DEPOCEN

Working Paper Series No. 2010/14

Modeling nonlinear and heterogeneous dynamic linkages in international monetary markets

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Abstract

In this paper we examine the dynamic linkages of international monetary markets over the 2004 - 2009 period using daily short-term interbank interest rates of three of the most advanced countries (France, United Kingdom and United States). Empirical results from vector error-correction models (VECM) and smooth transition error-correction models (STECM) indicate strong evidence of nonlinear and heterogeneous causalities between the three interest rates considered. We also find that exogenous shifts in the US short-term interest rate led those in France and in the UK within a horizon of one to two days. Finally, the national interest rate nexus appears to nonlinearly converge towards a steady state or a common long-run equilibrium because it is subject to structural change beyond a certain interest rate threshold. Our findings have important implications for the actions of leading central banks (ECB, Bank of England, and US Fed) since the behavior of short-term interest rates can be viewed as an indicator of the degree of central banks' policy interdependence.

Keywords: international monetary market relationships, short-term interest rates, VECMs and STECMs

JEL classifications: B22, C52, E63

1. Introduction

With the increasing trend of global financial and economic integration due mainly to the removal of capital controls and other investment barriers over the last three decades, an important body of the literature has been devoted to the investigation of the international interest rate linkages (see, e.g., Barassi et al., 2005; Wang et al., 2007). This issue has important policy implications for policymakers and market regulators as it deals directly with the extent of national central banks' monetary policy independence and coordination (see, e.g., Kirchgassner and Wolters, 1993; Benigno, 2002; Cuaresma and Wójcik, 2006). If there are strong spillovers of interest rates across countries, a particular country's interest rate would be highly sensitive to changes in interest rates of foreign markets, and as a result an independent monetary policy might not be conducted. Thus, an enhanced understanding of these causal linkages permits, for example, to minimize harmful impacts of crisis shock transmission by a series of central banks' coordinated actions or to better forecast the interest rate in a country. It is also commonly accepted that the linkages between interest rates reflect the degree of international capital mobility, which is crucial for global investors' investment-decision making and pricing of interest-rate sensitive products (Zhou, 2003).

The global financial crisis 2007-2009, which took roots from the US subprime and banking failures, has recently renewed the interest for studies of the international monetary market linkages since all countries are more or less affected via their multilateral economic and financial links. Meanwhile, international financial institutions such as the International Monetary Fund and the Bank of International Settlement appeared to demonstrate major failures in fostering global monetary cooperation, securing global financial stability, and providing a prudential framework for macroeconomic policies respectively. Moreover, central bank interventions are very weakly coordinated at the international level to effectively monitor the global matters of the crisis. Over the crisis period we mention that there is only one time where target interest rate cuts are announced jointly by the US Federal Reserve (Fed) and five other world's leading central banks on October 8, 2008, in an effort to calm down the financial market turmoil and to combat the significant deterioration of the main economic performance indicators. In this scheme of things, questions about the nature and dynamic characteristics of both short- and long-run relationships between international interest rates are of paramount importance.

In this study we analyze the dynamic linkages of short-term interest rates among France, the UK and the US, represented by the 3-month interbank offered rates respectively. The use of these interest rates is motivated by the fact that they capture both the time dynamics and magnitude of central banks' monetary interventions through changes in policy rates which have a considerable impact on financial market conditions and investment decisions (e.g., Rigobon and Sack, 2003; Bernanke and Kuttner, 2005; Chen, 2007; Ioannidis and Kontonikas, 2007). Being key inputs for economic activity and asset valuations, they further reflect changes in business cycle and macroeconomic fundamentals such as inflation and exchange rate equilibriums. Taking together we contribute to the related literature by not only investigating how different interest rates are linked in the short- and long-run, but also drawing more insights on the way according to which each central bank conducts its monetary policy with its peers.

Our empirical approach differs, however, significantly from the conventional VAR and cointegration methodologies used in the majority of previous studies in that we develop a nonlinear univariate and trivariate cointegration framework, jointly based on Vector Error-Correction Model (VECM) and Smooth Transition Error Correction Model (STECM) to assess the dynamics and the potential linkages of interest rates between three countries. While the VECM is commonly known to be useful for modeling and forecasting the long-run relationships between different nonstationary series, which is the case of interest rates, the STECM improves the dynamic adjustment process of interest rates to their long-run equilibrium over a certain threshold of interest rate deviations exceed. It should be noted that this nested setting is completely suitable for capturing asymmetric reactions and nonlinearities in the adjustment dynamics of interest rates towards their common equilibrium by allowing them to shift from one regime to another. Compared to past studies, our approach is in line with Mancuso et al. (2003), Holmes and Maghrebi (2004), Poghosyan and De Haan (2007), and Barassi et al. (2005) to a larger extent in the sense that these authors treat the nonlinear dynamics of interest rate linkages, but their research design and objectives were relatively different.

Over the period from December 31, 2004 to March 19, 2009 which is intentionally set to cover the major events related to the global financial crisis, we find strong evidence of nonlinear causal interactions of short-term interest rates among France, the UK and the US. In particular, exogenous shifts in the US interest rate are found to lead those in France and the UK within a horizon of one to two business days. It also appears that short-term interest rates in three considered countries have had similar behavior in recent months, which may be, in our opinion, due to the higher convergence to the long-run equilibrium of target interest rates, conducted by central banks of respective countries. This result is indeed supported by those of Scotti (2006) who investigates interest-rate feedback rules between the US Fed and the European Central Bank, using a combination of a bivariate autoregressive conditional hazard model and a conditional bivariate ordered probit model.

The remainder of the paper is organized as follows. Section 2 provides a brief review of theoretical arguments and empirical evidence related to the linkages of national interest rates. Section 3 presents our empirical methodology to examine the nonlinear interest rate interdependences and the data used. The main focus is on the econometric specification, implementation and estimation of the linear and nonlinear error-correction models. Section 4 reports and discusses the empirical results. Concluding remarks and policy implications are provided in Section 5.

2. International interest rate linkages: a short review of literature

Economic theory suggests several explanations why there should be interactions between short-term interest rates in international monetary markets (Barassi et al., 2005). If the interest rates are treated as analogous to any other financial securities, their levels and variations can be then interpreted as being driven by not only the supply-demand rule, but also profit-seeking financial flows. According to this view, in an international setting, any deviations from interest rate parity are likely to be exploited by arbitrage exchange, leading to the comovement of interest rates across countries. Alternatively, short-term interest rates of a particular country may be related to those in foreign countries as they are often used as monetary policy instruments to set exchange rate and inflation rate targets. Subject to changes in the market outlooks and macroeconomy, any modification of or deviation from the target would cause the interest rates to deviate from parity conditions. Therefore, there is likely to be considerable interdependence between national interest rates, and the linkage is expected to be stronger with the increasing integration of global financial markets. Also, it is possible to draw inferences on the degree of market integration by analyzing the cross-country interest rate nexus.

