Sovereign Default and Private Government Information

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Abstract

This paper develops a model of sovereign debt in which the government has some private information about the domestic economy. Sovereign borrowing is sustainable without enforcement, as repayment is a costly signal about the hidden economic fundamental. The signaling mechanism is able to jointly explain three empirical observations: (1) sovereign defaults usually happen in bad macroeconomic conditions, (2) foreign credit to the private sector contracts after the government defaults and expands after subsequent improvements in sovereign ratings, and (3) the current accounts and interest rates are usually counter-cyclical, especially in emerging markets. Furthermore the model reveals a novel positive externality of precautionary savings on the sustainability of sovereign debt. It also predicts that domestic agents, if left on their own, will under-insure against the risk of sovereign defaults. A numerical exercise shows that the information channel can explain a nontrivial level of sovereign borrowing. Overall, the paper argues that private government information plays an important role in understanding a country's ability to borrow from abroad.

Keywords: Sovereign debt, private external debt, asymmetric information, precautionary savings

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1 Introduction

There are two salient features of recent episodes of sovereign debt crises: (i) some governments manage to hide certain information about their economies from the market, and (ii) there is a close link between sovereign risk (i.e. government default risk) and credit to the private sector. This paper connects these two phenomena in a small open economy model. Its contribution is a framework through which three results that are consistent with empirical observations naturally emerge:

1. Sovereign governments repay debt in good times (when output is high) and default in bad times (when output is relatively low);

2. Foreign credit to the private sector is strongly correlated to ratings of sovereign government debt. In particular, foreign loans to the private sector dries up after a sovereign default, and flows again after improvements in the sovereign ratings.

3. The current accounts and interest rates are usually counter-cyclical, especially in emerging markets.

Furthermore, the model reveals a novel complementarity between buffer-stock savings and sovereign borrowing. Domestic agents do not internalize this positive externality on sovereign borrowing, and thus they under-insure against the bad state of sovereign defaults and drops in output. This inefficiency gives a rationale for governments to play an active role in accumulating foreign reserves. Finally, to evaluate whether the informational channel is quantitatively significant, the paper develops a simple numerical exercise in the infinite horizon. It shows that even if credit to the private sector is a small fraction of GDP, the informational channel can generate a post-default private credit contraction sufficient enough to sustain a significant level of sovereign borrowing.

There is widespread evidence that governments can hide certain information about the state of the domestic economy from the market. For instance, when the European debt crisis turned full-fledged in November 2009, the new Greek government revealed that the fiscal deficit was twice as large as previously believed. Governments' ability to hide poor macroeconomic performances has become a grave concern in Europe, and in other parts of the world. Reinhart and Rogoff (2010) provides a plethora of examples of governments' hidden debts and liabilities, ranging from the United States government's guarantee of Fannie Mae and Freddie Mac in the 2008-9 mortgage crisis, to Thailand's

hidden massive forward exchange market interventions in the 1997-8 Asian crisis.[1] The public seemed "blissfully unaware" of these bad information until solvency issues are exposed on the eve of a crisis. The time seems ripe the sovereign debt literature to pay new and critical attention to private government information and its implication(s).

On the other hand, the string of sovereign defaults in emerging markets (Russia, Ukraine, Pakistan, Ecuador, Argentina, Brazil, etc.) in the late 90s and early 2000s shares a common feature: sovereign defaults were often associated with strong contractions in foreign credit to the private sector, even after controlling for macroeconomic conditions, and the decline can persist for more than two years (see Sturzenegger and Zettelmeyer (2006); Arteta and Hale (2008); Trebesch (2009)). Traditional sovereign debt models are unfortunately not well-suited to explain this phenomenon. This is simply because they do not make a clear distinction between sovereign government debt and the private sector's debt. Our model clearly separates the two, and establishes an informational channel through which a sovereign default *causes* a drop in foreign credit to the private sector. A sovereign default thus causes an endogenous drop in domestic output (while the tradition of the literature is to treat this drop as exogenous[2]). On the opposite side of the story, the credible threat of an output drop deters a government from defaulting, and in fact a significant level of sovereign debt could be sustained alone by the informational channel.

To capture the observed asymmetry in information, the paper first develops a three period model where, in the middle period, the government has knowledge of a fundamental variable (say, the fiscal deficit) that the market only learns in the last period. This fundamental affects the productivity of domestic firms in the last period (for instance, a large deficit means that the government will have to raise corporate or capital taxes). There is an equilibrium in which the government can issue a positive amount of sovereign bond in the initial period, and the government fully reveals the fundamental in the middle period by its repayment decision. It defaults when the fundamental is bad, and repays when the fundamental is good. The debt payment is just sufficient enough to distinguish the two states of the private fundamental. Foreign creditors thus make smaller loans to the domestic private sector after a sovereign default, as they expect lower profitability. Interestingly, after imposing an intuitive criterion on what

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[1]In fact, Burnside et al. (2001) argue that hidden government guarantee of financial intermediaries is at the root of 1997-1998 Asian crisis.

[2]A majority of the sovereign debt literature follows Eaton and Gersovitz (1981), and assumes an exogenous fraction of output is lost by defaulting. See Arellano (2008); D'Erasmo (2008); Yue (2010); Alfaro and Kanczuk (2009) and the references therein.

off-equilibrium-path beliefs are reasonable, this equilibrium is unique.

There are three extensions to the model. In one direction, we introduce an exogenous shock to the GDP in the initial period. If this shock is positively correlated to the fundamental in the middle period, then the interest rate on sovereign debt is higher following a bad GDP shock, and lower following a good GDP shock. Thus for economies that are driven by persistent shocks), the sovereign risk premium is counter-cyclical. This is consistent with two empirical observations: emerging market business cycles experience persistent shocks (Aguiar and Gopinath (2004)), and the current account and interest rates in emerging markets are counter-cyclical (Aguiar and Gopinath (2004, 2006); Neumeyer and Perri (2005) and Uribe and Yue (2006)).

In another direction, we let domestic agents accumulate of foreign assets as a buffer stock against the bad fundamental shock in the middle period. If agents' utility exhibits diminishing marginal utility, then having a buffer stock of savings reduces the utility cost of repaying debt. Thus this buffer stock increases the sustainability of sovereign debt. This complementarity between savings and debt is a novel feature, and stands in contrast with the usual substitution effect. Furthermore, this complementarity is a positive externality of savings that individual agents do not internalize. Hence the decentralized economy will under-insure against future sovereign defaults. The model therefore provides a rationale for a government to compensate for this inefficiency by accumulating an appropriate level of foreign reserves.

Finally, we extend the three period model into the infinite horizon for a quantitative exercise. It shows that even if the size of foreign private credit is small (6% of GDP), the signaling mechanism is sufficient to generate a non-trivial ratio of sovereign debt (60% of GDP).

Related Literature

A number of recent papers have explored the impact of sovereign defaults on the domestic private sector (Guembel and Sussman (2009); Gennaioli et al. (2010); Broner and Ventura (2011)). A common feature of these models is that the government cannot discriminate between foreign and domestic debt holders when it defaults. Thus a sovereign default negatively affects the balance sheet of domestic banks holding government bonds, which in turn can lead to a domestic credit crunch and output drop. Our is different: domestic agents do not hold sovereign government bonds and there is no balance sheet effect.

Sandleris (2008, 2010) and Catão et al. (2009) have a similar signaling mechanism to ours to sustain sovereign repayment. In particular, Sandleris (2010) also studies the link between sovereign debt and private sector debt, but the private debt in his model is between domestic creditors and domestic borrowers, while the private debt in our model is external. All of these models are limited to the three period horizon, and thus are not fit for quantitative analyses, while we extend the basic model to the infinite horizon and a numerical example. Theoretically their models predict (infinitely) multiple equilibria, while we refine the perfect Bayesian equilibrium concept with Cho and Kreps (1987)'s intuitive criterion and produce a unique equilibrium. Last but not least, their models do not predict counter-cyclicality, complementarity between precautionary savings and sovereign borrowing, nor under-insurance.

To extend the basic three period model to the infinite horizon, we use insights from the earlier signaling model of Cole et al. (1995). They also refine equilibrium beliefs using Cho and Kreps (1987)'s intuitive criterion. However, they do not model public borrowing and private borrowing separately, while the emphasis of our paper is precisely this separation.

