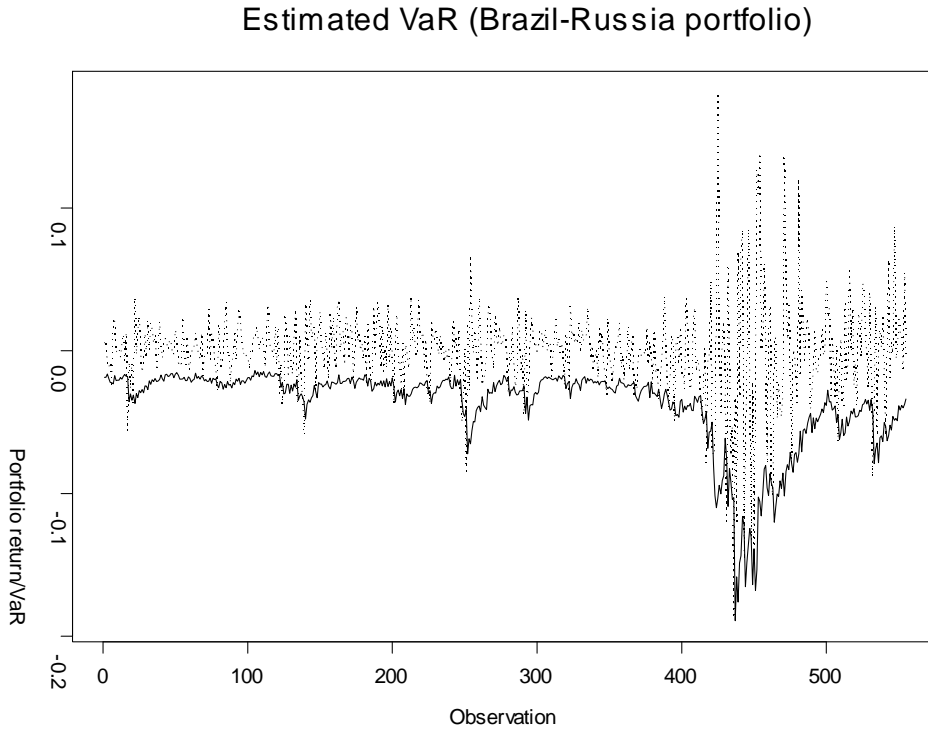


Fig. 9. Out of sample one step ahead estimated 5% portfolio VaR (Brazil-Russia) and observed returns.

and 63% of the respective economy's total exports). In other words, countries that are sensitive to commodity-price changes tend to comove closely with the US in both bull and bear markets. Thus, the heterogeneity of the economic structure and especially the trade profiles could, to this extent, be a relevant explanatory factor for the cross-market interdependences. For future research, it would be interesting to quantify the impact of different types of economic structure on market comovement by running cross-sectional studies.

#### 4 Concluding Remarks

Studies of the transmission of return and volatility shocks from one market to another as well as studies of the cross-market correlations are essential in finance, because they have many implications for international asset pricing and portfolio allocation. Indeed, a higher degree of comovement (or correlation) between markets would reduce the diversification benefits and imply that at least a partially integrated asset pricing model is appropriate for modeling the risk-return profile of the assets issued by the considered countries. With the advent of the current global financial crisis in the aftermath of the US housing market failures, not only academic researchers but also investors and



policymakers have shifted their attention to the extreme dependence structure of financial markets. This is explained by their shared concerns regarding the harmful consequences of contagion effects.

This paper employs a multivariate copula approach to examine the extreme comovement for a sample composed of four emerging markets and the US markets during the 2004-2009 period. The use of this method is advantageous in that it satisfactorily captures the tail dependencies between the markets studied, when univariate distributions are complicated and cannot be easily extended into a multivariate analysis (Jondeau and Rockinger, 2006). The copula functions also provide an interesting alternative to the traditional assumption of jointly normal distribution series, which appears to be unrealistic given the stochastic properties of the return data.

We first provide evidence of the superiority of a Student- $t$  GARCH-in-Mean specification which allows for leverage effects in explaining the time-variations of daily returns on stock market indices. When calibrating several well-known copulas based on the marginal distributions of the filtered returns from the selected GARCH model, we find evidence of extreme comovement for all market pairs both in the left (i.e., bearish markets) and right tails (bullish markets). Further, the results suggest that dependency on the US is higher and more persistent for Brazil and Russia, which are highly dependent on commodity prices, than for China and India whose economic growth is largely influenced by export price levels. Finally, the extreme dependence between emerging market pairs is found to be generally smaller in bearish markets than in bullish markets, which might indicate a low probability of simultaneous crashes. As a practical exercise to check the usefulness of the copula models developed in the paper, we estimate the value at risk for three equally-weighted portfolios for three couples of countries exhibiting tighter extreme comovements over the study period. The results indicate that the Copula-VaR model outperforms the analytical approach and historical simulation method. Undoubtedly, copula models fit at best financial data during widespread market panics and frictions, where the approximations of the usual probability distributions are likely to be strongly biased.

Given the increasing interest in detecting potential gains from international portfolio diversification in a globalized context, further investigation of stock market relationships is needed. Future extensions of this work could focus on an explicit explanation of extreme financial interdependence, using country-specific fundamentals.

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