An extensive literature has investigated the international linkages between short-term interest rates over both the short- and long-run. Earlier studies including, among others, Mishkin (1984), Mark (1985), and Cumby and Mishkin (1986) are mainly concerned by testing the hypothesis of real interest rate equality across countries, which is derived from uncovered interest rate parity, in order to shed light on the level of cross-market financial integration. They generally establish evidence against the equality of real interest rates and attribute the deviations from parity to the existence of a significant currency risk. Subsequent works focus essentially on the short- and long-run relations among various nominal and real interest rates rather than on their equalization using frequently VAR and cointegration frameworks (see, e.g., Awad and Goodwin, 1998; Bremnes et al., 2001; Yamada, 2002; Anoruo et al., 2002; Baum and Barkoulas, 2006; Wang et al., 2007). The reason is that the equality of real interest rates requires the purchasing power parity (PPP) to hold, while there is substantial evidence against the validity of PPP[1]. Main findings of these studies indicate that short-term interest rates tied up by a long-run phenomenon since they are often found to be cointegrated, with a dominant effect of the US interest rate. In addition, the transmission of interest rate events increases with the ongoing process of financial liberalization around the world.

In sum, while the studies cited above are useful for measuring the linear relationship between interest rates, they ignore the potential of asymmetry and nonlinearity in their dynamic adjustment process towards long-run equilibrium. Consistent with, among others, Mancuso et al. (2003), Barassi et al. (2005), and Poghosyan and De Haan (2007)

[1] See Rogoff (1996) for a survey of theoretical and empirical evidence on PPP.

who find evidence of nonlinearity and structural breaks in interest rate linkages, we develop a threshold multivariate cointegration framework to take this into account.

3. Threshold cointegration for international interest rate linkages

This section addresses the importance of appropriately modeling the short-run dynamics and long-run linkages of short-term interest rates. We begin with a description of the cointegration approach widely used in previous works to test the linkages between different interest rates. We then show how this basic empirical framework can be extended to capture asymmetry and nonlinearity in the time dynamics of interest rate deviations.

3.1 Linear adjustment dynamics

The linear cointegration framework, introduced by Granger (1981) and developed by Engle and Granger (1987), and Johansen (1988), among others, indicates that two integrated series of order one, *I(1)*, X_t and Y_t (i.e., two interest rate series) can evolve together in the long run if a linear combination between them is stationary. Two series are said to be cointegrated in this case and the theory suggests the existence of a long-run equilibrium to which the system converges over time. In addition, the following long-run relationship between X_t and Y_t must be verified:

$$Y_t = \alpha_0 + \alpha_1 X_t + z_t \tag{1}$$

In the above expression, z_t can be interpreted as the equilibrium error indicating the deviations of the system of interest rates from its long-run equilibrium at any point in time. Note also that z_t is an *I(0)* process and (α_0, α_1) defines the cointegrated vector.

Under the assumption of stationarity of z_t, the system's adjustment process may be reproduced using a Linear Error-Correction Model (LECM) and the dynamics of interest rate deviations from equilibrium is given by

$$\Delta z_t = \phi_0 + \lambda z_{t-1} + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \varepsilon_t \qquad (2)$$

where λ is the linear adjustment term ensuring the mean reversion to the equilibrium; ϕ_i are autoregressive parameters with $\forall i = 1, \ldots, p$; and ε_t is an error term which is supposed to follow a normal distribution with zero mean and variance of σ_ε^2, i.e., $\varepsilon_t \rightarrow N(0, \sigma_\varepsilon^2)$. Nevertheless, this econometric specification has a drawback according to which the adjustment process to the long-run equilibrium can be only linear, symmetric and continuous, with a constant adjustment speed measured by the coefficient λ. It naturally becomes inefficient whenever the adjustment process incorporates asymmetric and nonlinear patterns with a time-varying adjustment speed. In what follows we show that the linear framework described above can be improved to capture these stylized facets in a cointegrating system when it is extended to a nonlinear framework.

3.2 Modeling nonlinear interest rate dynamics with STECMs

We now focus on the introduction of the hypothesis of nonlinearity and switching regimes in the LECM specification. This yields a promising nonlinear framework given by the STECMs (*Smooth Transition Error-Correction Models*). Initially, the class of STECMs was introduced by Granger and Teräsvirta (1993), and applied for the first time by Van Dijk and Franses (2000). Their statistical properties and modeling approach were explicitly developed by Van Dijk, Teräsvirta and Franses (2002). One of the main advantages of this nonlinear specification is the ability of making the adjustment of interest rate dynamics nonlinear and asymmetric with time-varying adjustment speed. It particularly takes the smoothness in adjustment into account, and specifies the dynamic process depending on both the magnitude and the sign of disequilibrium associated with exogenous shocks and financial crises.

STECMs have recently been used in several studies including, among others, Anderson (1997), Balke and Fomby (1997), and Liu (2001) for interest rates; Van Dijk and Franses (2000), and Jawadi and Prat (2009) for stock prices, Jawadi, Bruneau and Sghaier (2009) for insurance premiums; Escribano (1997) for the money demands in the United Kingdom; and Rothman, Van Dijk and Franses (2001) for relationships between production and money demands. By conditionally defining an on-off adjustment dynamic process with respect to the sign and the size of the disequilibrium, these studies consistently suggest that STECMs are suitable for capturing nonlinearity and switching regimes, smoothness, persistence, discontinuities, structural breaks, inertia effects and asymmetry in the adjustment dynamics induced by market frictions.

Econometrically, the STECM constitutes an extension of the LECM to the nonlinear framework. As a prime example, a two-regime STECM can be specified as a combination of two LECMs insofar as it integrates two adjustment terms reproducing the speed of adjustment in the first regime and the intensity of error-correction in the second regime respectively. In practice, the extension to the nonlinear framework is made through the introduction of a nonlinear component defined as the product of a transition function and the adjustment term of the second regime. More specifically, we can set up a two-regime STECM for interest rate deviations as follows:

$$\Delta z_t = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \lambda_2 z_{t-1} \times F(z_{t-d}, \gamma, c) + \varepsilon_t \qquad (3)$$

where λ_1 and λ_2 are the adjustment terms in the first and second regimes respectively; z_{t-1} is the error-correction term; *F(.)* refers to the transition function; γ and *c* refer respectively to the transition speed ($\gamma > 0$) and the threshold parameters; *d* represents the delay parameter; and z_{t-d} denotes the transition variable[2]. Thus, the STECM explicitly describes two regimes corresponding to the extreme values of *F(.)*, i.e., one corresponding to *F(.)* = 0 and another to *F(.)* = 1, and an intermediate continuum state. Accordingly, the transition from one regime to another takes place smoothly, and the first regime is obtained when the interest rate adjustment dynamics is close to equilibrium, i.e., $F = 0$, that is:

$$\Delta z_t = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \varepsilon_t \qquad (4)$$

The dynamics of the second extreme regime is given by:

$$\Delta z_t = \phi_0 + (\lambda_1 + \lambda_2) z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \varepsilon_t \qquad (5)$$

In all cases, λ_1 and λ_2 constitute the most important parameters for this specification as their values and signs determine the adjustment dynamics of interest rates and their convergence speed towards equilibrium (Michael, Nobay and Peel, 1997). Indeed, even though λ_1 is positive, interest rates are nonlinearly mean-reverting and the STECM is stable only if λ_2 and ($\lambda_1 + \lambda_2$) are negative and statistically significant. This implies that for small deviations, interest rate movements may depart from the long-run equili-

[2] It should be noted that the difference between the STAR (Smooth Transition Autoregression) and the STECM is the transition variable, z_{t-d}. In a STAR model, z_{t-d} is a lagged dependent variable, Y_{t-d}, and in a STECM, z_{t-d} is represented by the error-correction term.

brium and would be characterized by explosive behavior or a unit root, while for large deviations, the adjustment process would be mean-reverting.

