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An Evolutionary Approach in Financial Forecasts

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1. **Introduction**
2. **Rethinking Efficient Markets**

2.1 Rational Expectations and Pricing Models

The Standard Financial Theory apparently relies on arbitrage mechanisms to set prices right. According to Modigliani-Miller Theorem (MM, 1958), while searching for the least expensive way to invest, rational investors leave no room for the firm to manipulate its value. The crux of this assertion is that, if there is a cheaper way to earn the same cash income in future, rational investors should abandon the more expensive financial instruments used by the firm and go for the cheaper ones, clearing any pricing discrepancies among assets or securities. It turns out that, such arbitrage arguments form the crucial logic in financial economics. Namely, The Efficient Market Hypothesis (EMH) which says: a security's price reflects its "fundamental value".

To see this, let us emphasize that a corollary of MM Theorems is this: A firm can create value only by proposing a risky project whose returns are sufficiently high to be attractive to investors. But risks and returns can only be realized in future. Thus, one can imagine that both the firm and investors, who buy and sell the firm's shares, interact in a finite horizon game. At each stage of the game, news, including those about the firm's projects, arrives to the market. Such flows of news, running from a remote past, form the information set of the current stage, at which investors update their beliefs about the distribution of the firm's future returns, consistent with Bayes' rule. Investors then act upon their beliefs. That is, they abandon any security, whose expected returns cannot meet or beat expected losses, while buying it more if the reverse holds true. Two important facts immediately follow: First, rational actions by investors tend to drive the security's price toward its fundamental value that confirms investors' beliefs. Second, in equilibrium, no security can yield excess risk-adjusted average returns, or average returns greater than are warranted for its risk. As already said, the first statement is EMH. The second one is known as Capital Asset Pricing Model (CAPM)[2]. We then see all three building blocks of the Standard Financial Theory, MM, EMH, and CAPM in one coherent setting.

A key telnet in efficient market arguments is the concept of rational expectation equilibrium or REE (Sargent, 1993)[3]. It states that individuals' actions are rational given beliefs, and beliefs are

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[2] The model was introduced by Treynor (1961, 1962), Sharpe (1964), Lintner (1965) and Mossin (1966) independently, building on the earlier work of Markowitz (1952) and Tobin (1958).
[3] REE corresponds to Weak Perfect Baysien Equilibrium or weak PBE (Myerson, 1991) in dynamic games.

consistent given facts. That is, the distribution agents use to forecast future realizations of returns is indeed the distribution that those realizations are drawn from.

When matching the theory with the real world, this consistency condition proves to be too demanding. Agents may not act in a manner consistent with REE. For instance, if share prices are an accurate estimate of value, it is appealing to use share price changes as an indicator to determine management compensation. But simply tying management pay to share -price performance would encourage managers inappropriately shift resources away from investments that ensure the firm's long-term profits. Such an incentive problem leaves open the possibilities that some investors may make mistakes in choosing their strategies. Even though, they still act rationally, given their beliefs, and these beliefs are still justified given facts. But the facts they used to form their beliefs are driven by moral hazard[4].

Thus, a share's price could deviate from its fundamental value. It is required then there are some effective institutions that evolve to mitigate incentive problems and to allow investors to get better access to crucial information over time. Meanwhile, irrational actions are unavoidable.

Suppose irrational actions, either by moral hazard or by misperceptions, appear with some probability, randomly tipping a security's price away from its value. According to Friedman (1953), such a pricing error will create an opportunity sensed by rational investors who will snap up the opportunity, thereby immediately correcting the mispricing. In reality, this price discovery process may take a few minutes, days, or even months, causing volatility clustering of stock prices, a phenomenon that gives birth to General Autoregressive Conditionally Heteroscedastic or GARCH family (Engle, 2004). The idea here is price shocks may be large at first, but they will gradually fade out, as sophisticated investors sooner or later will take advantage of pricing errors, driving share prices to the level consistent with CAPM. The concept of REE now appears to be too weak to characterize how market prices evolve. One needs to relax the consistency condition of REE. It now requires only that, there exists a sequence of actions (corresponding to investors' choices of portfolio), that are rational given investors' beliefs, and their beliefs are justified by some facts in which, with some small probability, they make mistakes in choosing their investment strategies. However, when time unfolds, this sequence will find its way to the state of REE, driving share prices to the level consistent with EMH. This Sequential Equilibrium concept is introduced by Kreps and Wilson (1982)[5]. It provides a theoretical basis for most pricing models.

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[4] We cast doubt on the strong form of EMH, should bounded rationality or asymmetry of information be in presence.

[5] By definition, a strategy profile and system of beliefs (σ, μ) is a *sequential equilibrium* of a dynamic game, if it has the following properties: (i) Strategy profile σ is sequentially rational given belief system μ. (ii) There exists a sequence of completely mixed strategies: $\{\sigma^k\}_{k=1}^{\infty}$, , with $\lim_{k\to\infty} \sigma^k = \sigma$, such that $\lim_{k\to\infty} \mu^k = \mu$, where μ^k denotes the beliefs derived from strategy profile σ^k, using Bayes' rule (Mas-Colell, Whinston and Green, Ch. 9, 1995).

2.2 A Learning Game Form of Efficient Markets

So far, pro- efficient-market arguments simply stay mute on the risks that rational investors or arbitragers may incur, when they attempt to exploit pricing errors. In reality, those risks can be large, so that strategies designed to correct mispricing may be rendered unattractive, leaving the pricing error remained. But then, the concepts of Sequential Rationality or Sequential Equilibrium could come under criticisms, that they no longer fully fit real world situations. Once again, we need to refine the concept of equilibrium, where agents or investors are not fully rational. They obviously do not devoid of rationality. But with some probability, they may make mistakes. And such mistakes can have a substantial and long-lived impact on prices.

Toward this end, we first mention the two most noticeable risks that arbitragers may incur. The first one is Fundamental Risk. If it is systematic, arbitragers can take advantage of pricing errors. But it comes at the cost of reducing the diversification of their portfolio. The second one is Noise Trader Risk. If it is related to a self-fulfilling misperception, it can force arbitrageurs to liquidate their positions early, bringing them potentially steep losses[6].

In facing those risks, an individual arbitrager faces yet another type of risk, that Abreu and Brunnermeier (2002) label synchronization risk. The fact is that the elimination of the mispricing now would require the participation of a sufficiently large number of separate arbitrageurs. Without knowing what others will be going to do, the arbitrageur may hesitate to exploit the mispricing because s/he does not know for sure how many other arbitrageurs have heard about the opportunity. Therefore s/he does not know for sure the best response: to wait for prices to revert to correct values or to correct the mispricing immediately.

Conversely, De Long et al. (1990b) consider the case, whereby an arbitrager has come to believe that a sizable fraction of noise traders keep pushing an asset's price above fundamental value. Acting rationally on that belief, the arbitrager will not sell or short the asset. Rather, s/he *buy* it, knowing that the earlier price rise will attract more feedback traders next period by contagion, leading to still high prices, at which point the arbitrageur can exit at a profit.

The situations mentioned above are just some possible states of a coordination game, in which, investors' attitudes toward risks may affect the outcome of the game. We wonder then, whether EMH is still valid or whether pricing errors still remain unchallenged.