On explaining the counter-cyclical capital flows to emerging markets and developing countries, the paper is related to the private information game theory model of Atkeson (1991). However his mechanism is based on moral hazard (the government commits an investment action that is not observed by foreign lenders), while our model is about signaling.

In addition, our paper is related to a branch of international finance literature on under-insurance in countries that less developed financial institutions. These papers show that domestic agents insufficiently hedge against exchange rate risks when they face either financial constraints (Caballero and Krishnamurthy (2003)), or are endowed with implicit government guarantees (Burnside et al. (2001)). There is no sovereign government debt in their models. Thus our message is very different: domestic agents under-insure against sovereign default.

Finally, our paper is linked to the vast literature on sovereign debt. Besides the papers cited above, classic models include Eaton and Gersovitz (1981); Grossman and Van Huyck (1988); Bulow and Rogoff (1989); Atkeson (1991); Cole et al. (1995), and more recent ones include Kletzer and Wright (2000); Kehoe and Perri (2002); Arellano (2008); Aguiar et al. (2009); Hellwig and Lorenzoni (2009); Yue (2010); Phan (2012). However these papers do not consider information asymmetry (with the exception of Phan (2012)) and they do not distinguish private debt from public debt.

Plan of paper

Section 2 lays out the environment of the three period model. It then establishes the informational channel that sustains sovereign repayment, shows how private external debt contracts after the government defaults, and provides a comparative statics exercise. Section 3 features the first extension, which shows that sovereign debt is counter-cyclical in equilibrium. Section 4 contains the second extension, showing how agents under-insure against sovereign default and the associated drop in output. Section 5 is the third extension: infinite horizon and a numerical example. The last section discusses limitations of the model and concludes.

2 Three period model

This section lays out a finite horizon small open economy model of international lending with asymmetric information. The model serves several purposes: first, it proves the sustainability of sovereign repayment as a costly signal of the private fundamental. Second, it establishes a connection between sovereign debt and private external debt via an informational channel. Finally, it serves as the basis for three extensions in subsequent sections of the paper.

2.1 Environment

Consider a three period ($t = 0, 1, 2$) economy with a good for consumption and a good for investment. There is a big player: a domestic government. And there are three types of small agents with a continuous population of measure one for each type: domestic households, domestic firms, and foreign creditors. Each household, firm, and foreign investor are assumed to be so small that they take aggregate allocations and prices as given.

Domestic households' preferences are given by

$$\tilde{U}(c_0, c_1, c_2) = \tilde{u}(c_0) + \beta\tilde{u}(c_1) + \beta^2\tilde{u}(c_2)$$

where c_t is the people's consumption in period t, and $\beta \in (0, 1]$ is the discount factor. The utility functions $\tilde{u}$ is strictly increasing, weakly concave and twice differentiable.

Domestic firms are competitive, and are owned by households. Their production will be discussed in the next subsection.

The government is benevolent in the sense that it aims to maximizes the utility of domestic households. It borrows from foreign creditors on behalf of the households to help them smooth their consumption. It also makes repayment/default decisions.

Foreign creditors are risk neutral, and competitive. They can trade a safe asset whose risk-free rate of return is $1 + r_f$, which is a given constant throughout the paper. Thus their preferences are given by their total expected utility:

$$U^f(c_0^f, c_1^f, c_2^f) = c_0^f + \frac{c_1^f}{1 + r_f} + \frac{c_2^f}{(1 + r_f)^2}$$

where c_t^f is their consumption in period t. As usual in the literature, foreign creditors are assumed to have "deep pockets" in the sense that they are endowed with sufficient

funds in each period to satisfy the borrowing needs of the domestic firms and the domestic government.

Production

The economy has two sources of income. The first is a production technology that turns imported investment good into consumption good. This production, as discussed below, requires working capital loans from foreign creditors. The second is a constant endowment of ω units of consumption good in each period. This source of income represents the sector of the economy that does not require trade credit, whose inner working the model abstracts away for simplicity.

Firms have access to the production technology mentioned above in periods $t = 0$ and $t = 1$. The production requires purchases of an imported investment good from a perfectly competitive world market. We normalize the price of a unit of the investment good at one unit of the consumption good. Production takes one period: investing k_t units of the investment good in period t results in $A_{t+1}f(k_t)$ units of the consumption good in period $t + 1$. The productivity shocks A_1, A_2 will be discussed shortly. The production function f is strictly increasing, strictly concave, twice differentiable and satisfies $\lim_{k \to 0+} f'(k) = \infty$. Firms maximize their profits and transfer all profits to households at the end of each period $t = 1$ and $t = 2$.

Following open economy models of Mendoza and Yue (2011); Uribe and Yue (2006); Neumeyer and Perri (2005), we assume firms' production require *trade credit:*

Assumption 1. *Firms must pay for the imports of the investment good with credit from foreign creditors.*

This assumption is important for our model. In particular, it implies that the country cannot default on sovereign debt, and divert the funds for repayment to self-finance its imports of the investment good. This is reasonable as in practice the government usually does not have the same expertise or efficiency as the financial intermediaries at providing working capital loans. On the other hand, the ability to self-finance investments would render sovereign debt theoretically impossible (see Bulow and Rogoff (1989)).[3] Note that the assumption should not be interpreted as all production inputs require foreign credit. Instead, k_t should be interpreted as the fraction of inputs that

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[3]Phan (2012) addresses the Bulow and Rogoff's puzzle (what sustains sovereign repayment if countries can self-finance all investments) in an infinite horizon model of asymmetric information.

need working capital loans from foreigners, and all other types of inputs (say labor and land) are supplied inelastically.

Information

The model's uncertainty lies in the productivity shocks. Recall that if a firm invests k_t in period t, it would get $A_{t+1}f(k_t)$ in period $t+1$. The productivity shock $A_{t+1} = A_{t+1}(\theta_t)$ is a function of the government's "private fundamental" θ_t in period t. The fundamental can take two possible values: good or bad, denoted by $\bar{\theta}$ and $\underline{\theta}$. The productivity shock in the good state $\bar{A} \equiv A(\bar{\theta})$ is higher than that in the bad state $\underline{A} \equiv A(\underline{\theta})$.

For simplicity, assume that the initial fundamental θ_0 is $\bar{\theta}$ and is common knowledge. That means everybody knows the economy begins with a good fundamental, and $A_1 = \bar{A}$. Thus the only source of uncertainty in this three period model is θ_1, the private fundamental in $t = 1$.

The following assumption lies at the heart of the model. It captures the asymmetry in the information regarding private fundamental θ_1:

Assumption 2. *The government learns the realization of private fundamental θ_1 at $t = 1$, but everybody else only learns the realization at $t = 2$ (through looking at the realization of productivity shock A_2).*

This assumption reflects the fact that governments usually has some private information about its own economy that the public does not know immediately (see the evidence mentioned in the introduction). For instance, a government can have have private information about its current fiscal deficit, which will affect its future tax policy, and thus will affect the productivity of domestic firms in the future.

For convenience, we sometimes call a government with fundamental $\underline{\theta}$ the "bad type", and a government with fundamental $\bar{\theta}$ the "good type".

We have so far laid out the physical environment of the model. Next we describe the contractual environment.

Private external debt contracts

In periods $t \in \{0, 1\}$, each firm enters a debt contract with a foreign investor. The contract specifies a loan of $k_t \geq 0$ from the foreign investor to the firm in period t, and a repayment $(1 + r_{t+1})k_t \geq 0$ from the household to the foreign investor in period $t+1$. Since A_{t+1} is observable in $t + 1$, we assume for simplicity that the interest rate r_{t+1}

can be contingent on A_{t+1}.[4] We make the following institutional assumption on private debt:

Assumption 3. *At most a fraction $\phi \in (0, 1]$ of each firm's output can be pledged as collateral for a private external loan.*

In other words, if payment in $t + 1$ is strictly less than the contracted amount of $(1 + r_{t+1})k_t$, at most fraction ϕ of the firm's output will be seized and transferred to the foreign investor. This type of constraint has been extensively used in international finance, particularly in the "sudden stop" literature (see, for instance, Mendoza (2002); Jeanne and Rancière (2006); Rancière et al. (2008)). Parameter ϕ can be interpreted as a measure of the quality of the country's financial institutions.