According to Teräsvirta and Anderson (1992), Granger and Teräsvirta (1993), and Teräsvirta (1994), *F(.)* can be either an exponential or a logistic function. A first-order logistic transition function is thus defined as follows:

$$F(z_{t-d}, \gamma, c) = \left[1 + \exp\left(-\gamma(z_{t-d} - c)\right)\right]^{-1} \quad (6)$$

Note that a first-order exponential transition function corresponds to:

$$F(z_{t-d}, \gamma, c) = 1 - \exp\left[-\gamma(z_{t-d} - c)^2\right] \quad (7)$$

The system composed of Equations (3) and (6) defines a LSTECM (Logistic STECM), whereas the system combining Equations (3) and (7) results in an ESTECM (Exponential STECM). Even though both models allow for a smooth transition between two distinct regimes and thus an asymmetric adjustment around the threshold parameter *c*, it should be noted that the ESTECM better captures the asymmetry inherent to the size of interest rate deviations, while the LSTECM is best suited for reproducing the asymmetry in the sign of interest rate deviations[3]. Further, the two interest-rate regimes are discriminated and associated with small and large values of the transition variable z_{t-d}. In the first regime, interest rate deviations are small and may be away from the equilibrium, uncorrected, near unit root and random, whereas in the second regime large interest rate deviations will be nonlinearly mean-reverting to equilibrium when they exceed a certain threshold and then approach a white noise.

According to Van Dijk, Teräsvirta and Franses (2002), the empirical modeling approach of STECMs is carried out in several steps. First, the empirical specification

[3] See Teräsvirta (1994) for more details regarding the statistical properties of these transition functions.

requires the definition of the explanatory variables, the determination of the lag number, linearity tests and the choice of transition function. Second, the STECM is estimated by the Nonlinear Least Squares (NLS) method based on a nonlinear optimization process[4]. Before moving to the analysis of the empirical results, we briefly present the linearity tests required for STECM modeling.

3.3 Nonlinear adjustment tests

These testing procedures aim to test the null hypothesis of linearity H_0 against its alternative of nonlinearity H_1. Under H_0, the interest rate adjustment deviation dynamics is reproduced using a LECM described by Equation (2), while a STECM given by Equation (3) is more appropriate under (H_1). However, the null hypothesis is defined differently and this can give rise to a problem of nuisance parameters, and the usual statistic inference is no longer available. To remedy this problem, Luukkonen et al. (1988) proposed replacing the transition function *F(.)* in Equation (3) by its Taylor development and applying Lagrange Multiplier (LM) tests to check for nonlinear adjustment. In the LM tests, their distribution is known under H_0 and follows a standard χ^2 distribution.

In order to apply these tests, we must determine the number of lags in the LECM, noted *p*, based on usual information criteria (AIC and BIC), the Ljung-Box test for serial autocorrelation, and the partial autocorrelation function. Next, a grid search defines the possible value for the delay parameter *d*. For example, plausible values that we consider for *d* include the following set [1,2,3,4,5] when using daily data. We then apply nonlinear adjustment tests for the possible values of *d*. The optimal value defining the transition in Equation (3) is the one for which linearity is most rejected.

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[4] See Jawadi and Prat (2009) for more details concerning the STECM modeling.

As pointed out by Luukkonen and Saikkonen (1988), the LM test implementation (LM_3 test) can be described in three main steps[5]:

Step 1: We estimate the LECM and compute the Squared Sum of Residuals under H_0 (SSR_0).

Step 2: We estimate the following auxiliary regression for each possible value of *d* and we compute the Squared Sum of Residuals associated with this regression (SSR_1):

$$\Delta z_t = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i} + \lambda_2 z_{t-1} \times z_{t-d} + \lambda_2 z_{t-1} \times z_{t-d}^2 + \lambda_2 z_{t-1} \times z_{t-d}^3 + \nu_t \qquad (8)$$

Step 3: We compute the Lagrange Multiplier statistics of LM_3 test as follows:

$LM_3(d) = T \times \dfrac{SSR_0 - SSR_1}{SSR_0} \rightarrow_{H_0} \chi^2(3, p)$, where p and T refer to the number of lags and the number of observations respectively.

In practice, the LM_3 statistics are computed for all possible values of *d*. The optimal value for *d* is the one whereby linearity is strongly rejected, or equivalently the value that should minimize the *p*-value of the LM_3 test.

4. Empirical results

4.1 Data and preliminary results

We investigate the international linkages of short-term interest rates using data from three of the most advanced countries: France, the UK and the USA. The data consists of

[5] Several LM tests were developed (LM_1, LM_2, LM_3, LM_4). For more details of these tests, see Van Dijk et al. (2002).

the daily three-month interbank offered interest rates obtained from Datastream International and cover the period from December 31, 2004 to March 19, 2009. Working with daily data is consistent with the fact that any monetary policy adjustments tend to be immediately incorporated in the short-term interest rates over a very short time spans. This also offers us the possibility to draw inferences on the responses of monetary policymakers to uncertainties and shocks related to the recent international financial crisis. Thus, the inclusion of France and the UK into sample countries is motivated by the fact that the financial crisis originating from massive failures in the US housing and banking markets was quickly transmitted to these countries, and makes it compulsory for their central banks to adjust the policy interest rates in order to reduce the negative effects of the said crisis.

[Please insert Figure 1 here]

As a preliminary step, we checked the integration order of the interest rate series by using several unit root tests such as the Dickey and Fuller (1981)'s augmented test, and Phillips and Perron (1988)'s test. The null hypothesis of unit root is not rejected in any of the interest rate series studied, as it can be visualized in Figure 1. However, given that these tests may be powerless when data is not generated by linear processes, we then apply the test of Zivot and Andrew (1992) which is robust to structural breaks. The latter also confirms the results of previous tests, indicating that the interest rate series are integrated of order one, noted *I(1)*, for the three countries in our sample[6].

In order to get an overview regarding the linkages of interest rates among France, the UK and the US under the effects of the recent financial crisis, we calculate the bila-

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[6] Results of unit root tests are not reported here to save space, but can be obtained upon request addressed to the corresponding author.

teral correlations between these interest rates over two subperiods, i.e., December 31, 2004 - July 31, 2007, and August 1, 2007 - March 19, 2009, and report the results in Panel 1 of Table 1.

[Please insert Table 1 here]

Accordingly, we observe a substantial increase in bilateral correlations over the post-crisis subperiod. This indicates that the crisis has yielded greater interdependence between monetary markets of the three countries considered. A potential explanation is the fact that central banks of respective countries may have a tendency to coordinate their policies in order to regulate and overcome the financial crisis. Albeit informative, these findings need to be improved using more parsimonious and robust modeling techniques because the correlation coefficients reflect simply a linear statistical association among the variables of interest.