To answer this question, let us construct a coordination game, based on the stories discussed above. For the sake of concreteness, we suppose there are two classes of investors. The first one is the class of professional agents, who are assumed to be able to get access to best available information and use sophisticated tools to make reasonable forecasts. The second one is the public. The key assumption here is, with some probability, both sides can make mistakes. Assume further that, at the current period, an arbitrager is drawn at random from the class of sophisticated investors. S/he decides whether to take advantage of the mispricing that s/he has

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[6] Noise trader risk is introduced by De Long et al. (1990a) and studied further by Shleifer and Vishny (1997).

sensed, or conversely, to act selectively, meaning to wait until the share's price is about to revert to its value or to "follow the herd". Either way, the selective action exacerbates the mispricing.

Without loss, we assume the following payoffs:

The Mispricing Game

Individual Investor: **Professional Agent:**	*Act rationally*	*Act Heuristically*
Act consistently with CAPM	(R, R)	$(-\varphi, -\gamma)$
Exacerbate the mispricing	$(-\gamma, -\varphi)$	(r, r)

Here, payoffs include: expected returns consistent with CAPM (R); expected returns, when the mispricing remains, (r); expected losses, associated with individual actions in correcting the mispricing (φ); and expected losses of making mistakes (γ). In line with CAPM, rationality yields the Pareto efficiency: $R > r$.

The question being asked is whether the arbitrager will take advantage of the mispricing or exacerbate it. As already suggested, the answer lies in the risk the arbitrager faces. If φ is very large, guessing many other arbitragers will exploit this mispricing opportunity is very risky, because the arbitrager suffers large losses, if s/he is wrong. It sounds like an echo of Abreu-Brunnermeier's synchronization risk, but it comes from a different perspective. Suppose either noise trader risk or fundamental risk is looming large, making φ very large. In this case, *Exacerbate* is the risk dominant strategy for the arbitrager. S/he will take it, expecting that heuristically feedback traders will push the mispricing far and further from its value (De Long et al, 1990b). The mispricing remains as the Risk Dominant Equilibrium (Harsanyi-Selten, 1988). Conversely, if φ is relatively small, agents will learn to act in a manner consistent with CAPM. Thus, the share's price can never be too far out of line, so the state consistent with EMH will emerge and dominate the market.

The next question then is how far a stock's price may deviate from its value, if φ is large; and how close it may converge to its fundamental value, if φ is small?

To answer this question, we first take a notice that, when φ is large, the risk dominant strategy for individuals is not necessarily a risk dominant strategy for the whole society. If pricing errors remain unchecked, but exacerbated by feedback investors; inefficient capital allocation that continues long enough may shift the economy into macro imbalances. Mounting sovereign debt, unexpected inflation and crises can be the results, as seen in Latin America in 1970s. Those costly experiences gave rise to the formation of institutions that aim to coordinate expectations and enforce incentives of arbitragers in order to combat pricing errors and to reduce information friction. Surprisingly, in the presence of such expectation-stabilizing institutions, the low equilibrium of the mispricing game will never be reached. Otherwise, the economy will be prone to chronic crises.

Similar arguments can be applied for the case, when φ is small. The closer individuals reach the ideal state of full rationality, the harder it is for them to face cognitive limitations of their own minds. Hence, the more likely they can make errors or engage in heuristic actions (Herbert Simon). Some forms of institutions then must be designed to push individuals to the frontier of their innovative abilities, while preventing them from crossing the line imposed by bounded rationality. Silicon Valley may still stand as the unique example of such market governance institutions.

Proposition 1: *In the presence of mechanisms designed to enhance efficient markets, a mixed strategy close to one of the pure equilibria will emerge.*

Corollary 1: *Let λ be the probability of playing the action consistent with CAPM in the mixed strategy. The closer the probability λ to 1, the more accurate a stock's price reflects its value. And the closer the probability λ to 0, the greater the stock's price deviate from its value.*

We shall call λ the *effective rate* of efficient markets, while $(1-\lambda)$ the *hazardous rate* of making pricing errors that cannot be eliminated by any market-enhancing institution that has come to the being through evolution. The mixed strategy implies the society accept some irrational actions that professional agents may make, while playing the role model. By contagion, their degree of rationality reflects that of the society, even though imperfectly. If the effective rate is high, we say the economy has a strong foundation and its growth is sustained. If this rate is low or the hazardous rate is high, we say the economy has a weak foundation and is prone to crises.

Proof of Proposition 1: Without loss, let us assume, initially φ is large, such that, the low equilibrium is risk dominant. If mispricing remains unchecked, the economy is at risks for heading into crises. Thus, the best response by a third party, that is, by the agent is mandated to protect the society's wellbeing, is to suppress any mispricing from early stages; not to allow it to mutate into a big chaos. For example, if there are some warning signs of housing bubble, tightening monetary policy must be imposed and financial institutions doing business in the real estate sector must be carefully regulated toward safety. Without going into details, we suppose that, the best response by the third party aims at altering the structure of the game in a way that coordinates individuals' actions to combat the mispricing. In particular, it reduces the risk an individual faces when combating the mispricing, φ, and increases the risk s/he incurs if exacerbating it, γ. We shall say the third party's best response is effective, if it produces a finite structural change: $\varphi_k \to \varphi_*$; $\gamma_k \to \gamma^*$, and $\varphi_{k+1} < \varphi_k, \gamma_{k+1} > \gamma_k, k = 1,2,..,K$. Here, φ_0, γ_0 are the initial values of the payoffs, whereas φ_*, γ^* are the lowest value of φ and the highest value of γ, respectively, that the society can construct through coordination. Further, and this is key, at the end of the structural change, the risk factors also change in a way that the high equilibrium now becomes the risk dominant equilibrium.

If time is left to unfold, the fraction of investors, who act in a manner consistent with CAPM will increase and eventually reach the threshold, λ. However, if this fraction increases further and beyond the threshold, λ, the parameter φ starts to rise, whereas γ starts to fall again. It is because the situation has become too complex to be handled optimally through coordination. Most market actors reach to the point where they strictly face cognitive limitations of their own minds, when processing information and making decisions. Again, the economy faces the risk of a suddenly jump back to the low equilibrium. A third party's best response, as seen in Silicon Valley, is to rein in this tendency, detecting and eliminating any potential error that may mutate into a large threat. This action is a form of screening mechanisms: Third parties, the capital firms, intentionally force the rate of rationally promising projects below the threshold, λ. But when they act in that manner, the transition toward the high equilibrium is interrupted and the evolution has to start all over again to bring the process closer to λ. Given that, the individual's best response is to act in a manner consistent with CAPM; but with the hazardous rate, $(1-\lambda)$, s/he may still make a mistake due to bounded rationality or asymmetry of information[7].