Sovereign debt contracts

Households would like to smooth their consumption via foreign borrowing. Following Mendoza and Yue (2011); Uribe and Yue (2006); Neumeyer and Perri (2005), we however assume that households cannot directly borrow from foreign creditors. Instead, the government acquires sovereign loans[5] from foreign creditors, then transfers to households in lump-sums. To repay the loans in the next period, the government can raise lump-sum taxes.

Lump-sum taxation allows us to abstract from the well-understood distortionary effect of capital income taxation and thus focus on optimal debt management. Indeed, we can replace lump-sum tax with distortionary tax and achieve the same qualitative results, but with unnecessary added complications.

Unlike private debt, government's debt contracts are sovereign in the sense that they are not enforceable by any international court. Following the literature on implicit debt contracts (see Kocherlakota (1996) and the references there-in), we model sovereign borrowing through a game. In period $t = 0$ foreign creditors lend G units of the investment good to the government. In period $t = 1$ the government decides an amount P of the consumption good to repay the foreign creditors. A puzzle is why the government will repay anything if there is no enforcement. This is the famous sovereign

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[4]Thus the debt contract is more similar to an equity contract than a standard (non-contingent) debt contract. The qualitative results of the paper still holds if contracts are non-contingent,.

[5]Throughout this paper, the terms sovereign debt and public external debt will be used interchangeably.

debt puzzle (see the introduction).[6]

Finally, this section makes the following simplifying assumption

Assumption 4 (Temporary). *The country does not save.*

We will allow the country to save in section 4.

Timing of events

1. $t = 0$:

   (a) Foreign creditors grants a sovereign loan B to the government. The government transfers B to households in lump-sums.

   (b) Foreign creditors make private loans k_0 to firms at rate $1 + r_1$. Firms then purchase k_0 units of investment good as input for production.

   (c) Households consume
   $$c_0 = \omega + B.$$

2. $t = 1$:

   (a) Private production yields $A_1 f(k_0)$ (recall $A_1 = \overline{A}$ with certainty).

   (b) Firms chooses how much to repay. Their aggregate repayment amount is denoted by p_1. Firms transfer profits to households.

   (c) Government privately learns the realization of fundamental θ_1.

   (d) Government chooses how much to repay. To repay, it taxes households in lump-sums. Government repayment is denoted by P.

   (e) Foreign creditors make private loans k_1 to firms at rate $1 + r_2$. Firms then purchase k_1 units of investment good from the world market as input for production.

   (f) Households consume

   $$c_1 = \omega + A_1 f(k_0) - p_1 - P.$$

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[6]Note that it is without loss of generality that we do not model sovereign loan between $t = 1$ and $t = 2$. The government would never have an incentive to repay at $t = 2$ when the game ends. Hence rational creditors would not lend to the government at $t = 1$.

3. $t = 2$:

   (a) Private production yields $A_2 f(k_1)$. (Recall $A_2 = A(\theta_1)$.)

   (b) Firms chooses how much to repay. Their aggregate repayment amount is denoted p_2. Firms transfer profits to households.

   (c) Households consume:
   $$c_2 = \omega + A_2 f(k_1) - p_2.$$

   The game ends.

Histories and strategies

A history h_1^g at $t = 1$ for the government consists of sovereign borrowing B, the aggregate distribution of private debt contracts k_0, r_1, and the realization of the fundamental θ_1. For the market (foreign creditors and firms), a history h_0^m at $t = 0$ consists of B, k_0, r_1. A market history h_1^m at $t = 1$ consists of h_0^m, aggregate private repayment p_1,[7] and government repayment P.

A strategy profile consists of: a sovereign loan $B \geq 0$ from each investor to the government, a private loan contract k_0, r_1 from each investor to a household, private repayment p_1 as a function of h_0^m, and government repayment P as a function of h_0^g, a private loan contract k_1, r_2 from each investor to a household as a function of market history h_1^m, and private repayment p_2 as a function of k_1, r_2 and h_1^m. We only consider symmetric strategies, i.e. strategies in which two different foreign investor plays the same strategy.

A belief system $\mu(h_1^m)$ for the a foreign investor specifies a probability distribution to each market history h_1^m. In particular, the foreign investor believes that the fundamental θ_1 is $\bar{\theta}$ with probability μ:
$$\Pr(\theta_1 = \bar{\theta}|h_1^m) = \mu(h_1^m).$$

A belief system is symmetric if it specifies the same probabilities for the same histories to any investor. We only consider symmetric belief systems. We sometimes call μ the "country rating", as the higher μ is the more likely foreign creditors presume that the borrowing country is in the good fundamental state $\bar{\theta}$.

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[7]Note that the histories only include the *aggregate* distribution of actions. Hence the action of each individual household, firm or foreign creditor is not recorded in history. These small players are thus "anonymous". See Mailath and Samuelson (2006) or Chari and Kehoe (1990); Chari et al. (1998) for discussions of anonymous players.

Equilibrium and refinement

Definition 1. A strategy-belief profile $\{B, k_0, r_1, p_1(h_0^m), P(h_1^g), k_1(h_1^m), r_2(h_1^m, A_2), \mu(h_1^m)\}$ constitutes a *perfect Bayesian equilibrium* if:

1. It maximizes the benevolent government's expected utility, taking as given foreigner's strategy, and

2. It maximizes each foreign investor's expected profits, taking as given the government's strategy, and

3. The belief system is consistent with Bayesian updating at any history that occurs with positive probability given the strategy profile.

From now on, we will simply refer to a perfect Bayesian equilibrium as an equilibrium.

Usually there are (uncountably) many equilibria in a signaling model, and such multiplicity weakens any policy implication. Thus for our analysis to have a coherent message, we will impose a restriction on what kinds of beliefs can be used when Bayesian updating is unfeasible. We adopt the *stability* refinement of Cho and Kreps (1987), which is usually known as the "intuitive criterion", and is common in signaling models (see Cole et al. (1995) for an early application in the context of sovereign debt).

In our environment, the refinement means the following:

Definition 2. An equilibrium fails is said to fail the *intuitive criterion* if there exists a deviation repayment P' for the government such that

1. The belief system specifies a positive probability to the event $\theta_1 = \underline{\theta}$ if the deviation repayment P' has been made, and

2. The $\underline{\theta}$ type *strictly* prefers the equilibrium strategy than to deviate to P', under *any* specification of off-equilibrium path beliefs, and

3. The $\overline{\theta}$ type (weakly) prefers to deviate to P' than to play the equilibrium strategy, under at least one system of beliefs.

Intuitively, this refinement rules out as implausible any equilibrium that requires a foreign lender to believe that an off-equilibrium-path move (to repay P' instead of P) by a borrowing government might have been made by a type (the $\underline{\theta}$ type) that could not possibly benefit by the move, given the equilibrium of the subgame following the deviation to P'.

2.2 Private external debt

We first solve for the equilibrium levels of private external debt. Since firms have no reputation concern (recall from footnote 7 that history does not record an individual firm's action), it is immediate that a firm will never repay more than the collateral value of $\phi A_{t+1} f(k_t)$ in any state of the world. Anticipating this, it is optimal for profit maximizing foreign creditors to set the private debt contracts such that in each state, the contracted repayment is equal to the collateral value. In other words, equilibrium private debt contract has the property:[8]

$$(1 + r_{t+1}(A_{t+1}))k_t = \phi A_{t+1} f(k_t). \tag{2.1}$$

The left hand side is the contracted repayment amount, and the right hand side is the collateral value. Given each loan size k_t, this equation pins down the equilibrium private interest rate $r_{t+1}(A_{t+1})$.

Now for each belief $\mu \in [0,1]$, let $k^*(\mu)$ be the size of a loan such that foreign creditors' expected profit from the loan is zero. The following equation pins down $k^*(\mu)$:

$$(1 + r_f)k^*(\mu) = \phi E_\mu[A]f(k^*(\mu)), \tag{2.2}$$

where

$$E_\mu[A] = \mu\overline{A} + (1 - \mu)\underline{A}. \tag{2.3}$$

The fact that $\lim_{k\to 0+} f'(k) = \infty$ guarantees that $k^*(\mu)$, for each μ, is unique.