The inspection of the descriptive statistics of interest rates gives rise to a number of interesting facts (Panel B of Table 1). On the one hand, the negative sign of the interest rates highlights their large decrease over the recent months, notably after the advent of the international financial crisis. The strong rejection of symmetry and normality as well as the leptokurtic character inherent to interest rate dynamics suggest, on the other hand, some evidence of nonlinear and asymmetric patterns in their time dynamics.

4.2 Bivariate and multivariate linear cointegration test results

Before moving to the investigation of the interest rate dynamics via nonlinear modeling, we propose to examine the linkages of short-term interest rates using two commonly used cointegration tests, i.e., two-step procedure of Engle and Granger (1987), and trace test procedure of Johansen (1988). Our findings can be then compared to those of the majority of previous works.

i) The Engle and Granger (1987) two-step procedure

We examine the long-run linkages between interest rates by using Equation (9) that is none other than an extension of the Equation (1). The cointegration relationship is checked by testing the stationarity of the model residuals. Indeed, the stationarity of the disequilibrium errors z_t is suggestive of the fact that interest rates are cointegrated and that they may be linearly mean-reverting.

$$i_t^j = \alpha + \beta i_t^k + \delta i_t^l + z_t \tag{9}$$

where i_t denotes the interest rate at time t, (α, β, δ) refers to the cointegrated vector and z_t correspond to model residuals measuring the magnitude of interest rate deviations from their long-run equilibrium. We consider three different cases: for j = France, k = USA and l = UK; for j = UK, k = USA and l = France; and finally for j = USA, k = France and l = UK.

[Please insert Table 2 here]

Results of Table 2 indicate that short-term interest rates in France, the UK and the US seem to be at least reasonably cointegrated at the 10% level, which is more or less consistent with the findings of previous studies (see, e.g., Awad and Goodwin, 1998). Accordingly, it is straightforward to model their linear adjustment dynamics and mean-reversion phenomenon through estimating a LECM, given by Equation (2) and recalled in the following formula:

$$\Delta z_t = \phi_0 + \lambda z_{t-1} + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \varepsilon_t$$

As discussed extensively in the financial econometrics literature, this modeling is powerful enough to reproduce any comovements between interest rates, potentially implied by the coordination of central bank interventions. It is also informative of how a central bank adjusts its target interest rates with respect to changes in policy rates conducted by the others. We carry out the estimation of this model using the Linear Least Squares Method (LLSM) for the three interest rate series studied, and report the results in Table 3.

[Please insert Table 3 here]

According to our findings, the linear adjustment term is negative, but it is only statistically significant at the 10% level for France and the US. This suggests that the linear error-correction mechanism is *a priori* not fully and continually activated, or rather misspecified. Otherwise, the significant dependence of interest rate deviations on the previous misalignments typically indicates some evidence of long-term persistence in interest rates.

One should however note that the Engle and Granger (1987) analysis of cointegration is only appropriate for examining a unique cointegrating vector at a time, and as a result information about the real linkages among considered interest rates may be lost due to the restriction of a bilateral relationship. For this reason, we propose to use the multivariate cointegration framework in order to exhaustively investigate the linkages between interest rates. Such multivariate cointegration tools, introduced by Johansen (1988), are more powerful in that they permit to simultaneously test for several cointegration relationships.

ii) The Johansen (1988)'s cointegration procedure

We firstly apply the trace test to check the null hypothesis of "no cointegration relationship" against its alternatives of at most one or at most two cointegration relationships, and report the results in Table 4. The findings indicate the rejection of the null hypothesis at the 5% level (55.86 > 35.19) and suggest the presence of at most one cointegration relationship (12.06 < 20.26).

[Please insert Table 4 here]

Table 5 reports the estimation of the cointegration relationship while normalizing the French interest rate. It means that the latter defines the endogenous variable in the studied cointegration relationship.

[Please insert Table 5 here]

In order to apprehend the mean-reversion process in interest rates within a multivariate framework, we estimated the dynamics of interest rate convergence towards equilibrium using a VECM with three equations (France, the UK and the US) and report the main results in Table 6. It is shown that the linear adjustment term is significant at the 5% level only for the US, while the French and British interest rates do not display linearly mean-reversion. This means that France and the UK may continue to decrease their interest rates in the future.

More interestingly, our results highlight strong evidence of interest rate linkages as for each country in the sample, the short-term interest rate deviations depend not only on their previous deviations but also on those of the other countries. This significant interdependence may last for two days, reflecting the temporal shift between countries, and the time required to decode, understand and react to the information contained in the interest rate changes by other countries. The low degree of interdependence between France and the US may be due to the differences in the monetary policies conducted by their central banks to manage the financial crisis. What can be observed is that the US Fed immediately decreased its target interest rate following the crisis occurrence and in several stages, whereas the ECB, supreme monetary authority for Bank of France, kept its key interest rate constant, even increased it, and only decreased it since November 12, 2008.

[Please insert Table 6 here]

Overall, both bivariate and multivariate cointegration techniques as well as correlation estimations provide some evidence of significant relationships among the interest rate series in France, the UK and the US. This may support the hypothesis of monetary policy interdependence between the ECB, the Bank of England, and the US Fed. However, the linear modeling may lead to biased and misspecified results due to the assumptions of both linearity and symmetry in the dynamic linkages of interest rates. A major consequence is that the adjustment process during periods of normal economic growth is similar to those during financial crises, while interest rate behavior is likely to be different. Additionally, the interest rate dynamics, depicted in Figure 1, seem to be neither linear nor symmetric. We may cite for instance that the intensity of interest rate increases in 2005 was less marked than that associated with interest rate decreases following the subprime crisis. All in all, the potential of asymmetry in interest rate adjustment dynamics may escape from the LECM represented by Equation (1).

[Please insert Table 7 here]

The above arguments are strongly justified in view of the stochastic properties of interest rate deviations z_t that we report in Table 7. Interest rate deviations are indeed negative on average for all three countries reflecting thus the wave of target interest rate cuts during the financial crisis period. The rejection of normality and symmetry hypothesis confirms ou intuition that interest rate adjustment dynamics is asymmetric and nonlinear, and consequently motivates the use of nonlinear error-correction models (NECMs) in modeling the interest rate dynamic linkages.

4.3 Estimation results of nonlinear STECM

We now turn to examine the adjustment dynamics of interest rate deviations for three considered countries using the STECM. In addition to the long-run cointegration relationships of interest rates described in Equation (9), we also introduce their short-run deviations into the nonlinear ECM in order to test how far one interest rate deviates following a deviation in the other interest rates. Accordingly, the formal representation of the STECM in Equation (3) can be rewritten as:

$$\Delta z_t = \phi_0 + \lambda_1 z_{t-1} + \sum_{i=1}^{p} \phi_{1,i} \Delta z_{t-i}^{F} + \sum_{j=0}^{p} \phi_{2,j} \Delta z_{t-j}^{US} + \sum_{k=0}^{p} \phi_{3,l} \Delta z_{t-k}^{UK} + \lambda_2 z_{t-1} \times F(z_{t-d}, \gamma, c) + \varepsilon_t \quad (10)$$

The above specification concerns the French case in which current and lagged US and UK interest rate deviations are introduced as explanatory variables in the nonlinear ECM. Similar STECM representations are also retained for the UK (respectively the US), while current and lagged French and US (respectively UK) interest rate deviations serve as explanatory variables.