Finally, when a third party, either in public or in private sectors is inefficient, it cannot bring φ down and γ up from any initial state of the economy, such that, the hazardous rate, $(1-\lambda)$, is too high for the economy to maintain a sustained growth path. Like many Latin American countries in 1970s, this economy exhibits what would appear to be *unwillingness* to create expectation-stabilizing institutions, with consequences of considerable uncertainty (Arrow, 1992) □

3. Forming Beliefs

3.1 The Adaptive Expectation Model

The Proposition1 gives us only a partial support for EMH. Although investors tend to behave consistent with CAPM, mispricing still remains (Milgrom, Ch. 12, 1992). And its size is proportional to the hazardous rate, $(1-\lambda)$, which in turn depends on the risk structure investors face when they try to coordinate their strategies. In the simplest form, the mixed strategy equilibrium of the mispricing game can be embedded in a "pricing model" as follows:

$$p_t = \lambda p_{t-1} + \beta(1-\lambda)m_t + (\varepsilon_t - \lambda\varepsilon_{t-1}) \tag{1}$$

Here, p_t is the change in price from one day to the next. On the right hand side, the first element, p_{t-1}, represents the knowledge gained from processing all available information which has been incorporated into yesterday price change. The second element, m_t, is the current market momentum, either a *bull* or *bear* market, that has not been analyzed. Yet, it does affect market's

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[7] Here we adopt the adaptive learning framework introduced by Young (1994). By construction, we have heterogeneity in payoffs, such that the absorbing state associated with the high equilibrium has the minimum stochastic potential, so long as the probability of individuals playing the strategy consistent with CAPM is still smaller than λ. Thus, the process tends to converge to this state, but it will be reverted, whenever the probability surpasses λ. For that reason, neither high equilibrium, nor the low one will be reached. Instead, the process frequently gets closer to the mixed Nash equilibrium, even though never lands on it.

expectation about how today price is going to vary. The third element, is MA(1) form of disturbances, where ε_t is assumed to be a white noise. Specification (1) implies that the full effect of any shock on a share's price will not be felt for sometime thereafter.

Koyck (1954) names equation (1) the Adaptive Expectation model. It consists of two elements: First, the regression equation: $p_t = \beta \hat{m}_t + \varepsilon_t$, it says investors should react, if rational forecasts indicate mispricing, denoted by $\hat{m}_t$. Namely, they should sell over-priced stocks, and buy under-priced ones. Second, as shown in the mispricing game, with probability $(1-\lambda)$, investors may make mistakes in forecasts or they just "follow the herd". Stated differently, with probability $(1-\lambda)$, the rational judgment on mispricing is revised by the current observation of market momentum, without investors having time to think about it: $\hat{m}_t = \lambda \hat{m}_{t-1} + (1-\lambda)m_t$. This is nothing but the mixed strategy equilibrium of the mispricing game. We shall delay any further specification of the model, until we gain a deeper understanding about the nature of market momentum, m_t. For now, we focus on some insights on price volatility that can be drawn from this model.

3.2 Volatility Puzzle:

In the Adaptive Expectation model, the mixed strategy equilibrium represents the weighted average of real investors' expectations. If the degree of rationality, λ, gets closer to 1, the model (1) converges to a Random Walk model or EMH[8]. Whereas, if λ gets closer to zero, the model (1) becomes $p_t = \beta m_t + \varepsilon_t$, which characterizes herding behavior. Since market momentum itself is a stochastic process, it means the weighted average will be more volatile than is justified by changes in fundamentals, (measured by the standard deviation of ε_t). Further, since the hazardous rate $(1-\lambda)$ is strictly greater than zero, one would expect volatility clustering. In some periods, markets are relatively steady - a characteristic of efficient markets. In other periods, bull or bear markets have prevailed. Nevertheless, in an economy with a strong foundation, random shocks should never be able to mutate into a large threat that can tip the economy into crisis. Yet when looking at statistics, something really puzzles us. Nothing seems to confirm our guesses.

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[8] When $\lambda = 1$, we then have : $p_t = \beta \hat{m}_t + \varepsilon_t$, where $\hat{m}_t = p_{t-1}$ by sequential rationality. Insert the latter into the former, and without loss, we can rewrite: $p_t = \varepsilon_t, \varepsilon_t \sim ARMA(p,q)$, which is a random walk model.

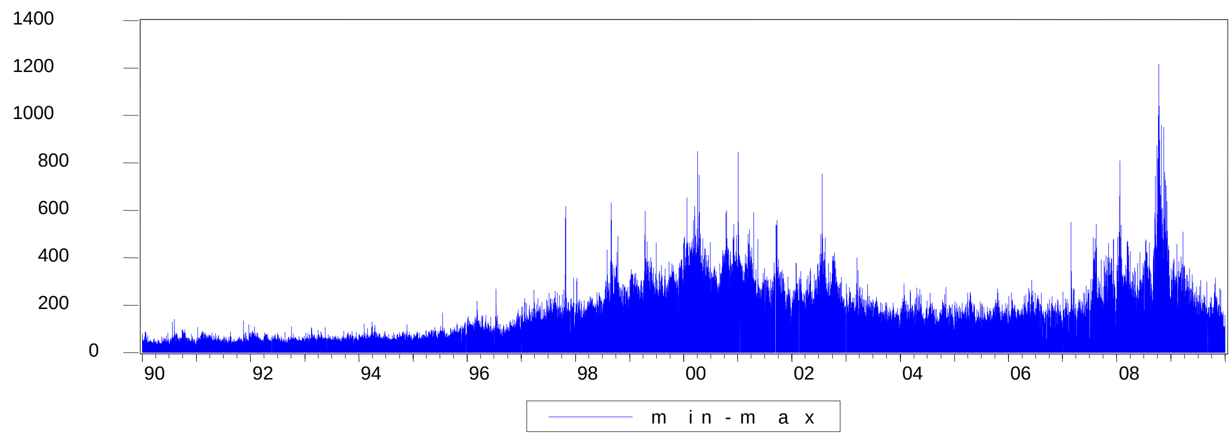

Figure 1: The lowest-highest price intervals of Dow Jones from January 1990 to January 2010.

We first take a notice that, Dow Jones prices seems to exhibit a *time-unvarying* variance from 1990 to 1995 during the golden age of the US economy with the boom of information technology and the Internet. This phenomenon suggests that, there might have existed a norm or spontaneous order (after Hayek) that kept volatility of market momentum, m_t, in check, without third-party enforcement[9]. However, from 1996 to 2009, the US economy seemed almost always to be beset by fads, booms, bubbles, and sudden crashes. It took roughly 13 years to re-inflate the economy, which was then struck down by the 2008-09 financial crises. This fact suggests that, the US economy must have had some errors or mutations, which had silently grown and changed the norm that kept m_t in check. One possibility is that some events had occurred that gradually raised the volatility of market momentum, m_t, associated with the irrational part in investors' minds. In a history perspective, this is the period that the US enjoyed (or suffered from) a massive and cheap lending, which induced the society to learn and adapt with new tastes and self-fulfilling beliefs. More precisely, the massive and cheap lending played a key role to induce people to learn and to adapt with positively autocorrelated price shocks, which investors perceived as the facts that fed the society's beliefs of becoming increasingly affluent.

Volatility *step-up* then became a fact: from 1995 to 2005 (the ten years between the time prior to dotcom bubble and its aftermath), price volatility increased by roughly 100%, and stayed relatively flat thereafter until the financial crisis broke out in 2008. (In order not to be in bubbles, realized returns from productivity growth must increase by the same amount, but obviously it was not the case). Comparing between dotcom and real estate bubbles, the time for the transition from fads to booms to bubbles and then the turn into a sudden crash was shortened for the real estate crisis, with the result that this crisis hit the US economy much harder, due to volatility

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[9] From equation (1), $Var(p_t \mid p_{t-1}, \varepsilon_{t-1}) = Var[\beta(1-\lambda)m_t + \varepsilon_t]$. Since, ε_t is a white noise, the confidence intervals of p_t, centered at $[\lambda p_{t-1} + \beta(1-\lambda)E(m_t) - \lambda\varepsilon_{t-1}]$, exhibit a *time unvarying* pattern, only if all elements of the covariant matrix of the random vector (m_t, ε_t) are constant. In this condition, the lowest-highest price intervals of Dow Jones can exhibit a time-unvarying pattern, from 1990 to 1995, as seen in *Figure 1*.

step- up. So why did volatility step-up keep evolving unnoticed? Was there anything to do with changes in the spontaneous order that had kept the economy in a stable state in the period 1990-95? And if that is the way the history has said, what is the reason behind the fact?