Finally, we assume that the collateral constraint ϕ is sufficiently tight that the private sector is borrowing-constrained in equilibrium:

Assumption 5. *[Technical]*

$$E_\mu[A]f'(k^*(\mu)) > \frac{1 + r_f}{1 - \phi}, \ \forall \mu \in [0,1].$$

We conclude this subsection with a basic result:

Lemma 1. *In any equilibrium, the equilibrium level of private loan at $t = 0$ is*

$$k_0^* = k^*(1)$$

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[8]Recall that r_{t+1} can be contingent on the realization of A_{t+1}.

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and at $t = 1$, *given country rating* μ, *equilibrium level of private loan is*

$$k_1^* = k^*(\mu)$$

that solves (2.2). Foreign credit to the private sector $k^*(\mu)$ *is increasing in country rating* μ.

Proof. Appendix. □

Corollary. *Let* $\overline{k} \equiv k^*(1)$ *and* $\underline{k} \equiv k^*(0)$. *A direct corollary of lemma 1 is that*

$$\overline{k} > \underline{k}. \tag{2.4}$$

Though simple, this inequality is a cornerstone of our model. The government has an incentive to keep a higher rating μ, as it allows for a higher level of private investment.

Single-crossing property

Lemma 1 shows that the country benefits from having a higher country rating, as there will be more private investment. However different types will benefit differently. The following assumption guarantees that the $\overline{\theta}$ type benefits more from a higher country rating than the $\underline{\theta}$ type would. This way, in a separating equilibrium, the $\overline{\theta}$ will repay and the $\underline{\theta}$ type will default, instead of the other way around.

Assumption 6. *[Technical]*

$$\frac{\overline{A}}{\underline{A}} > \frac{\tilde{u}'(\omega + (1-\phi)\underline{A}f(k))}{\tilde{u}'(\omega + (1-\phi)\overline{A}f(k))}, \forall k \in [\underline{k}, \overline{k}]. \tag{2.5}$$

For convenience, define the following functions:

$$
\begin{aligned}
u(c) &\equiv \tilde{u}(\omega + c) \\
g(k) &\equiv (1-\phi)f(k).
\end{aligned}
$$

Then (2.5) can be rewritten in a simpler form:

$$\frac{\overline{A}}{\underline{A}} > \frac{u'(\underline{A}g(k))}{u'(\overline{A}g(k))}, \forall k \in [\underline{k}, \overline{k}].$$
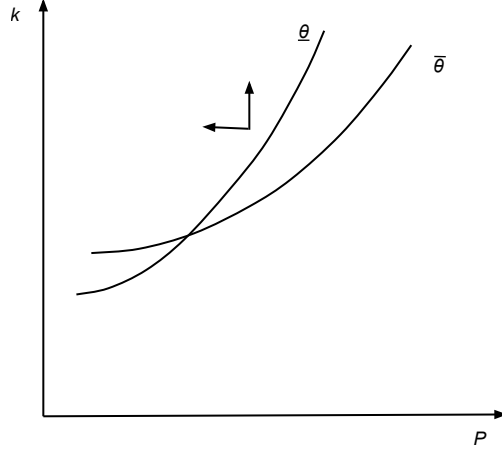
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Figure 2.1: Indifference curves of $U(P, k|\theta) = u(A_1 g(k_0^*) - P) + \beta u(A(\theta) g(k))$.

Let us gain a graphic intuition for this condition. Consider the difference curves of $U(P, k|\theta) \equiv u(A_1 g(k_0^*) - P) + \beta u(A(\theta) g(k))$, for each $\theta \in \{\overline{\theta}, \underline{\theta}\}$. Then under assumption 6, an indifference curve of the $\underline{\theta}$ type would intersect an indifference curve of the $\overline{\theta}$ type from below exactly once (thus giving name to the "single-crossing" property). The situation is depicted in figure 2.1.

Remark. Assumption 6 automatically holds, for example, if the utility function is linear. When the utility function is linear, the high fundamental type $\overline{\theta}$ always benefits more from higher investment as its production technology is more productive $(\overline{A} > \underline{A})$. When $\tilde{u}$ is concave, however, there is a second-order wealth effect: there is more output when the fundamental is good , and thus the welfare gain of having additional output is smaller. This wealth effect goes in the opposite direction as it makes the $\overline{\theta}$ type benefit less from higher investment. The assumption guarantees that the first-order effect dominates the second-order effect. For example, when $\tilde{u}$ takes the usual constant relative risk aversion (CRRA) functional form $\tilde{u}(c) = c^{1-\sigma}/(1-\sigma)$, where ω represents household's consumption from other sources of income not modeled here (for instance, households receive exogenous endowment ω in each period), inequality (2.5) reduces to:

$$\frac{\overline{A}}{\underline{A}} > \left(\frac{\omega + (1 - \phi)\overline{A} f(k)}{\omega + (1 - \phi)\underline{A} f(k)} \right)^{\sigma}$$

or equivalently, the risk aversion index is not too high, relative to other parameters:

$$\sigma < \frac{\log(\omega + (1 - \phi)\overline{A} f(k)) - \log(\omega + (1 - \phi)\underline{A} f(k))}{\log(\overline{A}) - \log(\underline{A})}.$$

2.3 Sovereign debt

This section shows that sovereign debt is sustainable in this environment without the need of enforcement or reputation. The reason is that a sovereign default will negatively affect foreign private investment, as a default signals a bad fundamental state. This provides the benevolent government an incentive to repay its debt. In equilibrium, the $\overline{\theta}$ type repays a positive amount P^*, while the $\underline{\theta}$ repays nothing. Foreign creditors will interpret that a government that repays P^* or more has a good fundamental state, and one that repays less than P^* has a bad fundamental state. Repayment is thus a costly signal of the hidden fundamental state. For this system of belief to be consistent, the repayment amount P^* must be sufficiently high so that the bad type finds it too costly to repay, yet has to be sufficiently low so that the good type is willing to pay it.

Proposition 1 (Equilibrium). *The following constitutes an equilibrium:*

- *Government defaults (repays nothing) in low fundamental state,*

- *Government repays P^* in good fundamental state. Repayment level P^* makes the $\underline{\theta}$ indifference between repaying and defaulting:*

$$
\begin{aligned}
& u(A_1 g(k_0^*) - 0) \quad + \beta u(\underline{A}g(\underline{k})) \\
= \ & u(A_1 g(k_0^*) - P^*) \quad + \beta u(\underline{A}g(\overline{k})).
\end{aligned} \tag{2.6}
$$

- *Foreign investor's belief system presumes that a government that repays any amount strictly less than P^* has bad fundamental, and thus lends only $\underline{k}$ to the private sector. However a government that repays P^* or more is presumed to have good fundamental, and thus its private sector receives $\overline{k}$ in investment.*

- *Sovereign loan at $t = 0$ is $B^* = \pi P^*/(1 + r_f)$.*

Proof. Appendix. □

The intuition for this theorem is the following. Given foreign investor's system of beliefs, it is suboptimal for the government to pay any amount other than 0 or P^*. It is optimal for the $\underline{\theta}$ type to default, as it does not gain from repaying P^* (equation (2.6) means that the bad type is indifferent between defaulting and receiving lower investment (the first line) and repaying P^* and receiving higher investment (the second line)). The single crossing property implies that the good type then strictly prefer to

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repay P^*. Hence the system of beliefs is consistent with Bayes law.

This equilibrium is fully separating in the sense that different types make different actions, and these actions thus serve as signals that allow the uninformed players (foreign creditors) to clearly distinguish the types. The signal it uses is the least costly, as the signal is just sufficiently high that it makes the $\underline{\theta}$ type indifferent, and any signal lower than this fail to fully separate the two types. Thus we call this the *efficient* separating equilibrium. Any separating equilibrium in which the equilibrium signal is larger than P^* is said to be *inefficient.*

A natural question arises: are there other equilibria? As aforementioned, one common feature in signaling models is the existence of too many equilibria. This weakens the power of the model's prediction or policy implication. Fortunately (and interestingly) the intuitive criterion helps rule out pooling, semi-separating and inefficient separating equilibria.