4.3.1 Specification

We employ several tests (Ljung-Box tests, autocorrelation functions and information criteria) to determine the LECM lag number for interest rate deviations. In accordance with our results, we retained $p = 3$ for the US and $p = 4$ for both France and the UK, indicating the importance of persistence and inertia effects in interest rate adjustment dynamics. In addition, for each interest rate in the sample we studied, we found that its adjustment dynamics is significantly affected by the current and previous deviations of the other interest rates with a slight dominance of the US rate. This potentially indicates significant linkages between central banks' policy decisions over the last few years.

[Please insert Table 8 here]

We then apply the nonlinear adjustment tests and Teräsvirta (1994)'s test to select the type of transition function. Table 8 reports the main results obtained[7]. Our findings highlight the rejection of the linear adjustment hypothesis for interest rates in all three countries, suggesting that they are nonlinearly mean-reverting. It is also worth noting that the Teräsvirta tests retain the exponential function, rather than the logistic function, to reproduce the transition between the interest rate regimes.

The results discussed above are highly interesting for several reasons. First, the rejection of linearity and the choice of an exponential transition function for all countries point to some common dynamic features between the three interest rates. Second, the acceptation of regime-switching hypothesis implies that the existence of at least two regimes characterizing the interest rate dynamics over the study period. The activation of these regimes and the transition from one regime to another depend closely on the intensity of interest rate changes in international monetary market or policy rate changes to a narrower extent, as well as on their interactions.

[7] More details about these tests may be obtained upon request addressed to the corresponding author.

4.3.2. Estimation and validation

Following the results of the specification tests, we estimate an ESTECM (3,4), ESTECM (4,1) and ESTECM (4,2) for the US, France and the UK respectively. The results reported in Table 9 indicate several important facts. First, all the autoregressive coefficients are statistically significant at either the 5% or the 10% levels, thus confirming the persistence effects suggested by linear modeling, and suggesting the continuity in interest rate cuts by central banks since July 2007. Second, for all interest rate series, both current and previous interest rate deviations have substantial information on the time dynamics of the interest rate considered. We also observe nonlinear continuous-time interdependence between interest rates. More interestingly, the dynamic adjustment process of interest rates is found to be significantly and negatively affected by their current deviations, but positively and nonlinearly correlated with their lagged deviations. This means that in times of turbulences marked by an international financial crisis, changes in one interest rate implies an immediate adverse reaction of the others, but after a certain time lag, they get positive feedback from the others once more information about interest rate changes have been extracted.

[Please insert Table 9 here]

Third, the parameters of the exponential function are statistically significant, confirming the Teräsvirta (1994)'s test and suggesting the presence of two regimes characterizing the dynamics of interest rate deviations. A central regime, in which the interest rate may deviate from the long-run equilibrium and be uncorrected until its deviations exceed a certain threshold, and an outsider regime describing the dynamics of the interest rate when it moves back to equilibrium thanks to the activation of the nonlinear adjustment terms $\hat{\lambda}_1$ and $\hat{\lambda}_2$. It is essential to note that these coefficients, being the most important parameters of the nonlinear adjustment model, are statistically significant at the 5% level. The second adjustment term $\hat{\lambda}_2$ is negative in all three cases and the sum ($\hat{\lambda}_1 + \hat{\lambda}_2$) is also negative. This means that even though the short-term interest rates may effectively deviate in the first regime from the equilibrium (i.e., $\hat{\lambda}_1 \geq 0$), they are nonlinearly mean-reverting over the long-run and the estimated ESTECM are stable over the study period.

Fourth, we check the robustness of these results by analyzing the statistical properties of the estimated residuals and show that they are symmetrical, stationary, and not autocorrelated. They are in addition characterized by an ARCH effect which reveals the time-varying volatility in the interest rate deviations.

Finally, to get more insights about the different regimes characterizing the interest rate adjustment dynamics, we depict in Figure 2 the estimated transition function (transition function with respect to transition variable) and the intertemporal transition function (transition function with respect to time factor) for the UK, the US and France respectively. We observe that the most important observations are asymmetrically distributed, confirming the choice of the exponential representation. The adjustment speed of the estimated transition functions varies and increases with the size of the interest rate deviations. The more the latter increase, the more rapid the interest rate mean-reversion is. Moreover, for all three countries, the values of the estimated transition function are very low and did not exceed 12% at most for the UK. This reflects on the one hand that the intervention of the UK monetary authority has been quite intensive in order to adjust the policy interest rate and thus to move the short-term interest rate back to its long-run equilibrium established with the US and French rates. On the other hand, this finding indicates that interest rate deviations are relatively near unit root and their dynamics may approach a random walk.

[Please insert Figure 2]

It is equally important to note that while the US monetary market is characterized by a more volatile transition function, perhaps because of the Fed's successive interventions, the UK market has a higher transition function, reflecting the highest interest rate decrease in 2009. More interestingly, a close examination of the graphs shows similar dynamics for the three interest rates considered, which is typically informative of some evidence of monetary policy interdependence among sample countries. Further, these functions consistently reflect the policy interest adjustments by the French, UK and US central banks over the recent period.

4.3.3. Interest rate undervaluation and overvaluation periods

The empirical model used in this paper also permits us to gauge the interest rate undervaluation and overvaluation phases as well as the speed of interest rate mean reversion and the rhythm of correction of interest rate misalignments. To do so, we use the two indicators developed by Peel and Taylor (2000)[8]. The first indicator corresponds to

$$\Omega(z_t) = 100 \times F(z_t) \times sign(z_t),\; sign(z_t) \equiv \frac{z_t}{|z_t|},\; -100 \leq \Omega(z_t) \leq 100 \qquad (11)$$

The above indicator reproduces the magnitude of the interest rate deviations from equilibrium. According to Peel and Taylor (2000), a positive (respectively negative)

[8] Note that Peel and Taylor (2000) develop these indicators to investigate the exchange rate adjustment towards purchasing power parity (PPP), while they are for example applied by Jawadi and Prat (2009) to examine the undervaluation and overvaluation of stock prices with respect to their fundamentals.

value of this indicator implies that interest rates are over-evaluated (respectively under-evaluated). Interest rates are in equilibrium when this indicator converges towards zero.

The second indicator is given by:

$$\Gamma(z_t) = 1 - F(z_{t-d}) \,,\; 0 \le \Gamma(z_t) \le 1 \qquad (12)$$

This indicator evaluates the interest rate adjustment speed. The more $\Gamma(z_t)$ tends towards 1, the more the interest rate deviations z_t approach a random walk, while the more $\Gamma(z_t)$ convergence towards the speed of adjustment increases and z_t converges towards a white noise (Jawadi and Prat, 2009).