3.3 Social Norms to Maintain Stability

The Adaptive Expectation model (1) nets out the effect on a share's price of a fundamental change, ε_t, from that of a spontaneous urge to action, whether it is due to hedonism or fear, after observing market momentum, m_t. The latter is known as Herd Behavior (Keynes, 1936). This notion describes how individuals in a group can act together without planned direction. Thus, it may help to illuminate how a spontaneous order may emerge to maintain stability; and why or how this norm may change, such that, some random errors may mutate into a great threat, tipping the economy into crises.

To begin, let us take a notice that a spontaneous urge may emerge when a group of individuals face a danger. For instance, evolutionary biologist Hamilton (1971) asserted that when being threaten by a predator, each individual group member reduces the danger to itself by moving as close as possible to the center of the fleeing group. In financial markets, it means, when short-term momentum, m_t, sends a share's price to a too high level than expected, aversion to risk rises. Thus, instead of buying more shares, investors *sell* them to move as close as possible to the center - the weighted average of investors' expectations, $[\lambda p_{t-1} + \beta(1-\lambda)E(m_t) - \lambda\varepsilon_{t-1}]$. As the consequence, the share's price is less volatile than it could have been. Psychologically, in the presence of such a spontaneous order - a natural check on risk-taking behavior - price volatility, $[Var(p_t \mid p_{t-1}, \varepsilon_{t-1})]^{\frac{1}{2}} = [Var[\beta(1-\lambda)m_t + \varepsilon_t]]^{\frac{1}{2}}$, tends to be confined in certain limits.

An insight can be drawn from this argument: Once price uncertainty is moderate, information incorporated in prices should allow professionals to make a good judgment on risk and returns and act upon it. Subsequently, a share's price may hardly deviate far from its value. A by-product is incentive problems also become less severe. As stock prices reflect more accurate value, company managers will be encouraged to undertake right projects in hopes for a long term return, without fearing that their actions will depress current prices, making the company more vulnerable to takeover. Quasi-efficient markets then emerge, as incentive mechanisms (or made orders) are embedded in more complex spontaneous orders or norms to maintain stability and efficiency.

So fear or hedonism is not necessarily bad as such instincts could serve to form the natural check on risk-taking behavior, from which a more elaborated institutional-arrangement emerges to nurture moral or professional codes of conduct – the rational side of the story. So how things could go wrong, as seen in the recent US bubbles and crashes?

In biology, when a big shock hits the system, like a global climate change, it may make new species mutate more easily to a large number, such that, they can tip the balance of nature into a new norm, that may be better or worse than the former.

In the US economy, a shock came in the form of massive and cheap foreign capital, flowing into the United States. It was due to the boom of new technologies, the collapse of the Soviet bloc, wars and conflicts, or a join of those events. And like in biology, this shock could make mutations or errors easier to spread, impairing the coherence of institutional arrangements that had created the US a golden age between 1990and 1995. As cause-effect, the US economy could become more prone to fads, booms. Crises would be the result, as already suggested.

But how can mutations mutate? Economists have long suggested that when the rate of capital inflows substantially surpasses the rate of productivity growth, it causes positive covariance among stock prices, which tend to perform extraordinarily in early stages of bubbles. Two effects then follow: First, short-sighted management may be prevailed. Firms that focus on speculation for short-term gains from bubbles will expand, whereas the ones that focus on creating bases for a long-enduring future may be among those most subject to takeover attempts (adverse risk selection). Second, as stock prices run up substantially, financial institutions, such as investments banks, see this phenomenon as a real opportunity for making money. They will lobby for stripping away regulations, so that they can well engage in extremely risky activities. They do so on the twin presumptions that: (1) favorable outcomes, when stock prices keep going up, will lead to unusually large profits for the banks; whereas (2) unfavorable outcomes, when bubbles burst, the banks walk away from heavy losses, which must be picked up by the government, which is responsible for the safety of the financial system. This is what Stiglitz – Weiss (1981) call "incentive effect"; Kane (1985), and Mc Kinnon (1991) call this "moral hazard in the banks"; Roubini (2010) and Stiglitz (2010) call it "socialization of private losses"[10].

The more the capital flow in to fuel market booms, the more easily the problems of adverse selection and moral hazard spread. These market failures, otherwise should be noises, now become a norm, although the bad norm. This is the outcome of the game, where many individuals interact in circumstances they are "teased" to take more risk. As capital inflows create series of positive shocks, the natural check on risk-taking behavior becomes weaker step by step. Large positive price shocks then get incorporated into expectations, [as modeled in equation (1)]. The expectation that prices will continue to rise becomes justified, making it self-fulfilling. This gives financial institutions incentives to create more instruments, with more leverage, more risk taking than ever. We then see speciation in the financial world - the emergence of new species (or financial innovations) - as distinct from their ancestors and from one another in competing for making money out of bubbles. Thus, the system looks like in equilibrium, but in fact, it constantly adapts toward more risk-taking behavior. This process can not be in "equilibrium", unless on the other side of the equation, foreign capital keeps pouring in to pump bubbles. Debts build up, as cause-effect. When debts reach an unbearable level, capital inflows dry out, and bubbles burst. To prevent the system from collapse, the bank bailouts are needed, which confirms the banks' beliefs that they are too big to fail.

A question arises: If the government is committed to take bailout option in the event of crisis; then why isn't government regulation stringent enough to prevent crises from happening.

To answer, let us focus only on the investment banks that act as underwriters to mitigate the problem of adverse selection. This monitoring is necessary because it transmits to the public the information about the companies, who have engaged in shoddy or unscrupulous business practices. Such information allows investors to avoid excessive risk taking, thus making fads, booms, and bubbles harder to go on.

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[10] For a rigorous analysis of Moral Hazard in Banking, see for example, Hellmann, Murdock, and Stiglitz (2000).

However, even if most of the banks work diligently, still there are some banks may enjoy moral hazard. As the natural check on risk taking is loosened, investors are willing to take more risk, Ponzi schema may mutate among the banks themselves[11]. When reverberating, these mutations may diversify into a large number of complex instruments, making the normal scrutiny dysfunctional. Further, such diversification creates momentum to turn public opinions against government's regulation for the safety of the economy. Excessive risk-taking then spread in the favorable climate of the foreign capital inundation. Of course, in anticipating what will happen, once capital inflows increase at the macro level, some people may raise their voices. But they risk losing their jobs and become "extinct". We now summarize our discussion in a very costly proposition below[12]:

Proposition 2: *In the presence of macroeconomic instability-cum-deregulation that aggravates adverse selection and moral hazard, the economy will deviate from the state of quasi-efficient markets and converge to the state of crises with probability 1.*

To prove this proposition, it is convenient to introduce the game below:

The Moral-Hazard Game

Individual Investor: **Investment Bank:**	*Avoid excessive risk-taking*	*Engage in excessive risk-taking*
Manage Risk	(R, R)	$(-\varphi, -\gamma)$
Create Risk	$(-\gamma, -\varphi)$	(r, r)

Here, the investor buys instruments issued by the bank. These instruments are either stocks of the firms for which the bank is the underwriter (the bank manages risk); or they are mortgage-backed securities, and the like, that the bank has created (the bank creates risk). Needless to say, this is a specific case of the Mispricing Game, which has some critical changes in the payoff structure: First, thanks to positive covariance among stock prices, which tend to perform extraordinarily in bubbles, expected losses of correcting mispricing, φ, increase. Second, due to deregulation, expected losses of taking excessive risk, γ, decrease. (That is, we observe a step-wise change in payoffs, such that: $\varphi_k \to \varphi_*$; $\gamma_k \to \gamma^*$, and $\varphi_{k+1} > \varphi_k, \gamma_{k+1} < \gamma_k, k = 1,2,.., K$). Third, because of moral hazard (or socialization of private losses), the expected returns from taking excessive risk are greater than the ones consistent with CAPM: $r > R$. Finally, since the change in payoffs is made step-wise, at the limit, we shall have a *unique* payoff structure, such that the low equilibrium has an evolutionary advantage: $(R - \varphi_*) < (r - \gamma_*)$.