Proposition 2 (Uniqueness)**.** *The least-cost separating equilibrium is the* unique *equilibrium that satisfies the intuitive criterion.*

Proof. Appendix. □

Intuitively, what explains this uniqueness? First of all, any separating equilibrium that does not use the efficient payment P^* fails the intuitive criterion. Suppose $P^e > P^*$ is the equilibrium payment amount (figure 2.2b). Foreign creditors in this equilibrium associate any government that repays less than P^e with the bad type $\underline{\theta}$. Now consider a deviation to repay $P' \equiv (P^e + P^*)/2$. Since $P' > P^*$, the bad type strictly loses from this deviation, regardless of the system of belief. On the other hand, the good type gains from deviating to P' under the separating belief system described in the theorem statement. The government thus, under the stability refinement, should be able to deviate to P' and still successfully signal its type. Thus any separating equilibrium in which $P^e > P^*$ is unstable.

We can eliminate more equilibria in a similar fashion. In figures 2.2 and 2.3, consider the region of points (P, k) below the $\underline{\theta}$ type's indifference curve that associated with the bad type's equilibrium payoff, but above the $\overline{\theta}$ type's indifference curve associated with the good type's equilibrium payoff. The shaded "wedges" in the figures are representations of this region in different equilibria. Any equilibrium in which this region intersects horizontal line $\{(P, k)|k = \overline{k}\}$ at more than one point fails the intuitive
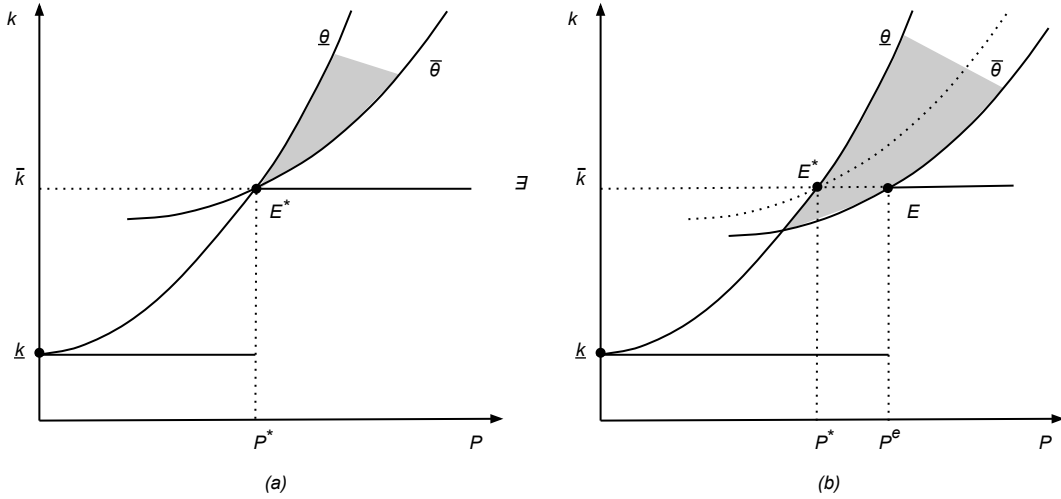
18

Figure 2.2: (a) Efficient separating equilibrium and (b) Non-least cost separating equilibrium

criterion This is because in such an equilibrium, there always exists a deviation (to repay P', the payment amount associated with any point E' in the intersection) that the bad type can never benefit from (the outcome of this deviation always lies below the bad type's equilibrium indifference curve), while the good type strictly gains from this deviation as long as the belief system specifies $\Pr(\overline{\theta}|P') = 1$.

Remark 1. Note that the only time the region intersects the upper half plane at a single point is in the efficient separating equilibrium (figure 2.2a).[9]

Remark 2. If $u(c) = c$ then there is a straightforward closed-form solution for equilibrium payment:

$$P^*(s) = \beta \underline{A}(g(\overline{k}) - g(\underline{k})).$$

The right hand side is equal to the $\underline{\theta}$ type's gain in output (discounted by β) from having level of investment $\overline{k}$ compared instead of $\underline{k}$. The left hand side is the cost of repayment. This is a clear manifestation of the fact that for the bad type, the cost of equilibrium repayment exactly cancels out the gain.

Remark 3. Information asymmetry plays a crucial role in sustaining sovereign debt in this model. If θ_1 were common knowledge at $t = 1$ then sovereign repayment is

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[9]In fact, there is an interesting mathematical observation. If we collect all possible equilibria, and for each equilibrium we draw the associated "wedge", then the intersection of all the wedges is precisely the wedge associated with the efficient equilibrium (figure 2.2a).
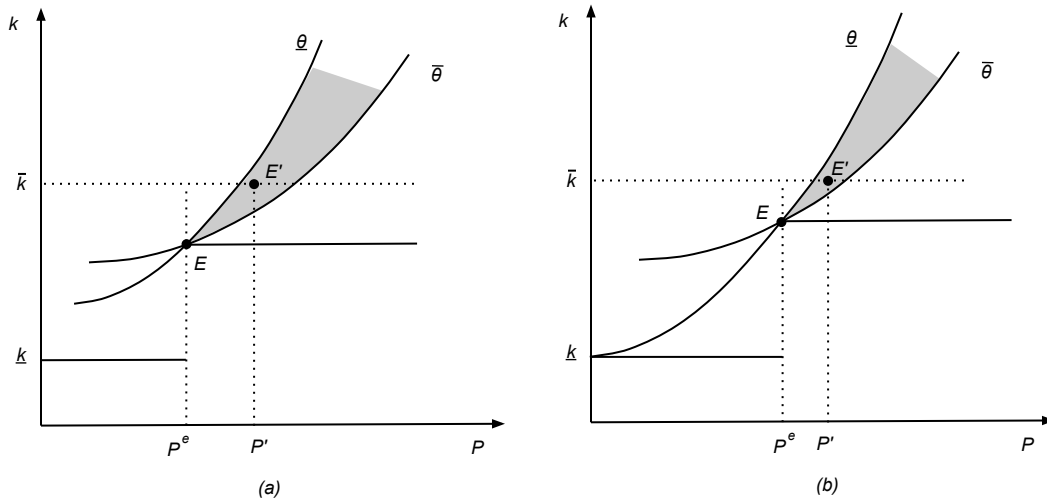
Figure 2.3: (a) Pooling equilibrium and (b) Semi-separating equilibrium

unsustainable in any subgame perfect equilibrium. Repayment no longer serves any signaling purpose.

2.4 Co-movement in public-private debt

2.4.1 Public default and private crunch

As discussed in the introduction, a salient common feature of recent sovereign default episodes is a strong decline in foreign credit to the private sector in the aftermath of sovereign defaults or sovereign debt restructuring, even after controlling for macroeconomic conditions. Furthermore, Borensztein et al. (2007) show that sovereign ratings are a significant determinant of credit ratings assigned to corporations in emerging market economies. Kohlscheen and OConnell (2008) document that the volume of trade credit provided by commercial banks falls sharply after sovereign defaults (the median drops are about 35% after two years and 51% after sovereign defaults).

Our model is consistent with this empirical observation. In particular, a direct corollary of proposition 1 above is that sovereign defaults precede declines in credit to the private sector:

Proposition 3 (Association between sovereign and private debt). *On the equilibrium path, private credit declines (from $\bar{k}$ to $\underline{k}$) after a sovereign default (which is accompanied by a downgrade in country rating from $\mu = 1$ to $\mu = 0$).*

In equilibrium, a sovereign defaults causes a drop in private credit through the channel of information: the former leads foreign creditors to downgrade their beliefs over the country's fundamental, and thus they expect less revenues from their investments to the private sector. This causes them to reduce lending to domestic firms.

2.4.2 The role of institutions

This subsection provides an exercise in comparative statics. Its predictions are consistent with the empirical observations on the effect of institutions external debt. Countries with better institutions have higher "debt tolerance", i.e. they can issue higher levels of sovereign debt *and* private external debt, and they default on sovereign debt less often. Institutions include fiscal institutions that promotes responsible fiscal policies, and legal institutions that protect creditors' rights (see Reinhart et al. (2003)).