[Please insert Figure 3]

The two indicators discussed above are computed for each of the three countries in our sample and presented in Figure 3. As it can be observed, the evolution of the second indicator highlights a high adjustment speed, thus indicating that interest rate adjustment is highly activated for all three monetary markets since it is close to the unity almost all the times. This also confirms the low estimated value for the transition functions. Moreover, we observe clear phases of interest rate overvaluation for the US and the UK, and an undervaluation phase for France in 2005. These phases are then followed by an interest rate deviation correction phase, which appears however to be more agitated at the end of the estimation period, probably because of the effect of the global financial crisis. Meanwhile it is worth noting that the most recent developments of the undervaluation and overvaluation indicator have shown several similarities in interest rate adjustment dynamics for France, the UK and the US, which once again suggests some indication of their higher monetary policy linkages over recent years.

5. Concluding remarks

Within the context of the recent global financial crisis and economic meltdown, this paper investigated the dynamic linkages of short-term interest rates in international monetary markets using data from France, the UK and the US. To address this issue, we used different linear and nonlinear econometric techniques including bivariate and trivariate cointegration tests, VECMs and STECMs. These techniques particularly enabled us to examine short-run dynamics and long-run relationships of the variables of interest. They also appear to be best suited for capturing several forms of potential asymmetry, nonlinearity and structural changes in their adjustment dynamics.

We mainly find strong evidence of nonlinear and heterogeneous linkages between the three interest rates considered. In particular, exogenous shifts in the US short-term interest rate are found to lead those in France and in the UK within a horizon of one to two days. The results also establish that over the study period, the time deviations of interest rates are subject to a regime-switching behavior and their multivariate nexus converge towards a common long-run equilibrium in a nonlinear way. If we can consider that deviations of short-term interest rates reasonably reflect changes in target interest rates, the convergence of short-term interest rates towards a common equilibrium over recent years may be explained by more coordination of the ECB, US and UK central bankers in an effort to manage the crisis issues together. Note finally that, in order to precisely confirm this intuition, an extension of this work can be made and consists of examining the interdependence of monetary policy surprises.

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Table 1. Correlation matrix and descriptive statistics

This table reports the bilateral correlations between interest rates, as well as their descriptive statistics of interest rates. DIF, DIU and DIUK designate interest rate changes for respectively France, the USA and the UK.

	DIF	**DIU**	**DIUK**
Panel A: Correlation matrix			
First subperiod: December 31, 2004 – July 31, 2007			
DIF	1.00	0.02	0.07
DIU		1.00	-0.02
DIUK			1.00
Second subperiod: August 1, 2007 – March 19, 2009			
DIF	1.00	0.38	0.58
DIU		1.00	0.40
DIUK			1.00
Panel B: Descriptive statistics: December 31, 2004 – March 19, 2009			
Mean ($\times 10^5$)	-0.49	-1.15	-2.80
Std. dev. ($\times 10^2$)	0.02	0.04	0.04
Skewness	-1.11	-2.66	-16.13
Kurtosis	12.72	38.21	410.42
Jarque-Bera	4542.35	57982.80	7634819.00
Probability	0.00	0.00	0.00

Table 2. Linear cointegration relationships

This table reports the results from the linear cointegration tests applied to three interest rate series according to Engle and Granger (1987) two-step procedure. t-statistics are given in parenthesis.

	France	**UK**	**US**
α ($\times 10$)	-0.05	0.10	0.03
	(-5.06)	(25.70)	(2.05)
β	-0.22	0.30	-0.53
	(-12.07)	(22.4)	(-12.07)
δ	0.96	0.61	1.09
	(39.90)	(39.90)	(22.39)
R^2	0.60	0.69	0.33
ADF	-3.54	-3.63	-3.49

Table 3. LECM estimation results

We report parameter estimates of the linear error-correction model for three interest rate series using linear least squares method. ϕ_0, ϕ_1 and λ refer to constant term, autoregressive parameter and linear adjustment term. * indicates significance at the 1% level. Standard errors are given in parenthesis.

	France	**UK**	**US**
ϕ_0 ($\times 10^5$)	1.69	-1.80	1.20
	(1.62)	(-1.67)	(0.82)
ϕ_1	0.14	0.16	-0.30
	(4.58)	(5.4)	(9.66)
λ ($\times 10$)	-0.03*	-0.02	-0.03*
	(-1.84)	(-0.77)	(-1.89)
R^2	0.02	0.03	0.08

Table 4. Johansen cointegration test

We report the results of the Johansen cointegration test. The null hypothesis of no cointegration is tested against its alternatives of at least one or at least two cointegration relationships. The reported trace statistics are compared to critical values at the 0.05 level of significance to make decisions. The rejection of the null hypothesis leads to favor the hypothesis of cointegration between three interest rate series. * denotes rejection of the null hypothesis of none, at most one, and at most two cointegration relationships at the 5% level.

Hypothesized number of CE(s)	Eigenvalue	Trace statistics	Critical value at the 5% level	Probability
None*	0.039	55.86	35.19	0.00
At most 1	0.007	12.06	20.26	0.44
At most 2	0.003	3.67	9.16	0.46

Table 5. Cointegration relationship estimation

This table reports the estimation results of the cointegration relationship while defining the French interest rate as the endogenous variable. INTF, INTUK and INTU refer respectively to the French, British and American interest rates. Empirical t-statistics are given in parenthesis. ** and *** indicate significance at the 5% and 1% levels.

Normalized cointegrating coefficients of one cointegrating equation(s)			
INTF	INTUK	INTU	C
1.00	-2.41***	-0.73**	0.12***
	(-6.70)	(-2.81)	(6.67)
Log-likelihood value		22798.18	

Table 6. VECM estimation results

We report the parameter estimates of the three-equation vector error-correction model that we employed to examine the dynamics of the interest rate convergence toward long-run equilibrium. Empirical t-statistics of the estimates are given in parenthesis. ** and *** indicate significance at the 5% and 1% levels.

Error-correction estimation	D(ZF)	D(ZU)	D(ZUK)
CointEq1 ($\times 10^2$)	0.03	-0.30	0.10
	(0.29)	(-2.42)	(0.69)
D(ZF(-1))	-0.23	-0.43	0.75
	(-1.03)	(-1.40)	(3.21)***
D(ZF(-2))	0.76	1.26	-0.68***
	(3.35)	(4.05)	(-2.90)
D(ZU(-1))	-0.01	0.47	0.02
	(-0.29)	(7.04)	(0.40)
D(ZU(-2))	-0.06	-0.04	0.08
	(-1.30)	(-0.63)	(1.58)
D(ZUK(-1))	-0.37	-0.10	0.89***
	(-1.59)	(-0.30)	(3.69)
D(ZUK(-2))	0.65	1.16	-0.55
	(2.76)	(3.60)	(-2.24)
C	1.6E-05	1.6E-05	-1.7E-05
	(1.60)	(1.12)	(-1.59)
Adj. R-squared	0.03	0.11	0.04
Log likelihood	7190.81	6839.36	7138.58

Table 7. Descriptive statistics for interest rate deviations

This table presents summarized statistics for the interest rate deviations from the long-run equilibrium. They are negative on average for all three countries, indicating the intensity of interest rate decreases within the financial crisis.