Proof of Proposition 2: The game has two norms: First, the efficient one, denoted by E_1, which corresponds to the state of quasi- efficient markets; and second, the bad norm, E_2, corresponds

[11] This is what has been confessed by Mr. Madoff, who was charged with setting Ponzi schema that consumed about \$20 billion in lost cash and almost \$65 billion in paper wealth (*The New York Time*, Feb 15, 2011).
[12] Mc Kinnon (Ch.7, 1992) introduces the core idea of this proposition.

to excessive risk taking and debt buildup. Due to mutation, there are probabilities that the system can be transformed from one norm to another, and vice versa. These transition probabilities form a Perturbed Markov process, P^{δ}, which is *irreducible* (Young, Ch.3, 1998). [Here, δ is probability of making "innovation" or mutation by players]. The question is, why under the climate of large capital inflows *plus* deregulation, mutation may cause the system to transform from E_1 to E_2, but unlikely to go in the opposite direction.

To see this, let $v_{12}^{t}, \mu_{12}^{t,\delta}$ be probability of reaching E_2 from E_1 in exact *t* periods, and the probability of visiting E_2, starting from E_1 during *t* - periods, respectively. We have $v_{12}^{t} = P_{12}^{t,\delta} \approx \delta^{r_{12}}$ (multiplied by a constant), where, $P^{t,\delta}$ is the *t*-fold product of P^{δ}; and r_{12} is an integer defined by an *adaptive learning* rule, such that there is a one-to-one correspondence between r_{12} and the risk factor of E_2 (Young, Ch.4, 1998). By the law of large number, when *t* is large, $v_{12}^{t} \approx \mu_{12}^{t,\delta}$. Further, $\mu_{12}^{t,\delta} \approx \mu^{\delta}(E_2)$, which is a stationary distribution of the perturbed Markov process, P^{δ}, measured at E_2. It implies, when δ becomes small, $\mu^{\delta}(E_2)$ approaches to $\mu^{0}(E_2)$, at a rate that is approximately equal to $\delta^{r_{12}}$. Thus, the resistance of moving from E_1 to E_2 is r_{12}. By the same chain of reasoning, the resistance of moving from E_2 to E_1 is r_{21}, which corresponds to the risk factor of E_1. The key point in Proposition 2 is that, under capital inflows-cum- lax regulation, moral hazard in the banks is easier to spread. In a Markov process, that means moving from E_1 to E_2 is less resistant than going in the reverse direction, due to $r_{12} < r_{21}$. Or equivalently, when $\delta \to 0$, then $\mu^{\delta} \to \mu^{0}$, which is a stationary distribution of the process P^{0}, such that, $\mu^{0}(E_2) > 0$, and $\mu^{0}(E_1) = 0$ □

During a relatively short period in the modern US history, the system have transformed from the state of quasi-efficient markets, E_1, to an unsustainable state, E_2, causing volatility step-up, excessive risk-taking, and debt buildup. As a result, the 2008-2009 became unavoidable, with probability 1.

Corollary 2: *When the natural check on risk-taking behavior is loosened, price, p_t, exhibits volatility-clustering. However, the longer the time period, the higher the frequency of extremely large volatility will be. This excessive risk-taking norm will remain, until bubbles burst.*

Proof of Corollary 2: Let $p_t = u_t$, where u_t is a linear combination of m_t and ε_t, according to the specification of equation (1). From *Figure 2*, we shall assume, $E(u_t) = 0, Var(u_t) = \sigma_t^2$. Since ε_t is a white noise, volatility clustering of p_t is attributed to shifts in norms of risk taking that causes variance of m_t to change over time.

More specifically, volatility of p_t mirrors that of m_t (plus a constant, which is the standard deviation of ε_t). Once the natural check on risk-taking behavior is relaxed (exactly, after 1995);

risk taking behavior tends to oscillate between periods in which the first norm, E_1, and then the other norm, E_2, is dominant. Those shifts in risk-taking norms cause variance of m_t to change, such that, p_t exhibits volatility clustering[13]. However, if E_2 has an evolutionary advantage; then in the long run, extremely large volatility will occur with a high frequency, until bubbles burst□

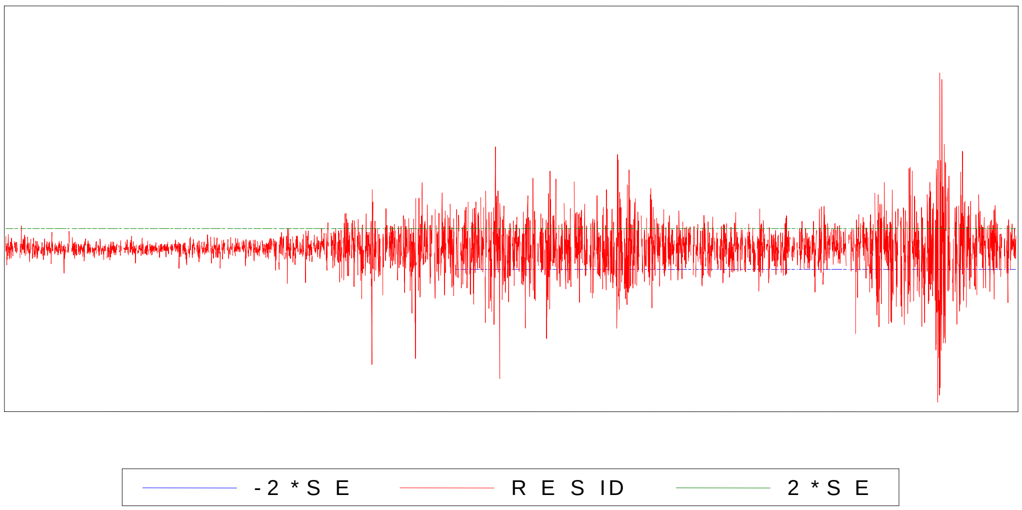


Figure 2: Volatility of p_t, from Jan. 1990 to Jan. 2010; the estimation uses data from 1928 to 2010.