Proposition 4 (Comparative statics). *If country a has stronger financial institutions for private external debt then country b, then country a not only can sustain higher levels of private external debt*

$$\phi^a > \phi^b \Rightarrow k^{*a}(\mu) > k^{*b}(\mu), \forall \mu \in [0, 1],$$

but can also sustain higher levels of sovereign debt

$$B^{*a} \; > \; B^{*b}.$$

Also, if country a has a better distribution of the private fundamental ($\pi^a > \pi^b$), then it default less often, and can issue sovereign debt at lower interest rates

$$\frac{1+r^*}{\pi^a} < \frac{1+r^*}{\pi^b}.$$

Proof. Appendix. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Note that the complementarity between public and private debt depends heavily through the informational channel. If θ_1 were common knowledge at $t = 1$ then both countries receive the same level of private investment in each possible state of period $t = 1$ ($k^{*a}(\overline{\theta}) = k^{*b}(\overline{\theta})$ and $k^{*a}(\underline{\theta}) = k^{*b}(\underline{\theta})$), and they trivially receive the same level of sovereign debt ($B^{*a} = B^{*b} = 0$).

3   Counter-cyclical interest rate

This section extends the basic three period model above to explain the counter-cyclicality of the current account and sovereign interest rate that emerging market governments face, as documented by Neumeyer and Perri (2005); Uribe and Yue (2006); Aguiar and Gopinath (2004) and Mendoza and Yue (2011). One salient feature of emerging markets' business cycles is the persistence of their output shocks (Aguiar and Gopinath 2004). The following argument shows that the combination of persistent shocks and private information can help explain the counter-cyclical flow of foreign capital.

Suppose the GDP of the economy in period $t = 0$ is random. That is households endowments ω at the beginning of the game is a strictly increasing function $\omega(\epsilon_0)$ of a random output shock ϵ_0, which has a non-degenerate distribution F over support $\mathcal{E}$.[10] We assume that the realization of ϵ_0 is common knowledge at $t = 0$ (so the only private information is still only the realization of θ_1 at $t = 1$). Second, we assume that a good fundamental shock at $t = 1$ is more likely to follow a good output shock at $t = 0$: If $\epsilon_0^h, \epsilon_0^l \in \mathcal{E}$, and $\epsilon_0^h > \epsilon_0^l$ then

$$\underbrace{\Pr(\theta_1 = \overline{\theta}|\epsilon_0^h)}_{\pi(\epsilon_0^h)} > \underbrace{\Pr(\theta_1 = \overline{\theta}|\epsilon_0^l)}_{\pi(\epsilon_0^l)}. \tag{3.1}$$

We can apply the same arguments for proposition 1 to each realization of ϵ_0. Notice that equilibrium repayment P^* is independent of probability $\pi(\cdot)$ (a common property of separating signaling equilibria), but the interest rate on sovereign loans are inversely related to π:

$$\frac{P^*}{B^*(\epsilon_0)} = \frac{1 + r_f}{\pi(\epsilon_0)}.$$

Hence it is straightforward that if (3.1) holds then the interest rate on sovereign loans as well as the size of the loan (and hence the current account) are lower, following the event $\theta_0 = \overline{\theta}$ then following $\theta_0 = \underline{\theta}$.

Proposition 5 (Counter-cyclicality). *If the $t = 0$ shock are persistent (inequality (3.1)), then the size of sovereign loans and the interest rate on them are counter-cyclical.*

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[10]In a dynamic setting, we can introduce ϵ shocks in periods $t \geq 1$ in the following way: the productivity shock A_{t+1} in each period is a function $A(\theta_t, \epsilon_{t+1})$ of θ_t and ϵ_{t+1}, where ϵ_{t+1}s are independently and identically distributed according to a stationary distribution over $\mathcal{E}$. Unlike θ_ts, there is no information asymmetry regarding ϵ_{t+1}s: all agents can immediately infer ϵ_{t+1} at $t+1$, from the realizations of A_{t+1} and θ_t.

4 Self-insurance

In this paper, sovereign default is not only a risk to foreigners who are lending the government, but also a risk to households, as a default triggers a drop in foreign investment into their firms. From the perspective of an individual household, this is an exogenous risk. How would a rational household insure itself agains this risk? So far in the paper we have excluded households from the asset market for the sake of simplicity. This section relaxes this constraint, and lets households buy the international risk-free assets in period $t = 0$ to build a stock of savings for period $t = 1$.[11] This buffer stock of assets helps cushion the fall in household wealth that follows a sovereign default. The analysis here can be extended to a richer class of assets (whose returns are contingent on government defaults, for instance, credit default swaps on sovereign bonds), but for simplicity, we assume that households only have access to the safe asset whose rate of return is $1 + r_f$.

Two interesting results emerge. First, unlike typical sovereign debt models, sovereign borrowing and precautionary saving (in the form of asset accumulation) are not substitute, but *complementary*. In fact, the buffer stock of savings increases the sustainability of sovereign debt. Second, this increase in the country's debt tolerance is not internalized by individual households. Thus the aggregate stock of savings across households is suboptimal. In other words, this is a new channel through which households *under-insure* against the sovereign default state. Two assumptions are essential for these results: private information, firms' investment requires working capital loans from foreign creditors. The second assumption implies that households use the buffer stocks of safe assets at $t = 1$ for consumption, rather than for financing firms' investments.

Throughout this section, we assume domestic households have strictly concave utility function: $u'' < 0$. For clarity, we do not consider shocks at $t = 0$ (as in section 3).

4.1 Precautionary savings and sovereign risk

The following proposition shows that savings increases the sustainability of sovereign debt. The intuition is that savings creates a buffer stock that raises the income in the bad state ($\theta_1 = \underline{\theta}$). Given concavity in the utility function, this makes it less costly (in

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[11]Whether households can save at $t = 1$ for consumption in $t = 2$ is qualitatively unimportant for the results in this section, as there is no sovereign borrowing between $t = 1$ and $t = 2$. We thus abstract away from households savings at $t = 1$ for simplicity.

terms of utility) for the government to repay the sovereign loan. Realizing this, foreign creditors are willing to make larger sovereign loans.

Proposition 6 (Effect of savings on sovereign debt). *Given any level of savings s at $t = 0$, the equilibrium repayment level $P^*(s)$ in the least-cost separating equilibrium is increasing in s. As a consequence, equilibrium sovereign debt level $B^*(s)$ is increasing in s.*

Proof. The equilibrium repayment level $P^*(s)$ in the least-cost separating equilibrium makes the bad type indifferent between repaying and defaulting. Hence $P^*(s)$ solves:

$$u((1 + r_f)s + A_1 g(k_0^*) - 0) \qquad + \beta u(\underline{A} g(\underline{k}))$$
$$= u((1 + r_f)s + A_1 g(k_0^*) - P^*(s)) \quad + \beta u(\underline{A} g(\overline{k})). \tag{4.1}$$

Recall that $\overline{k}$ and $\underline{k}$ are derived from the fact that competitive foreign creditors earn zero expected profits from loans to the private sector. Since in our model, households' savings do not fund firms' investment, $\overline{k}$ and $\underline{k}$ are independent of s.

Let

$$c^D(s) \equiv (1 + r_f)s + A_1 g(k_0^*)$$
$$c^P(s) \equiv (1 + r_f)s + A_1 g(k_0^*) - P^*(s).$$

Applying implicit differentiation to equality (4.1) yields:

$$(1 + r_f)u'(c^D(s)) = (1 + r_f - \frac{d}{ds}P^*(s))u'(c^P(s)),$$

which in turns implies

$$\frac{d}{ds}P^*(s) = \frac{(1 + r_f)\left(u'(c^P(s)) - u'(c^D(s))\right)}{u'(c^P(s))}.$$

Since $c^D(s) < c^P(s)$, and u' is strictly positive and strictly decreasing (recall that $u' > 0$ and $u'' < 0$), it follows that $\frac{d}{ds}P^*(s) > 0$. Hence $P^*(s)$ is increasing in s.

Consequently, $B^*(s) = \frac{\pi}{1+r_f}P^*(s)$ is increasing in s. $\qquad \square$

4.2 Underinsurance

The previous proposition shows a positive impact of savings on sovereign borrowing. This positive impact is however an externality that households do not internalize, because each individual household takes what happens in the sovereign credit market as given. As a result, households' stock of savings is suboptimal. This subsection formalizes this intuition.