	ZUK	*ZU*	*ZF*
Mean ($\times 10^{18}$)	-5.25	-1.70	-0.93
Std. dev. ($\times 10$)	0.05	0.10	0.07
Skewness	-0.43	-0.24	-0.76
Kurtosis	3.45	1.70	2.86
Jarque-Bera	43.49	87.60	107.80
Probability	0.00	0.00	0.00

Table 8. LM linearity test

This table reports the results of the Lagrange multiplier test for normality. p is the number of lags in the change of deviation. $\hat{d}$ refers to the delay number defining the transition variable z_{t-d}.

Delay	**US**	**France**	**UK**
p	3	4	4
$\hat{d}$	4	1	2
p-value	(0.00)	(0.00)	(0.00)
Teräsvirta's test conclusion	ESTECM	ESTECM	ESTECM

Table 9. ESTECM estimation results

This table reports the parameter estimates of the ESTECM that allows for nonlinearity and regime shifts. The values in parenthesis are the t-statistics. Q(12) is the Ljung-Box statistics. DW, ADF and ARCH are the empirical statistics of the Durbin Watson, ADF and ARCH tests. $\overline{R}^2$ denotes the determination coefficient of the regression model. * and ** indicate significance at the 10% and 5% levels respectively.

Variables	**US**	**France**	**UK**
p	3	4	4
$\hat{d}$	4	1	2
$\hat{\gamma}$	11.42**	13.28**	100.47*
	(3.01)	(2.22)	(1.79)
c ($\times 10^2$)	-0.04*	0.11**	0.08**
	(-1.94)	(1.73)	(10.55)
$\hat{\phi}_0$ ($\times 10^6$)	-7.60	-5.20**	2.00
	(-1.06)	(-2.50)	(-1.16)
$\hat{\lambda}_1$	-0.04	0.007**	0.03**
	(-3.36)	(2.44)	(3.65)
$\hat{\lambda}_2$	-0.05	-0.01**	-0.04**
	(-3.55)	(-2.69)	(-3.80)
$\hat{\phi}^{UK}{}_{1,0}$	-2.06**	-0.99**	-
	(-16.20)	(-11.70)	
$\hat{\phi}^{UK}{}_{1,1}$	1.34**	0.52**	0.52**
	(8.60)	(16.80)	(17.20)
$\hat{\phi}^{UK}{}_{1,2}$	0.61**	0.07**	0.09**
	(4.20)	(2.20)	(2.53)
$\hat{\phi}^{UK}{}_{1,3}$	0.12**	0.06**	0.11**
	(3.30)	(2.10)	(3.44)
$\hat{\phi}^{UK}{}_{1,4}$	-	0.09**	0.09**
		(3.30)	(3.30)
$\hat{\phi}^{USA}{}_{2,0}$	-	-0.05**	-0.09**
		(-7.03)	(-16.30)
$\hat{\phi}^{USA}{}_{2,1}$	0.52**	0.03**	0.05**
	(17.10)	(5.10)	(6.84)
$\hat{\phi}^{USA}{}_{2,2}$	0.04	-	0.006
	(1.13)		(0.87)
$\hat{\phi}^{USA}{}_{2,3}$	0.12**	-	0.02**
	(4.60)		(3.30)
$\hat{\phi}^{F}{}_{3,0}$	-0.94**	-	-0.93**
	(-7.03)		(-117.20)
$\hat{\phi}^{F}{}_{3,1}$	0.83**	0.49	0.51**
	(5.60)	(16.3)	(16.4)
$\hat{\phi}^{F}{}_{3,2}$	0.61**	0.07**	0.08**
	(4.32)	(2.20)	(2.51)
$\hat{\phi}^{F}{}_{3,3}$	-	0.07**	0.09**
		(2.04)	(2.81)
$\hat{\phi}^{F}{}_{3,4}$	-	0.01	0.09**
		(3.31)	(3.22)
$\overline{R}^2$	0.82	0.96	0.92
DW	2.04	2.02	1.99
ADF	-17.66	-15.25	-15.20
Q(12) p-values	0.00	0.00	0.00
ARCH (q)	62.60**	49.30**	79.20**
	(q =2)	(q =1)	(q =1)
Number of iterations	9	32	20

Figure 1. Time dynamics of daily short-term interest rate for France (INTF1), the US (INTU1) and the UK (INTUK1)

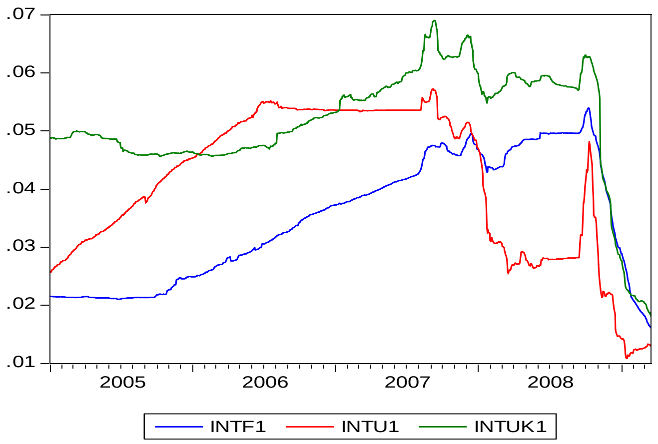
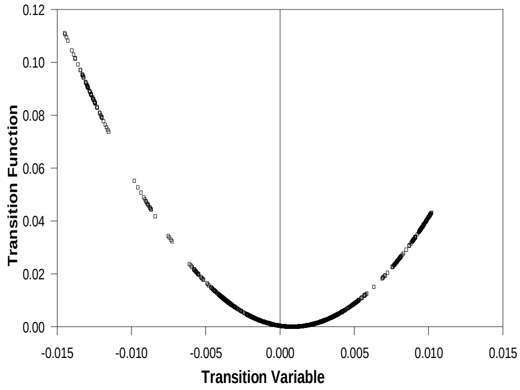
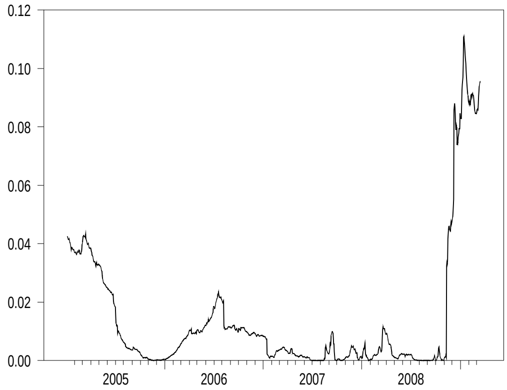


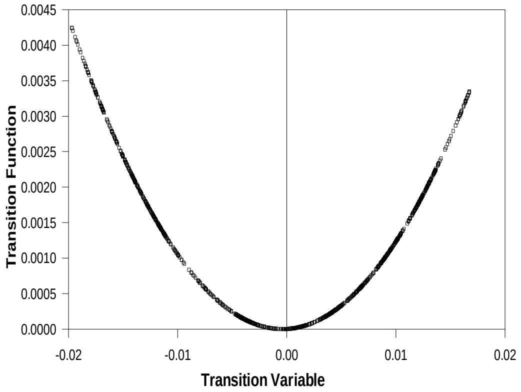
Figure 2. Estimated transition functions of the ESTECMs for the UK, US and French interest rates