For clarity's sake, we shall divide the modern history of Dow Jones into three periods. The first period, from 1990 to 1995, when volatility of m_t (plus that of ε_t) is confined within the two-standard deviation interval. The second period is from 1996 to 2004. This period, in turn, is divided into two subsets, with the first one from 1996 to 2000, when dotcom bubbles was gradually pumped up until it finally burst in 2000. The second one is from 2001 to 2004, when volatility of p_t gradually becomes less extreme. A more prudent regulation had some role to play here to change the course of evolution in the norms of risk taking[14]. The third period is from 2005 to 2010, when excessive risk taking became dominant. As time unfolded, large volatility appeared with a high frequency, until the financial crisis happened. History then provides

[13] As already said, volatility clustering is the key phenomenon that gives birth to GARCH family. The simplest version is characterized by Auto Regressive, Conditionally Heteroscedasticity: $\varepsilon_t = u_t[\alpha_0 + \alpha_1 \varepsilon_{t-1}^2]^{1/2}$, where u_t is standard normal.

[14] In the wake of terrorist attacks in the United States in 2001, companies who had engaged in shoddy or questionable bookkeeping were essentially caught in a series of government investigations. The loss of investor faith in the tech industry after this event then helps to put an end on otherwise an enduring dotcom bubbles.

evidence for the logic underlying the model. That is, shifts in norms of risk-taking drive price volatility.

Corollary 3: *In short terms, temporary shifts in risk taking norms follow some specific rule.*

Proof of Corollary 3: Without loss, suppose the current state is at the prudent norm, E_1. Suppose further that, the resistance from E_1 to E_2 is greater than one and exactly one mutation occurs. The state of the game is still in the basin of attraction of E_1. Thus, after this one-time-shock, the process will eventually revert to E_1. However, stochastic perturbations may not be isolated events. They can occur in succession (or with a high frequency) *and* within one class of players, such that, the number of mutations is greater than r_{12}. Given a specific rule of adaptive learning, these mutations may be sizable enough, such that the process is pushed out of the basin of attraction of E_1 and reaches a transient state. From that, the system can transit to E_2 with probability one. The same logic can be applied for a transition from E_2 to E_1, [even though, in the long run, it is harder for the system to follow this path than the other way around, if the resistance $r_{21} > r_{12}$]□

We then observe two opposite forces that work in any short period of time: First, *local pull* that keeps the system within the basin of attraction of one norm. When one isolated perturbation hits the system, the process will drift back to the original state. Second, *local push*, when one type of mutants occurs with a high frequency in a very short period, such mutations can be sizable enough to overcome gravity or local pull. Thus, they can tip the system away from the current norm and out of its basin of attraction. As a result, the process can transit to the other norm.

Once again, we can see, when positively autocorrelated shocks appear frequently, adaptive learning process will surely lead people to accept more risk or larger price volatility in a step-up schema. An important implication is, once we have identified a specific pattern of unexpected shocks or momentums, we can predict a near future price move accordingly.

4. Price Forecasts

4.1 Specification : Autoregressive Distributed Lag Model

To pave the way to a forecast model, let us zoom in on *Figure 2* to observe the reflection of price volatility, caused by autocorrelated shocks in a short period of time.

In Figure 3a below, a one-time-shock happened on 3/11/1996. The system then immediately reverted to the current norm, such that, the price change p_t was kept within the two-sigma interval. As time passed, the price change p_t tended to converge to zero.

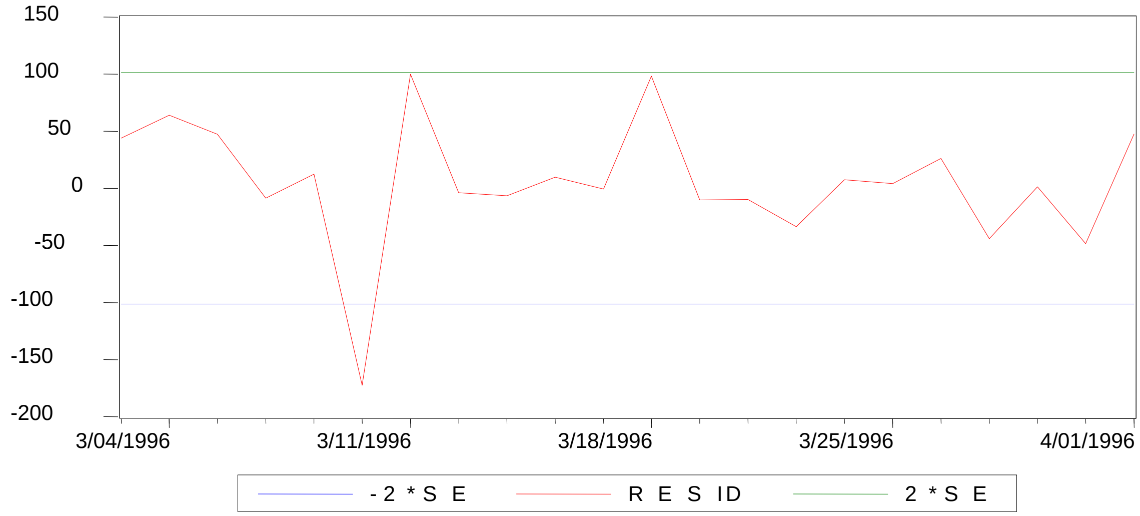


Figure 3a: Volatility of p_t, in March 1996. The estimation uses data from 1928 to 2010.

In Figure 3b, instead, positively autocorrelated shocks appeared frequently in a very short period, just a few days or a week, that raised investors' appetite for taking excessive risk. As a result, p_t became more volatile and frequently broke the two-sigma interval.

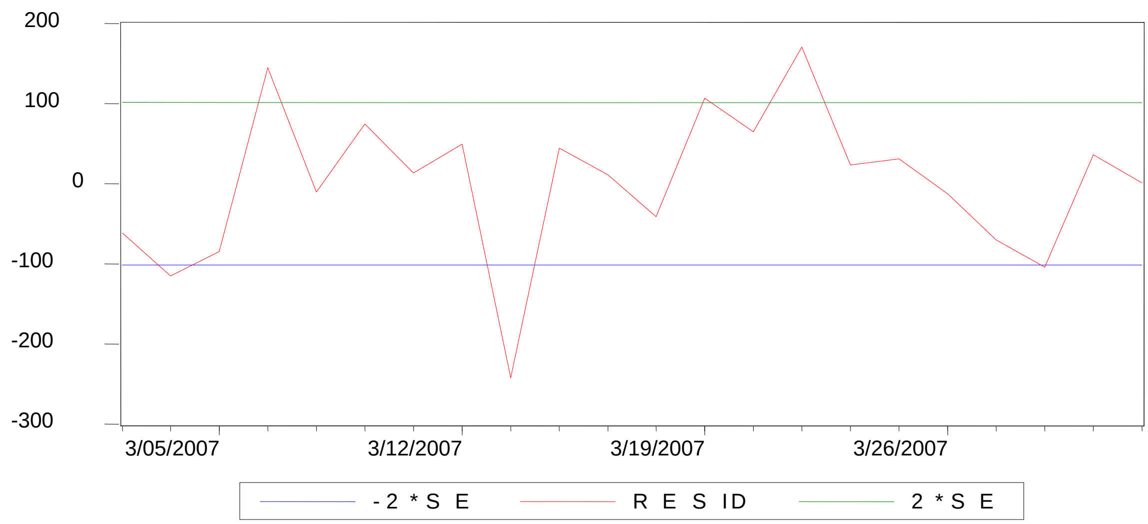


Figure 3b: Volatility of p_t in March 2007. The estimation uses data from 1928 to 2010.