Definition 3. S^o *is* an *optimal level of savings* if it solves the social planner's savings problem:

$$\max_{S \geq 0} \quad u(c_0(S)) \quad +\beta\pi \left(u(c^P(S)) + \beta u(\overline{A}g(\overline{k})) \right)$$
$$+\beta(1 - \pi) \left(u(c^D(S)) + \beta u(\underline{A}g(\underline{k})) \right)$$

where $c_0(S) \equiv B^*(S) - S$.

The first order condition associated with S^o is:

$$u'(c_0(S^o)) \geq \beta \left(\pi u'(c^P(S^o)) + (1 - \pi)u'(c^D(S^o)) \right) \tag{4.2}$$
$$+ \left(\pi \frac{dP^*}{dS}\big|_{S^o} \right) \left(u'(c_0(S^o)) - \beta u'(c^P(S^o)) \right)$$

and equality holds if $S^o > 0$.

In comparison, the equilibrium level of (decentralized) households' savings is s^* that solves the following first order condition:

$$u'(c_0(s^*)) \geq \beta \left(\pi u'(c^P(s^*)) + (1 - \pi)u'(c^D(s^*)) \right) \tag{4.3}$$

and equality holds if $s^* > 0$.

Equation (4.3) reflects the usual "precautionary motive" to accumulate savings (see Kimball and Weil (2009); Jeanne and Rancière (2006); Alfaro and Kanczuk (2009)). Households simply save to self-insure against the bad state in $t = 1$. They do not internalize the positive externality of savings on sovereign debt (which is captured by the second term of equation (4.2)). Proposition 7 then follows naturally:

Proposition 7 (Underinsurance). *Households under-insure against the sovereign default state ($s^* < S^o$) if the government is borrowing constrained at the optimal savings level S^o.*

Proof. If the government is borrowing constrained at S^o in a separating equilibrium then

$$u'(c_0(S^o)) > \beta u'(c^P(S^o)).$$

Hence the right hand side of the first order condition for S^o is strictly larger than that of the first order condition for s^*. Consequently, $s^* < S^o$. $\qquad\square$

There are two interesting implications from the results in this section. First, proposition 7 gives a rationale for a benevolent government to either complement households' savings by building a stock of foreign reserves. Alternatively, the government can subsidize households' savings, for instance, by selling domestic government bonds at an interest rate higher than the world's safe rate, and committing to repay these domestic bonds. However, if the government cannot commit to either of these policies, then whether the country can achieve the optimal buffer stock of savings requires further research.

Second, there is a branch of international macroeconomics literature that allows countries to endogenously choose *both* the level of sovereign borrowing and the level of reserve accumulation. In a recent paper, Alfaro and Kanczuk (2009) uses a small open endowment economy model of complete information to quantify the optimal level of reserves. They find that the precautionary motive to save (as captured by first order condition (4.3)) is can only support insignificant levels of reserves, inconsistent with empirical evidence. In their model (and much of the literature), reserves and debt are substitutes, because instead of accumulating more savings the country could reduce their sovereign borrowing. Consequently, they miss out the complementarity as captured by the last term of first order condition 4.2. As mentioned in the beginning of this section, the complementarity depends on two assumptions: the borrowing government has private information, and the reserves of the country cannot be used to finance working capital loans.

5 Infinite horizon model and quantitative exercise

This section extends the previous three period model to the infinite horizon. Its main aim is to provide a quantitative exercise that would have otherwise been inappropriate with the three period horizon. The exercise shows that even if trade credit is small (6% of GDP), the incentive of a higher level of trade credit can sustain a significant level of sovereign debt (60% of GDP).

5.1 Environment

All the agents in the economy now live infinitely in periods $t = 0, 1, 2, \ldots$. The utility of a representative household over a sequence of consumption good $\{c_t\}_{t=0}^{\infty}$ is

$$\tilde{U}(\{c_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \cdot \tilde{u}(c_t).$$

Households receive an endowment of ω in each period.

Foreign creditors are risk neutral, competitive, have deep pockets, and they can trade a safe asset that gives a risk-free interest rate of $1 + r_f > 1$.

In each period t, domestic firms borrow k_t from foreign creditors, then invest this amount to produce $A_{t+1} f(k_t)$ in period $t + 1$. For $t \geq 0$, the productivity shock in each period $t + 1$ is a function $A_{t+1}(\theta_t)$ of private fundamental θ_t in period t. The fundamental can take two possible values: good and bad, denoted by $\overline{\theta}$ and $\underline{\theta}$. The productivity shock in the good state $\overline{A} \equiv A(\overline{\theta})$ is higher than that in the bad state $\underline{A} \equiv A(\underline{\theta})$.

The private fundamentals $\{\theta_t\}_{t \geq 0}$ follow a Markov process whose transitional probability matrix is

$$\begin{pmatrix} \pi(\overline{\theta}|\overline{\theta}) & \pi(\underline{\theta}|\overline{\theta}) \\ \pi(\overline{\theta}|\underline{\theta}) & \pi(\underline{\theta}|\underline{\theta}) \end{pmatrix}.$$

The initial fundamental is $\theta_0 = \overline{\theta}$, and is common knowledge. For $t > 0$, the government learns θ_t in period t, but everybody else learns θ_t only in $t + 1$.

In each period $t \geq 0$, each firm receives a loan contract from a foreign creditor, by pledging up to a fraction ϕ of its final output. The contract can be made contingent on realizations of output shock A_{t+1} (which becomes common knowledge in $t+1$). The government receives sovereign loans from foreign creditors, and transfers the revenues to households in lump-sums.

For simplicity, assume the country does save.

Timing of events

1. $t = 0$:

   (a) Foreign creditors lend B_0 to government, who then transfers to households in lump-sums.

   (b) Foreign creditors make private loans k_0 to firms at rate $1 + r_1$. Firms then purchase k_0 units of investment good as input for production.

   (c) Households consume
   $$c_0 = \omega + B_0.$$

2. In each period $t \geq 1$:

   (a) Private production yields $A_t(\theta_{t-1})f(k_{t-1})$.

   (b) Firms repay p_t, and transfer profits $A_t(\theta_t)f(k_{t-1}) - p_t$ to households.

   (c) Government privately learns the realization of fundamental θ_t.

   (d) Government repays P_t by taxing households in lump-sums.

   (e) Foreign creditors make private loans k_t to firms at rate $1 + r_{t+1}$. Firms then purchase k_t units of investment good as input for production.

   (f) Households consume
   $$c_t = \omega + A_t(\theta_{t-1})f(k_{t-1}) - p_t - P_t.$$

For convenience, denote $u(c) \equiv \tilde{u}(\omega + c)$ and $(1 - \phi)f(k) \equiv g(k)$.

Finally, the following list summarizes the essential assumptions of the model.

Assumption 1. Domestic production requires imported investment goods that firms must pay for in advance with funds raised from *trade credit* from foreign creditors.

Assumption 2. For all periods $t > 0$, the government learns the realization of θ_t at t, but other agents only learn the realization at $t + 1$.

Assumption 3. At most a fraction $\phi \in (0, 1]$ of the firm's output can be pledged as collateral for the private external loan.

Like the three period model, we make two further technical assumptions:

Technical Assumption 1. The collateral constraint ϕ is sufficiently tight so that firms are credit constrained:

$$E_\mu[A]g'(k^*(\mu)) > 1 + r_f, \ \forall\mu \in [0,1]$$

where $k^*(\mu)$ solves the binding collateral constraint condition given country rating μ

$$(1 + r_f)k^*(\mu) = \phi E_\mu[A]f(k^*(\mu)).$$

Technical Assumption 2. The "single-crossing" property holds:

$$\frac{\overline{A}}{\underline{A}} > \frac{u'(\underline{A}g(k))}{u'(\overline{A}g(k))}, \forall k \in [\underline{k}, \overline{k}]. \tag{5.1}$$

Equilibrium and refinement

We use the concept of stationary Markov perfect Bayesian equilibria. A stationary Markov strategy for foreign creditors is a time-independent function of the belief μ (which we will refer to as the "country rating"), government payment P, realization of fundamental θ_- from the previous period, realization of the current productivity shock A and private sector's investment k_- from the previous period. A stationary Markov strategy for the government is a time-independent function of the realization of previous period's fundamental θ_- and current fundamental θ (that the foreigners and firms do not know yet), current productivity shock A and private sector's investment k_- from the previous period.