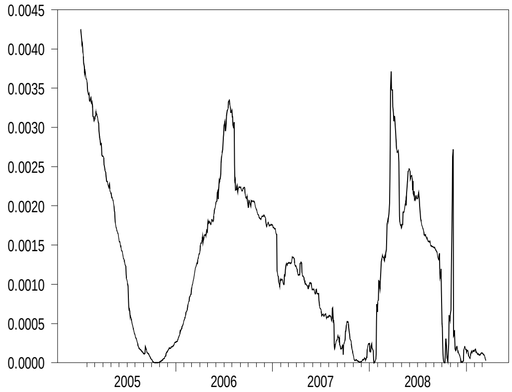
a) Transition function for the UK



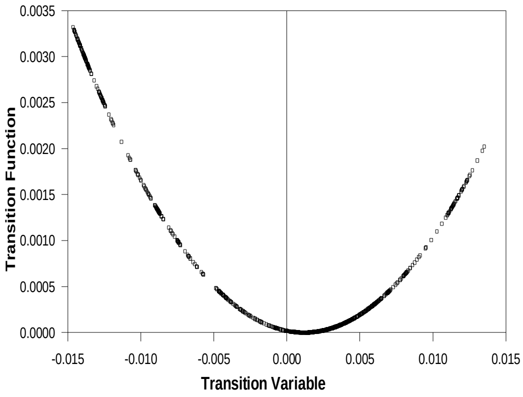
b) Intertemporal transition function for the UK



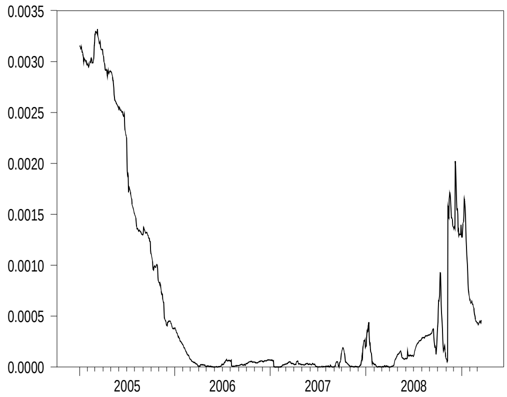
c) Transition function for the US



d) Intertemporal transition function for the US

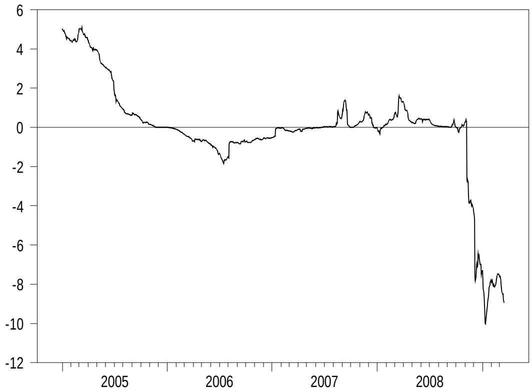


e) Transition function for France

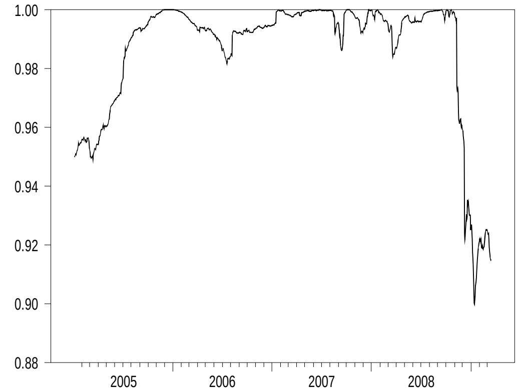


f) Intertemporal transition function for France

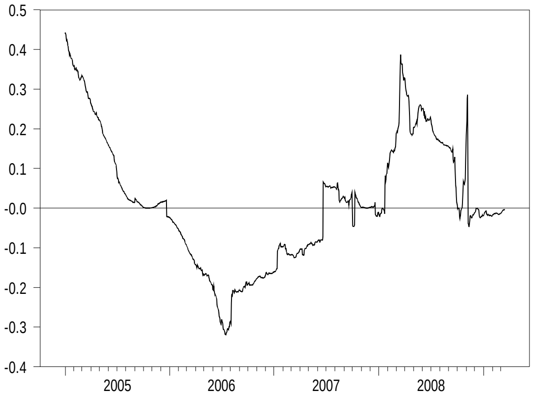
Figure 3. Undervaluation, overvaluation and mean reversion in the interest rate adjustment dynamics for the UK, US and French interest rates



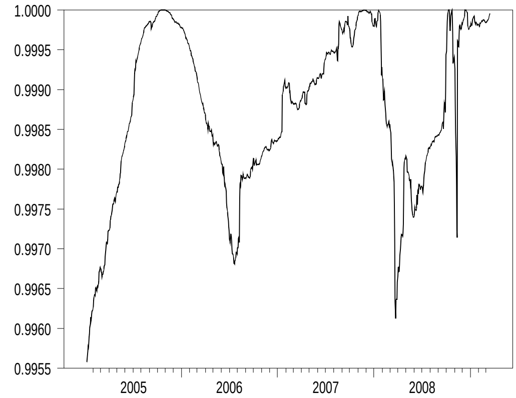
a) Under- and overvaluation phases for the UK



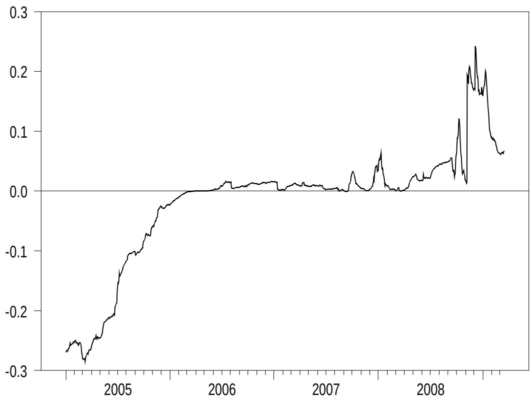
b) Mean reversion indicator for the UK



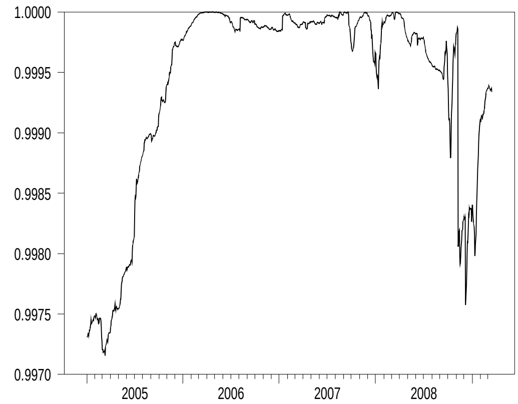
c) Under- and overvaluation phases for the US



d) Mean reversion indicator for the US



e) Under- and overvaluation phases for France



f) Mean reversion indicator for France