Nevertheless, some common rule seems to apply for both cases. We first take a notice that, in any moment *t*, once an unexpected shock hits the system, which is materialized in the direction and the magnitude of the residual; it tends to cause p_{t+1} to drift back in the opposite direction and by the same magnitude. This phenomenon is clearest, once a single shock hits the system hard enough to push p_t to go beyond the two-sigma interval. This drift-back effect is nothing but local pull or gravity. In the same time, accumulation of shocks from the most recent past works in a way that garthers momentum to push p_{t+1}. This accumulation of forces creates local push, once positively autocorrelated shocks occur with a high frequency in just a few days. Thus, the sign and the magnitude of p_{t+1} is influenced by two opposite forces. The current shock has a negative effect on p_{t+1}, whereas accumulation of shocks in *a few days* before has a positive effect on p_{t+1}.

Note that, residuals of estimate of p_t reflect time-varying variance of m_t. The rule of price motion discussed above suggests a general specification: $A(L)p_t = B(L)m_t + D(L)\varepsilon_t$. Or equivalently:

$$p_t = \alpha_1 p_{t-1} + \alpha_2 p_{t-2} + ...\alpha_p p_{t-p} + \beta_0 m_t + \beta_1 m_{t-1} + .. + \beta_K m_{t-K} + \varepsilon_t - \theta_1\varepsilon_{t-1} - ... - \theta_q \varepsilon_{t-q} \qquad (2)$$

This is a *finite* distributed lag model or the Autoregressive Distributed Lag (ARDL) model, which is a generic form of the adaptive expectation model mentioned earlier.

Given the rule of price motion, we shall restrict that $\beta_0 > 0, \beta_1 < 0, \beta_k > 0, k = 2,..,K$, and $|\beta_0| = |\beta_1|$. To simplify notations, we omit parameters associated with λ. Further, we still assume that, the disturbance ε_t in (2) is taken to be a serially uncorrelated, constant variance random variable. The fact is that, if the original disturbances were subject to autoregression, equation (2) would become the result of partial defferencing, which has removed that part of the disturbance process (Green, Ch.20, 2008). Provided ε_t is a white noise, Nonlinear Least Squares estimator will be consistent and asymtotically normally distributed[15].

The price forecast is then:

$$\hat{p}_{t+1} = \hat{\alpha}_1 p_t + \hat{\alpha}_2 p_{t-1} + ...\hat{\alpha}_p p_{t-p+1} + \hat{\beta}_0 \hat{m}_{t+1} + \hat{\beta}_1 m_t + .. + \hat{\beta}_K m_{t-K+1} - \hat{\theta}_1 \hat{\varepsilon}_t - ... - \hat{\theta}_q \hat{\varepsilon}_{t-q+1} \qquad (3)$$

---

[15] The sum to be minimized is $S(\alpha, \beta, \theta) = S(\Phi) = \sum_t \varepsilon_t^2$. The interation is $\hat{\Phi}_{s+1} = \hat{\Phi}_s - [G_s' G_s]^{-1} G_s' \hat{\varepsilon}_s$

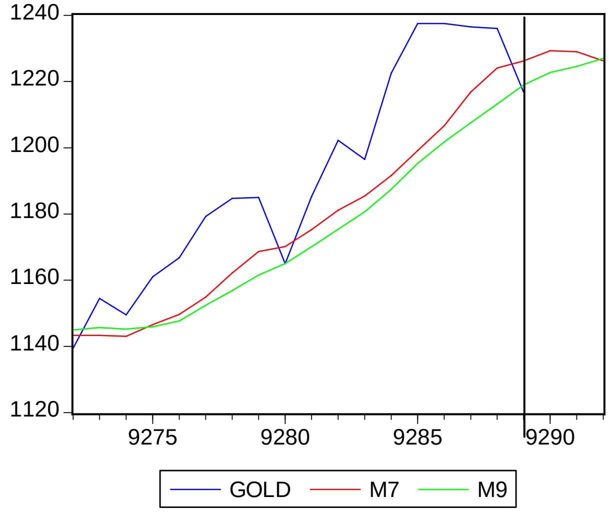
Here, G_s is a $(T - p - q) \times (p + K + 1 + q)$ matrix of derivatives of residuals with respect to the parameters: $\alpha = (\alpha_1,...,\alpha_p)', \beta = (\beta_0,..,\beta_K)', \theta = (\theta_1,...,\theta_q)'$; and $\hat{\varepsilon}_s$ is vector of residuals at step s.

An example of regression equation (2) and its forecast (3) are given in Appendix 1. Given the expected square forecast errors, $MSE(\hat{m}_{t+1} \mid t)$ and $MSE(\hat{p}_{t+1} \mid t)$, and given the foecast of price direction, one can calculate an approximation of tomorrow's price quotation. Notice further that, Figure 1 *does* suggest a practical way to make a fair judgment on how large is the price volatility on a daily basis. (In many cases, such a pragmatic method is not worser than what one can get from a tedious computation of volatility, using standard deviation). Given those facts, one should be able to predict the min – max interval for the next trading day.

4.2 Using Patterns in Forecasts

Like any specific form of equation (2), the distributed lag model given in Appendix 1 (model A1 for short) makes use of most recent unexpected shocks, materialized in lagged variables Lm_t, to estimate the change in price, p_t. These variables are very noisy. Therefore, the forecast $\hat{m}_{t+1}$ is sufferred from an attenuation effect and it may have a wrong sign. Since $\hat{m}_{t+1}$ enters into forecat equation (3), the prediction $\hat{p}_{t+1}$ is suffered from the same problems as well. When it has a wrong sign, the approximation mentioned earlier can not fix the attenuation effect. Instead, it exaggerates the forecast error. Therefore, simply using the forecast of model A1 is often misleading.

To overcome this problem of precision, let us take a closer look at two series of rolling averages, M7 and M9, whose difference forms one term in the Lm_t part in model A1. They seem to incorporate most recent unexpected shocks rather smoothly (see figure A1). Therefore, their *3* – step - ahead forecast may characterize a pattern that allows us to study how unexpected shocks affect expectation of price move. Thus, when such a pattern repeats itself, it will allow us to learn which specific form of equation (2) is suitable for the forecast of p_t. To see how this procedure may work, let us look at Figure A1 again:



For clarity, let us divide Figure A1 into two parts: before and after date *t*, May 18, 2010 (or the sample 9289). It is the time when markets have observed p_t , while guessing p_{t+1}. By definition, most recent unexpected shocks have been incorporated in series M7 and M9. Thus, their forecasts reflect how local push affects the market's expectation of price move. In this particular case, the local push is powerful enough to bend down both M7 and M9. By backward induction, at time *(t+1)*, the local push tends to force the price to go down.

Next, before date *t*, markets expected that there would have a slowdown in the temporarily upward trend of prices. And exactly at date *t*, the actual price has confirmed this market's expectation. By forward induction, rational expectation should nullify the force of local pull. Now, combining both forward and backward inductions, one can judge that local push outweighs local pull and the price should go down at date *(t+1)*.

Obviously, market prices are so noisy and we often cannot have a clear idea whether local pull will outweigh local push or the reverse holds true. But one rule seems to emerge from this simple example. When positively autocorrelated shocks appear with a high frequency in a short period of time, the three-step-ahead forecasts of M7 and M9 will embody all available information about how the local push affects the market's expectation of price move. The magnitude of local pull, instead, has been defined by the actual price move at date *t*. Thus, by properly adding series [M7 - M9] into equation (2), the problem of precision should be mitigated. An example is given in Appendix A2.