A (Markov perfect Bayesian) equilibrium consists of a strategy profile and a system of beliefs such that, at every possible state (including those that only occur out of equilibrium), players' beliefs over the fundamental θ of the current period are consistent with Bayes' rule wherever possible. Given these beliefs, each player's strategy specifies actions that are best responses to other players' strategies.

5.2 Efficient separating equilibrium

The counter-part to the least-cost separating equilibrium from the three period model in this infinite horizon has a new interesting feature: the endogenous exclusion of a defaulting government from sovereign credit, and its endogenous re-entry after a set-

tlement payment of the defaulted debt has been made. By settling the old loans, the governments in default signal that their economy is again healthy. [12]

In equilibrium the government always repays zero in the bad fundamental state, and always makes a positive payment as a signal in the good fundamental state, and after a default, if the fundamental becomes good again, the government makes a settlement payment as a signal.

Bayesian updating is straightforward: if a sufficient payment was made in the current period, then the belief (or country rating) is good: $\mu = 1$. If the payment is insufficient then the belief is bad: $\mu = 0$. The belief remains at $\mu = 0$ until a signaling payment of a sufficient size is made, at which time the country rating gets "upgraded" to $\mu = 1$.

The equilibrium sovereign lending strategy is to lend a positive amount to a government with good rating $\mu = 1$, and to lend nothing to one with bad rating $\mu = 0$. Perfect competition among foreign creditors will pin down the amount of loan. Lending to the private sector is $\overline{k}$ if $\mu = 1$ and $\underline{k}$ if $\mu = 0$. Recall that $\overline{k}$ and $\underline{k}$ are found by solving bind collateral constraint conditions:

$$
\begin{aligned}
(1 + r_f)\underline{k} &= \phi \underline{A} f(\underline{k}) \\
(1 + r_f)\overline{k} &= \phi \overline{A} f(\overline{k}).
\end{aligned}
$$

Proposition 8. *There is a separating (stationary Markov perfect Bayesian) equilibrium in which*

1. *The government begins with good country rating ($\mu = 1$).*

2. *If the country currently has good rating ($\mu = 1$),*

    (a) *The government repays P^* in good fundamental state. Repayment level P^* makes the $\underline{\theta}$ type indifferent between repaying and defaulting.*

    (b) *The government defaults (repays nothing) in low fundamental state.*

    (c) *Foreign creditors associate belief $\mu = 0$ to a government that repays any amount strictly less than P^*, and $\mu = 1$ to a government that repays P^* or more.*

3. *If the country currently has bad rating ($\mu = 0$),*

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[12]This endogenous re-entry is not a new result. See Cole et al. (1995).

(a) *The government makes settlement repayment $P_{set}^* \in (0, P^*)$ in good states.*

(b) *The government does not make any late repayment, and remains in default.*

(c) *Country rating is upgraded back to $\mu = 1$ after any settlement payment is at least P_{set}^*, and stays at $\mu = 0$ if the settlement payment is smaller than P_{set}^*.*

Proof. Appendix. □

5.3 Numerical example

Here we compute the least-cost separating equilibrium above. We assume that the utility function has constant relative risk aversion: $u(c) = c^{1-\sigma}/(1-\sigma)$, and the production function is $f(k) = k^\alpha$. Table 1 shows the chosen parameter values, and table 2 shows the values of the equilibrium outcomes. The target statistics are taken from Mendoza and Yue (2011) and Jeanne and Rancière (2006).

Calibrated parameters		Value	Target statistics
Time period		one year	
Risk aversion index	σ	2	Standard RBC[13] value
Production function parameter	α	0.3	Standard capital share in GDP
Discount factor	β	0.9	Standard RBC value
Risk-free interest rate	r_f	0.5	Standard RBC value
Endowment	ω	1	Denominator for rescaling
Probability of good fundamental	$\overline{\pi}$	0.9	Probability of recession (10%)
Probability of good fundamental	$\underline{\pi}$	0.5	Average stay in default (2 years)
Good productivity shock	$\overline{A}$	2.939	Working capital to GDP (6%)
Bad productivity shock	$\underline{A}$	2.499	Output drop following default (13%)
Fraction of pledge-able output	ϕ	0.10	Jeanne and Rancière (2006)

Table 1: Parameter values

Sovereign repayment (face value of debt)/GDP	$P^*/\overline{y}$	0.6176
Settlement payment/original debt	P_{set}^*/P^*	0.6009
Risk premium on sovereign loan	$\frac{P^*}{B^*(1+r_f)}$	1.0091
Risk premium on new sovereign loan after settlement	$\frac{P_{set}^*}{B_{re-entry}^*(1+r_f)}$	1.0270

Table 2: Equilibrium outcomes

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6 Conclusion

This paper provides a tractable framework to analyze the role of private government information on the inter-relationship between sovereign debt, private debt, and savings. The signaling mechanism in the model is able to explain and predict several results. First, sovereign debt is sustainable without enforcement. Second, governments repay in good times and default in bad times. Third, foreign capital flow to the private sector dries up after the government defaults, and increases after the government makes sufficient settlement payments and regain good credit rating. Fourth, precautionary savings increases the government's ability to borrow from abroad. And fifth, domestic agents under-insure against sovereign defaults, and thus the paper gives a rationale for the government to actively manage foreign reserves. Over-all, the paper shows the importance of private government information in the three-way relationship between public external debt, private external debt and the country's stock of precautionary savings.

For tractability, the model has made several strong assumptions. First, we have assumed all taxes are non-distortionary. If taxes were distortionary, then repaying sovereign debt would become more costly. The restriction on the gap between firms' productivity in the good state vs. that in the bad state would have to be more stringent, in order to satisfy the single crossing property. But once the restriction is met, the equilibrium level of sovereign debt would actually be higher, as sovereign repayment is now a costlier signal.

Second, we have assumed households' savings does not finance domestic investment. If households' savings can decrease domestic firms' dependence on foreign credit, the savings can be a substitute for external debt. However, the paper wants to show that *precautionary* savings, not savings for investment, is complementary to sovereign borrowing. Thus it abstracts away from savings for investment. In fact, most of the quantitative literature on sovereign debt do not model savings all-together. As an exception, Alfaro and Kanczuk (2009) studies a model where a country simultaneously optimizes sovereign borrowing and precautionary savings. Their analysis shows that, without adding ad-hoc assumptions, the model predicts the optimal level of foreign reserves (interpreted as a form of precautionary savings) is close to zero, and the optimal

level of sovereign borrowing is much lower than what observed in data. Our model suggests that by allowing for asymmetric information, our signaling mechanism can significantly alter the results. The extent to which private government information can explain simultaneously high levels of foreign reserves and sovereign borrowing in emerging market economies remains a question for future research.

Third, we have abstracted away from domestic private debt. In fact, the results on the sustainability of sovereign debt and the co-movement between public and private debt still hold if there are a group of risk-neutral domestic creditors, who can provide firms with working capital loans, and who also do not learn the fundamental until production matures. The informational channel that explains why private external debt declines applies equally well to private domestic debt.

The model's numerical example should not be taken too literally. It only serves to illustrate: (a) how one could embed the signaling mechanism and the public-private debt symbiosis in the infinite horizon, and (b) these two factors can play quantitatively significant roles. It remains an avenue for future research to embed the signaling mechanism and the public-private debt relationship into an extensive stochastic dynamic equilibrium model. Existing quantitative models (for instance Alfaro and Kanczuk (2005, 2009); Aguiar and Gopinath (2006); Arellano (2008); Yue (2010); Mendoza and Yue (2011)) have yet been able to fully explain the observed high levels of sovereign borrowing.

As the current sovereign debt crisis continues to unravel across the world, there is an increasing need to understand the linkage between different borrowing relationships, as well as the implications of private government information. This paper has only scratched the surface of the what a model that combines asymmetric information and sovereign risk can potentially explain. Among many diverse avenues for future research, it would be interesting to explore the combination of sovereign debt of different maturities and costly signals of higher dimensions. For instance, an illiquid government can still distinguish itself from an insolvent one by signaling to the market their willingness to make sacrifices to repay by going through a painful austerity program. A two dimensional signal can consist of how much debt has been repaid and whether an austerity program has been implemented.

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