Conjecture 1: *For any pattern formed by the three-step-ahead forecasts of M7 and M9, we only need a finite number of specific forms of equations (2) to capture the information about how the balance between local pull and push has been made by precedents.*

Conjecture 2: *There is a one-to-one correspondence between each pattern and a specific form of equation (2) that allows us to overcome the problem of precision*.

To argue why these conjectures should hold, we shall take several notices. First, no matter how markets may be, the two opposite forces, local pull and push, always work simultaneously to govern market prices. Thus, there are only a finite number of situations that reflect the balance between two forces. Second, no matter what situation may emerge, the best estimate of the local pull and push can only be made by the market's forecasts of series M7 and M9. As a matter of facts, a shorter rolling average, such as M3, is too noisy, while a longer one, like M40, is too irrelevant. They will not work for our purpose of estimate of changes in the balance between the two opposite forces. Third, since the improvement of precision requires series M7 and M9, there are only a finite number of specific forms of equation (2) that one can try. And finally, through learning by doing, one should be able to discover the link between each pattern with one specific form of equation (2) that gives us the best estimate of price move.

5. Conclusions

Appendix 1: A Distributed Lag Model for Changes in Kitco Gold Price:

Table A1: Estimate of a Distributed lag Model for changes in Kitco's gold price, using Eviews.

Dependent Variable: D(GOLD)
Method: Least Squares
Date: 05/19/10 Time: 22:42
Sample: 100 9289
Included observations: 9190
Convergence achieved after 16 iterations
Backcast: 91 99

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(GOLD(-1))	-1.000969	0.001892	-528.9798	0.0000
D(GOLD(-2))	-1.184721	0.001576	-751.6584	0.0000

D(GOLD(-3))	0.176734	0.001733	102.0075	0.0000
D(GOLD(-4))	0.170227	0.002354	72.31546	0.0000
M3-M9	4.387544	0.003377	1299.131	0.0000
M3(-1)-M9(-1)	-4.211463	0.004687	-898.5474	0.0000
M7(-1)-M9(-1)	2.026977	0.007125	284.4953	0.0000
M7(-2)-M9(-2)	0.700128	0.003117	224.5949	0.0000
M7(-1)-T40(-1)	-0.373119	0.007742	-48.19487	0.0000
M7(-2)-T40(-2)	0.350322	0.007407	47.29808	0.0000
MA(1)	-1.087693	0.010590	-102.7060	0.0000
MA(2)	1.036716	0.013761	75.33477	0.0000
MA(3)	0.241308	0.016735	14.41926	0.0000
MA(4)	-0.533850	0.017062	-31.28879	0.0000
MA(5)	0.647388	0.016527	39.17231	0.0000
MA(6)	-0.038465	0.017011	-2.261239	0.0238
MA(7)	-0.252714	0.016639	-15.18786	0.0000
MA(8)	0.434830	0.013614	31.94031	0.0000
MA(9)	-0.356030	0.010216	-34.85171	0.0000

R-squared	0.994074	Mean dependent var	0.119587
Adjusted R-squared	0.994063	S.D. dependent var	6.735960
S.E. of regression	0.519037	Akaike info criterion	1.528380
Sum squared resid	2470.658	Schwarz criterion	1.543113
Log likelihood	-7003.908	Durbin-Watson stat	2.258056

Inverted MA Roots	.72	.71+.65i	.71-.65i	.38-.92i
	.38+.92i	-.10-.84i	-.10+.84i	-.82+.28i
	-.82-.28i			

Here, GOLD is Kitco's PM quotation; local pull (or gravity) is reflected by negative weights at lag of one, [M3(-1)-M9(-1)] and [M7(-1)-M40(-1)]. Local push is reflected by positive weights at lags [M7(-1)-M9(-1)], [M7(-2)-M9(-2)], [M7(-2)-M40(-2)], where $M\kappa = \frac{1}{\kappa}\sum_{\tau=t-\kappa+1}^{t} GOLD(\tau)$. So m_t is nothing but the gap between two rolling averages that allows us to measure the magnitude of short term momentum at time *t*.

Table A1: Actual Kitco's gold prices on May 19, 2010

Table A2: Kitco's quotations on gold prices, May 17 - 19, 2010

Date	obs	PM (GOLD)	Min - Max
May 17, 2010	9288	1236	1220 - 1242
May 18, 2010	9289	1216.75	1208 - 1229
May 19, 2010	9290	1195	1289 - 1220

Figure A1: PM quotation (GOLD, until May 18, 2010) and T+3 short term momentum forecasts

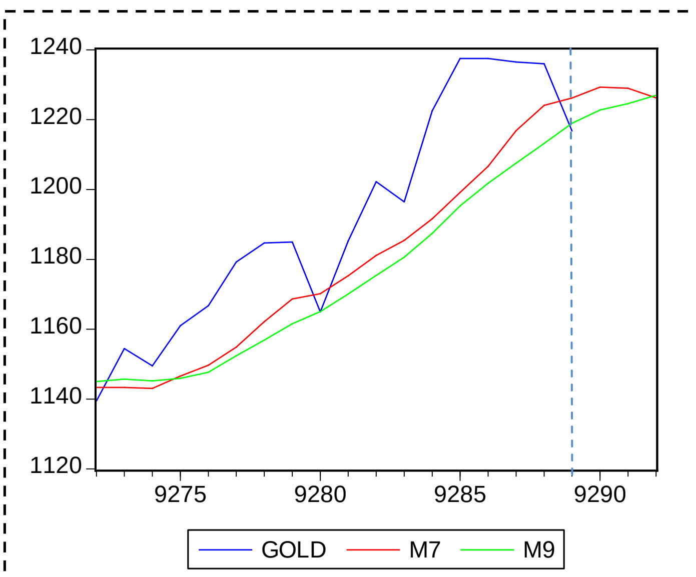
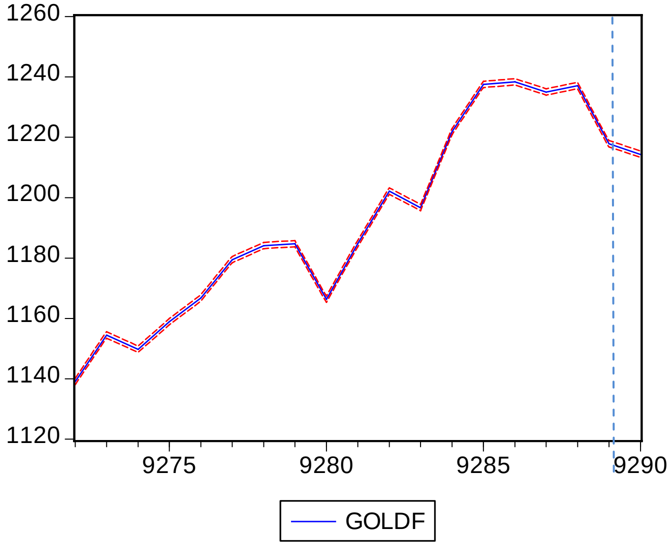


Figure A2: Price forecast for May 19, 2010 (observation 9290)



Forecast: GOLDF
Actual: GOLD
Forecast sample: 9272 9292
Adjusted sample: 9272 9290
Included observations: 18

Root Mean Squared Error	0.843696
Mean Absolute Error	0.619631
Mean Abs. Percent Error	0.051796
Theil Inequality Coefficient	0.000354
Bias Proportion	0.002037
Variance Proportion	0.023952
Covariance Proportion	0.974010